Compound Utility and Asset Pricing

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Abstract: Compound utility theory (CUT) offers an alternative to prospect theory, modelling nonlinear preferences without probability transformation. Applying CUT to portfolio choice and asset pricing, this paper investigate implications of nonlinear preferences for the structure of stochastic discount factor under various assumptions. Testable hypotheses are derived that seem to enrich current understanding in asset pricing relations significantly. Unlike psychologically motivated approaches that often seems to suggest investor irrationality, we show that most of the empirical "anomalies" can be rationally accommodated by the assumption that investors' have quasiconcave preferences in probabilities. In addition, this study leads to a convenient framework for empirical investigations of the structure and behavior of stochastic discount factors under upper- and lower-market conditions separately.

Key words: compound utility, reference point, consumption, investment, asset pricing, disutility aversion, upper-market beta, lower-market beta.

JEL Classification numbers: D46, D81,G10, G11, G12.

1 Introduction

Experimental studies have established strong evidence that people tend to have nonlinear preferences in probabilities while making decisions under risk.[1] If those evidences are valid, then one would expect nonlinear preferences to show up also in real-world data such as asset prices. Alternatively, since nonlinear preference violates any theory that assumes preference to be linear in probabilities, asset pricing models derived from those theories (including expected utility) are likely to encounter empirical difficulties. The equity premium puzzle (Mehra and Prescott, 1985; Kocherlakota, 1996; Campbell, 1999) is a notable example.

Among decision models that assume nonlinear preferences,[2] prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman,1992) is commonly viewed as empirically most successful. Apart from modelling preference through expected utility for gains and

[1]For instance, the Allais (1953, 1979) paradox, the Ellsberg (1961) paradox, and other types of behavioral patterns as documented in Kahneman and Tversky (1979), Tversky and Kahneman (1992), Lattimore et al. (1992), Gonzalez and Wu (1999), Tversky and Fox (1995), Abdellaoui (2000), Bleichrodt and Pinto (2000), van de Kuilen et al. (2006), and Zou (2006a).

[2]E.g., rank-dependent theories (e.g., Quiggin, 1982; Yaari, 1987), subjective probability theories (e.g., Gilboa, 1987; Schmeidler, 1989), mixture symmetry and quadratic utility theory (Chew et al., 1991), ranked-weighted theories (Luce and Fishburn, 1991; Marley and Luce, 2001), cumulative prospect theory (Tversky and Kahneman, 1992; Wakker and Tversky, 1993; Chateauneuf and Wakker, 1999), and multiple-priors theories (Jaffray, 1989; Gilboa and Schmeidler, 1989). For more recent contributions, see, e.g., reviews by Starmer (2000), Sugden (2000), Schmidt (2000), and Bell and Fishburn (2000). See also Harless and Camerer (1994) and Hey and Orme (1994) on comparing theories of choice under risk and uncertainty.

losses (rather than utility for wealth), prospect theory further assumes that people use their own *decision weights*, instead of true probabilities, to compute expected utility. The decision weights are derived by way of a probability weighting function that captures nonlinear preferences in probabilities.

Applications of nonlinear representation theories to the study of portfolio choice and asset pricing are still quite limited, however. Behavioral models that are influenced by prospect theory typically focus on the preference for gains and losses, or the way a reference point is determined that determines choice behavior. For instance, Benartzi and Thaler (1995) offer a solution to the equity premium puzzle by arguing that people tend to be myopically averse to losses. They are irrational in the sense that if they can buy and hold for longer period of time – a pure commitment problem – then they are willing to take on more risk. Shefrin and Statman (2000) assume that people have different aspiration levels. They attempt to find an optimal trade off between expected wealth and probability of losses. In Shefrin and Statman's model, investors with high aspiration levels could prefer casino-type of securities even where the expected wealth were negative. Barberis et al. (2001) set a loss-aversion model in a dynamic context. Through a detailed calibration exercise, they explore to what extent past performance could influence the degree of loss aversion and explain some of the asset pricing anomalies. Barberis and Huang (2001) also derive cross-sectional implications of their dynamic loss-aversion model, arguing that the representative investor is likely to be prone to "individual-stock mental accounting" rather than "portfolio accounting."

Except Benartzi and Thaler (1995) none of the above applications of prospect theory assumes probability transformation, however. Admittedly, the probability weighting function is somehow difficult to apply – especially in a portfolio context where covariances between assets' returns need to be fully understood. Should one use the true (empirical) distributions or to use the transformed distributions that could be different from person to person? Compound utility theory (CUT) recently developed by Zou (2006a) offers an alternative to prospect theory, which models nonlinear preferences without probability transformation. CUT is shown to accommodate a large class of empirical anomalies for EUT, and it has an intuitive axiomatic foundation.

The aim of the present paper is to apply CUT to portfolio choice and asset pricing, and investigate implications of nonlinear preferences for the structure of stochastic discount factor under various assumptions. We derive testable hypotheses that seem to enrich current understanding in asset pricing relations significantly. Unlike psychologically motivated approaches that often seems to suggest investor irrationality, we show that most of the empirical "anomalies" can be rationally accommodated by the assumption that investors' have quasiconcave preferences in probabilities. In addition, our study offers a convenient framework for empirical investigations of the structure and behavior of stochastic discount factors under upper- and lower-market conditions separately. There is an emerging literature showing asymmetric stock correlations with the market under upper-market and lower-market conditions (e.g., Ang and Chen, 2001; Ang, Bekaert and Liu, 2002; Campbell, Koedijk and Kofman, 2002). The models derived in this paper do allow this difference in correlation conditional on the market being up or down.

The next section describes the model and defines the basic concepts. It starts from a single-period consumption-investment problem and characterizes the optimal decisions and the SDF. Section 3 studies the cross-sectional relations between assets' expected returns, where a general-beta CAPM is derived. Section 4 extends the analysis to an intertemporal consumption-investment problem with an infinitely living representative agent. The general SDF is shown to be a weighted average of two functions, one of the return on the market portfolio and one of the intertemporal consumption ratio. Section 5 concludes the paper with a number of suggestions for empirical studies in the future. The proofs of the propositions are relegated to the Appendix.

2 The Model

Let $t = 0, 1, 2, ...$ denote the dates and Ω_t the investment opportunity set – the set of all feasible investment strategies – at time t. Each element $p \in \Omega_t$ is called an asset or portfolio, broadly understood as any finite combination of available assets, including options, futures, other financial products, long or short positions, etc. We assume that Ω_t is closed under portfolio formation in that $p = \sum_{i=1}^{n} \theta_i A_i \in \Omega_t$ whenever $A_i \in \Omega_t$, where θ_i is the weight of the portfolio in asset A_i satisfying $\sum_{i=1}^{n} \theta_i = 1$.

For all $p_t \in \Omega_t$, it is assumed that their gross returns r_{t+1} (1 plus the rates of return) at time $t+1$ are continuously distributed almost everywhere even though their realized values are discrete. Further assume that the capital market is perfectly competitive, there is no transaction costs and tax, and that investors can borrow and lend at a risk-free (gross) interest rate $r_{0,t+1}$. The excess return is denoted by x_{t+1} $(= r_{t+1} - r_{0,t+1})$.

2.1 Investor preference

Consider a representative agent (henceforth, investor) whose choice behavior somehow reflects the aggregation of information, beliefs, and preferences of investors as a whole. Let $\overline{w}_t$ denote the investor's wealth at time t. The investor's decision involves choosing a consumption level $c_{t+1} = \overline{w}_t - w_t$ for consumptions in period $[t, t+1]$ and investing the remaining wealth w_t in an optimal portfolio $p_t \in \Omega_t$ which generates random wealth w_{t+1} $(= w_t r_{p,t+1})$ at time $t+1$. From now on, unless needed for clarity, the subscripts t and $t+1$ will be dropped to ease notation.

Let $\omega = wr_0$ be the "reference level" where $w > 0$ is the invested capital. Let wr_p denote the random wealth level where r_p is the gross return on a portfolio p. Define "utility for gains" by a function $U : \mathbb{R} \to \mathbb{R}^+$ satisfying $U(wx_p) = 0$ for all $x_p \leq 0$ and the derivative $U'(wx_p) > 0$ for all $x_p > 0$. Define "disutility for losses" by a function $D : \mathbb{R} \to \mathbb{R}^+$ satisfying $D(-wx_p) = 0$ for all $x_p \geq 0$ and $D'(-wx_p) > 0$ for all $x_p < 0$. Let $u_p = EU(wx_p)$ and $d_p = ED(-wx_p)$ denote the expected values of U and D, and call u_p and d_p the "utility-reward" and "disutility-risk" of portfolio p, respectively.

In this context, we say that compound utility theory (CUT) holds if the investor's preference over risky returns on investments $p \in \Omega$ can be represented by a compound utility (CU) function $V : (\omega, u_p, d_p) \in \mathbb{R}^3_+ \to V(\omega, u_p, d_p) \in \mathbb{R}$. This function is generally nonlinear in its variables, increasing in utility-reward u, and decreasing in disutility-risk d (Zou, 2006a).

Incorporating the consumption choice, the investor's problem is now given by

$$\max_{0\le w\le \overline{w},\ p\in\Omega} V_0(c)+V(\omega,u_p,d_p) \tag{1}$$

$$\begin{aligned} \text{where } \omega &= wr_0,\ c=\overline{w}-w \\ u_p &= E\left[U(w(r_p-r_0))\right] \\ d_p &= E\left[D(w(r_0-r_p))\right] \end{aligned}$$

Assume that all functions V_0, V, U and D are twice continuously differentiable (except for U and D at point 0). They are assumed to satisfy

Assumption 1 *(i) $U, U' > 0, U'' \le 0$ for $r_p > r_0$; $D, D' > 0, D'' \ge 0$ for $r_0 > r_p$; and $U'(0^+) = D'(0^+) = 1$. (ii) $V_0' > 0$ and $V_0'' < 0$. (iii) $\partial V/\partial\omega > 0$, $\partial V/\partial u > 0, \partial V/\partial d < 0$, and V is (quasi) concave on $\mathbb{R}_+^3$.*

We have defined the functions of U and D solely on the gains and losses in order to bring our model closer to prospect theory. The magnitude of these potential gain or loss, however, can be influenced by the investment capital that is the investor's choice variable. It is shown in Zou (2006a) that U and D are invariant up to a positive ratio scale in ranking the "more rewarding" and "more risky" relations. Hence without loss of generality they are normalized here by $U'(0^+) = D'(0^+) = 1$. Interpretations of the rest of the assumption are straightforward.

Conditional on ω, the set of reward-risk combinations of the feasible strategies is defined by

$$\Phi(\omega) = \{(u,d)\in\mathbb{R}_+^2 : u = EU(w(r-r_0)),\ d = ED(w(r_0-r))],\ \text{for some } p\in\Omega\}.$$

Since Ω and $\Phi(\omega)$ are essentially the same – except that their elements have different "labels" – we may call $\Phi(\omega)$ the investment opportunity set as well. An advantage that $\Phi(\omega)$ gives us is the familiar graphical interpretations of *indifference curves* and *efficient frontier* of the opportunity set (see Figure **??**).

Finally, define the investor's degree of *disutility-risk aversion with respect to utility reward* (henceforth, *disutility aversion*) by

$$\rho(\omega, u, d) = -(\frac{\partial V(\omega, u, d)}{\partial d})/(\frac{\partial V(\omega, u, d)}{\partial u}). \tag{2}$$

This measure tells how much marginal increase in utility-reward is required for each marginal increase in disutility-risk if the investor is to be indifferent. Graphically, $\rho(\omega, u, d)$ is the slope of the indifference curve at point (u, d). Note that the class of lower partial moment models is nested as a special case of CUT by defining $U = (w - \omega)^+$, $D = (\omega - w)^+ + \lambda[(\omega - w)^+]^n$, and $V = \omega + EU - ED$ for arbitrary order $n \geq 0$. These models, however, imply a *linear* functional form of V in u $(= EU)$ and d $(= ED)$ in that the degree of disutility aversion is constant and equal to one $(\rho \equiv 1)$. We shall see that empirical data seem to suggest a quasiconcave functional forms of V.

2.2 Optimal consumption-investment decision

Assuming that there exists an optimal solution $w \in (0, \overline{w})$ and $m \in \Omega$ to the problem (**??**) satisfying $E(r_m) > r_0$, we obtain the following proposition.

Proposition 1 *Under Assumptions 1, the solution $w_m \in (0, \overline{w})$ and $m \in \Omega$ to (1) is characterized by*

$$r_0 \frac{\partial V(\omega, u, d)}{\partial \omega}|_{(u,d)=(u_m d_m)} = V_0'(\overline{w} - w) \tag{3}$$

and for all $i \in \Omega$

$$E[U'(w(r_m - r_0))(r_i - r_0)] \quad = \quad \rho_m E[D'(w(r_0 - r_m))(r_0 - r_i)] \tag{4}$$

$$\textit{where } \rho_m \quad = \quad -\frac{\frac{\partial V(\omega,u,d)}{\partial d}}{\frac{\partial V(\omega,u,d)}{\partial u}}|_{(u,d)=(u_m d_m)} \tag{5}$$

If we assume that the market is in equilibrium so that long and short positions in all derivative securities (including cash) are equal, then we may interpret r_m as the return on a market portfolio – i.e., the value-weighted portfolio of all risky assets. More generally, we can just treat r_m as the return on a benchmark portfolio or simply a benchmark.

2.3 The stochastic discount factor

It is well-known (e.g., Harrison and Kreps, 1979) that provided Ω_t does not permit arbitrage, the prices of all assets $p_t \in \Omega_t$ at time t can be expressed as the expectation of their time $t+1$ prices, p_{t+1} (including dividends),[3] through a common stochastic discount factor (SDF) $\delta_{t+1} > 0$:

$$p_t = E_t[\delta_{t+1} p_{t+1}] \tag{6}$$

where E_t is the expectation operator conditional on the information available at time t.

Proposition 1 implies a quite general SDF that is consistent with no-arbitrage. We first specify the structure of this SDF in the next proposition, and then derive a general expected return – beta relation. Let G_t denote the "upper-market event $x_{m,t} > 0$" and L_t the lower-market event $x_{m,t} \leq 0$. To ease notation define

$$\overline{x} = \begin{cases} x \text{ if } G \\ 0 \text{ if } L \end{cases} ; \quad \underline{x} = \begin{cases} -x \text{ if } L \\ 0 \text{ if } G \end{cases} . \tag{7}$$

[3]Without ambiguity, we let p_t denote either an asset or the price of the asset.

We call $\overline{x}$ the asset's *upper-market "gain"* and $\underline{x}$ the asset's *lower-market "loss"* (neither need be positive). Note that $E(\overline{x}) - E(\underline{x}) = E(x)$ is the asset's expected excess return or risk premium. The subscript t reappears in the next proposition for clarity.

Proposition 2 (dichotomous SDF) *Conditions (3) and (3) of Proposition* **??** *imply a dichotomous SDF δ_{t+1} that prices all assets $p_t \in \Omega_t$ according to (6). It is given by*

$$\delta_{t+1} = \begin{cases} B_t U'(w_t \overline{x}_{m,t+1}) \text{ if } G_t \\ \\ B_t \rho_t D'(w_t \underline{x}_{m,t+1}) \text{ if } L_t \end{cases} \tag{8}$$

where

$$B_t = \frac{1}{r_{0,t+1} E_t[U'(w_t \overline{x}_{m,t+1}) + \rho_t D'(w_t \underline{x}_{m,t+1})]} \tag{9}$$

$$\rho_t = \frac{E[U'(w_t \overline{x}_{m,t+1}) \overline{x}_{m,t+1}]}{E[D'(w_t \underline{x}_{m,t+1}) \underline{x}_{m,t+1}]} \tag{10}$$

and w_t satisfies condition (3).

We now move on to the pricing model in terms of expected returns.

3 Cross-section of expected returns

Since for all $p \in \Omega$, $E(\delta x) = E(\delta \overline{x}) - E(\delta \underline{x}) = 0$, there exists a constant number φ_i for every asset i such that

$$\varphi_i = \frac{E(\delta \overline{x}_i)}{E(\delta \overline{x}_m)} = \frac{E(\delta \underline{x}_i)}{E(\delta \underline{x}_m)} \tag{11}$$

Equivalently,

$$E(\delta \overline{x}_i) = \varphi_i E(\delta \overline{x}_m) \text{ and } E(\delta \underline{x}_i) = \varphi_i E(\delta \underline{x}_m) \tag{12}$$

The sign and magnitude of φ indicate how the asset's excess return tends to behave (i.e., in what direction and by what degree) in upper- and lower-markets.

By the Mean-value Theorem,

$$U'(w\overline{x}_m) = U'(0^+) + U''(\kappa_1)w\overline{x}_m \quad 0 \leq \kappa_1 \leq w\overline{x}_m \tag{13}$$

$$D'(w\underline{x}_m) = D'(0^+) + D''(\kappa_2)w\underline{x}_m \quad 0 \leq \kappa_2 \leq w\underline{x}_m. \tag{14}$$

Define $\lambda^+ = -U''(\kappa_1)w$, and $\lambda^- = D''(\kappa_2)w$. In general, since κ_1 and κ_2 are random variables the defined λ^+ and λ^- are random variables as well. Recall that $U'(0^+) = D'(0^+) = 1$, it follows without loss of generality that the SDF can be written as

$$\delta_{t+1} = \begin{cases} B_t(1 - \lambda^+_{t+1}\overline{x}_{m,t+1}) \text{ if } G_{t+1} \\ B_t\rho_t(1 + \lambda^-_{t+1}\underline{x}_{m,t+1}) \text{ if } L_{t+1} \end{cases} \tag{15}$$

where

$$B_t = \frac{1}{r_{0,t+1}[\Pr(G_{t+1}) + \rho_t \Pr(L_{t+1}) - E_t(\lambda^+_{t+1}\overline{x}_{m,t+1} - \rho_t\lambda^-_{t+1}\underline{x}_{m,t+1})]} \tag{16}$$

$$\rho_t = \frac{E[(1 - \lambda^+_{t+1}\overline{x}_{m,t+1})\overline{x}_{m,t+1}]}{E[(1 + \lambda^-_{t+1}\underline{x}_{m,t+1})\underline{x}_{m,t+1}]} \tag{17}$$

Substituting (15) into the equations in (12) yields

$$E(\overline{x}) - E(\lambda^+\overline{x}_m\overline{x}) = \varphi[E(\overline{x}_m) - E(\lambda^+\overline{x}_m^2)] \tag{18}$$

$$E(\underline{x}) + E(\lambda^-\underline{x}_m\underline{x}) = \varphi[E(\underline{x}_m) + E(\lambda^-\underline{x}_m^2)]. \tag{19}$$

A generalization of the traditional CAPM is in order.

Proposition 3 (general beta CAPM) *Assume that* $\Pr(\lambda^+ + \lambda^- = 0) < 1$*. Then the dichotomous SDF of Proposition 2 implies for all* $p \in \Omega$*,*

$$E(x) = \beta E(x_m) + (\rho - 1)[E(\underline{x}) - \beta E(\underline{x}_m)] \tag{20}$$

where

$$\rho = \frac{E(\overline{x}_m) - E(\lambda^+ \overline{x}_m^2)}{E(\underline{x}_m) + E(\lambda^- \underline{x}_m^2)} \tag{21}$$

$$\beta = \frac{E(\lambda^+ \overline{x}_m \overline{x}) + \rho E(\lambda^- \underline{x}_m \underline{x})}{E(\lambda^+ \overline{x}_m^2) + \rho E(\lambda^- \underline{x}_m^2)} \tag{22}$$

Since the lambdas in (21) and (22) can be random as well as the excess returns, the above general beta CAPM is indeed very general. Practically, one could assume that the lambdas are constant because the second moments of excess returns tend to be very small (e.g., typically less than one percent for monthly returns). They can then be estimated using empirical data. Note that (20) has two other equivalent expressions, each having its own meaning.

$$E(\overline{x}) - \beta E(\overline{x}_m) = \rho_m [E(\underline{x}) - \beta E(\underline{x}_m)]$$

$$E(\overline{x}) - \rho_m E(\underline{x}) = \beta [E(\overline{x}_m) - \rho_m E(\underline{x}_m)]$$

We can now easily derive a number of approximate pricing models as special cases by specific assumptions about λ^+, λ^-, and ρ. In all the following cases we assume that the lambdas are constant real numbers.

3.0.1 The mean-variance model

Assume $\lambda^+ = \lambda^- = \lambda > 0$ and $\rho \equiv 1$. Then the expected return − beta relation in (20) and the general beta in (22) reduce to the best-beta CAPM (BCAPM) (Zou, 2006b):

$$E(x) = \beta E(x_m) \text{ where } \beta = \frac{E(x_m x)}{E(x_m^2)} \tag{23}$$

The BCAPM is equivalent to the mean-variance CAPM when the model holds exactly, but has less pricing errors if the CAPM is mis-specified. In this case the market risk premium is determined by $E(x_m) = \lambda E(x_m^2)$, as implied by $\lambda^+ = \lambda^- = \lambda$ and $\rho \equiv 1$ in (21).

3.0.2 The lower-semivariance model

Assume now that $\lambda^+ = 0$, $\lambda^- > 0$, and $\rho \equiv 1$. This model has been studied, e.g., by Bawa and Lindenberg (1977), and Harlow and Rao (1989) among others, where (20) and (22) simplify to

$$E(x) = \beta E(x_m) \text{ where } \beta = \frac{E(\underline{x}_m \underline{x})}{E(\underline{x}_m^2)} \tag{24}$$

The market risk premium is now determined by $E(x_m) = \lambda^- E(\underline{x}_m^2)$, as implied by $\lambda^+ = 0$ and $\rho \equiv 1$ in (21). It is slightly misleading to call (24) a semivariance model because the betas are defined by the lower second moments.

3.0.3 The upper-semivariance model

Naturally, one may be interested in the symmetrical case where $\lambda^+ > 0$ and $\lambda^- = 0$. Again assume that $\rho \equiv 1$. Then (20) and (22) reduce to

$$E(x) = \beta E(x_m) \text{ where } \beta = \frac{E(\overline{x}_m \overline{x})}{E(\overline{x}_m^2)} \tag{25}$$

The market risk premium is now determined by the upper-second moments: $E(x_m) = \lambda^+ E(\overline{x}_m^2)$ from (21).

3.0.4 The gain-loss model

If we assume that $\lambda^+ = \lambda^- = 0$, then Proposition 3 cannot be used directly because its assumption does not hold. However, from (18) and (19) we find that

$$E(\overline{x}) = \varphi E(\overline{x}_m) \tag{26}$$

$$E(\underline{x}) = \varphi E(\underline{x}_m). \tag{27}$$

This is the case where investors evaluate reward and risk by the expected gains and losses (with respect to r_0) respectively. All assets' expected excess returns conditional on the market being up and down are now linearly related to the market through the assets' φ. In this case, $E(x_m) > 0$ if and only if $\rho_m > 1$, i.e., the risk premium of the capital market is explained solely by the investor's disutility aversion. The upper-market and lower-market SDFs also become strikingly simple:

$$\delta_{t+1} = \begin{cases} \frac{1}{r_{0,t+1}[\Pr(G_{t+1})+\rho_t \Pr(L_{t+1})]} & \text{if } G_{t+1} \\ \frac{\rho_t}{r_{0,t+1}[\Pr(G_{t+1})+\rho_t \Pr(L_{t+1})]} & \text{if } L_{t+1} \end{cases} . \tag{28}$$

$$\rho_t = \frac{E(\overline{x}_{m,t+1})}{E(\underline{x}_{m,t+1})} = Z_{m,t} \quad \text{(gain-loss ratio of } m\text{)} \tag{29}$$

The pricing model (26)-(27) with the SDF given by (28)-(29)is a good alternative for the MV models when applied to *high-frequency* trading environments. Studies of portfolio decisions based on expected losses can be traced back to Domar and Musgrave (1944). The asset pricing model with this simple form appears first in Bawa and Lindenberg (1977), although they do not derive the SDF of the model as we do here. An important advantage of the gain-loss model over the other models above is its consistency with no-arbitrage and with more general probability distributions. It can thus be applied to areas such as performance evaluation or portfolio choice where options are actively traded.

It is worth remarking that we have derived the gain-loss asset pricing model under the more general assumption that investors have quasiconcave preferences over expected gain and loss. Traditional expected utility approach has to assume that utility is a piece-wise linear function of the form

$$U(w|\omega) = \begin{cases} w - \omega \text{ if } w > \omega \\ \lambda(w - \omega) \text{ if } w \leq \omega \end{cases}$$

where λ measures degree of loss (or risk) aversion. A serious limitation of this assumption is its difficulty to obtain equilibrium under different degrees of loss aversion. To see this, let $u = E\max(w - \omega, 0)$ and $d = E\max(\omega - w, 0)$ so that $V = u - \lambda d$. It follows that $\rho = -(\partial V/\partial d)/(\partial V/\partial u) = \lambda$ is a constant. Since equilibrium requires $\lambda = Z_m$ as in (29), the equation cannot hold for different lambdas. The only possibility, then, is where all investors are identical as assumed in Barberis et al. (2001). This difficulty does not arise if investors have the more general quasiconcave preferences over u and d (e.g., Zou, 2000).

3.0.5 The dichotomous asset pricing model

Instead of restricting the investor's preferences, one could also restrict the joint distributions of asset returns. For instance, if assets' returns satisfy Ross's (1978) two-fund separability then all expected utility maximizers will choose the same optimal risky portfolio m. Zou (2005) shows further that such optimal portfolio will also be gain-loss efficient in that it has the highest gain-loss ratio Z_m. Therefore, Ross's two-fund separability condition will allow separation not only for expected utility preferences but also for (at least a class of) compound utility preferences, such as quasiconcave preferences over utility-reward and disutility-risk measured by expected gain and loss respectively.

Assuming two-fund separation, then the dichotomous asset pricing model (DAPM) holds which predicts (Zou, 2005)

$$\begin{aligned} E(\overline{x}) &= \beta E(\overline{x}_m),\ E(\underline{x}) = \beta E(\underline{x}_m),\ E(x) = \beta E(x_m) && (30) \\ \text{where } \beta &= \frac{E(\overline{x}_m \overline{x})}{E(\overline{x}_m^2)} = \frac{E(\underline{x}_m \underline{x})}{E(\underline{x}_m^2)} = \frac{E(x_m x)}{E(x_m^2)} = \varphi && (31) \end{aligned}$$

To the author's best knowledge, (30) and (31) are the strongest implications of two-fund separation documented to date.

Deriving more refined approximate models by higher-order Taylor's expansions of U and D is straightforward. But since the model presented above is already much richer than existing models, it may be a good resting point for now.

4 Intertemporal Consumption, Investment, and Asset Prices

In this section we apply CUT to an infinitely lived representative agent model. The model is similar in spirit to Barberis et al. (2001) in that the agent is assumed to derive utility not only from consumption but also from utility-reward and disutility-risk for future investment outcomes. The agent's objective is assumed to maximize

$$E\left(\sum_{t=0}^{\infty} \phi^t \left(V_0(c_t) + b_t V(\omega_t, u_t, d_t)\right)\right), \quad 0 < \phi^t < 1 \qquad (32)$$

where $b_t > 0$ is a scaling factor, indicating the weight of compound utility V in the overall utility for consumption and investment pairs. For tractability, we assume that V_0 and V have the power functional forms given by

$$V_0(c_t) = \begin{cases} \frac{c_t^{1-\gamma}}{1-\gamma} & \gamma \neq 1 \\ \ln(c_t) & \gamma = 1 \end{cases} \tag{33}$$

$$V(\omega_t, u_t, d_t) = \begin{cases} \frac{[(w_t - c_t) r_{0,t+1}((1+u_t)^a - (1+d_t))]^{1-\gamma}}{1-\gamma} & \gamma \neq 1 \\ \ln\left[(w_t - c_t)\left((1+u_t)^a - (1+d_t)\right)\right] & \gamma = 1 \end{cases}, \quad 0 < a \leq 1 \tag{34}$$

$$u_t = E_t U(x_{t+1}) \equiv E_t U(\max(r_{t+1} - r_{0,t+1}, 0)) \tag{35}$$

$$d_t = E_t D(-x_{t+1}) \equiv E_t D(\max(r_{0,t+1} - r_{t+1}, 0)) \tag{36}$$

where r_{t+1} is the gross portfolio return on $p \in \Omega_t$, the agent's choice at time t. Now $w_t - c_t$ is the invested capital at time t (so that $\omega_t = (w_t - c_t)r_{0,t+1}$) and w_t denotes the time-t wealth that evolves following the random process

$$w_{t+1} = (w_t - c_t)r_{t+1} \tag{37}$$

In (33)-(36) the functions U and D are assumed to satisfy Assumption 1, and the parameter a measures the investor's degree of disutility tolerance:

$$\begin{aligned} \frac{\partial V}{\partial u} &= \omega^{1-\gamma} \times a(1+u)^{a-1} > 0 \\ \frac{\partial V}{\partial d} &= -\omega^{1-\gamma} < 0 \\ \rho(\omega, u, d) &= \frac{(1+u)^{1-a}}{a} \geq 1 \end{aligned}$$

Therefore the higher is a the lower is the agent's disutility aversion.

Our agent's preference differs from that of Barberis et al. (2001) in many aspects through the functional form of V. For instance, we do not assume that past performance affects the agent's preference; we allow V to be generally nonlinear (indeed, concave) in (u, d)

and hence in probabilities; and we allow utility-disutility functions for gains and losses to be more general as well, which is important for understanding the cross-section of expected returns as shown in the previous section.

The approach we adopt differs from Barberis et al. (2001) in that we do not assume any specific stochastic processes (e.g., of consumption growth, dividend growth, etc.). Instead, we use the recursive analysis similar to Epstein and Zin (1991).[4] At any time t, let $\overline{V}_t$ denote the agent's lifetime utility given by

$$\begin{aligned} \overline{V}_t &= E_t\left(\sum_{j=0}^{\infty}\phi^j\left(V_0(c_{t+j}) + b_{t+j}V(\omega_{t+1+j}, u_{t+j}, d_{t+j})\right)\right) \\ &= V_0(c_t) + b_t V(\omega_{t+1}, u_t, d_t) + \phi E_t \overline{V}_{t+1} \end{aligned} \tag{38}$$

Let J denote the optimal value of utility in (38) as a function of current wealth w_t and current information I_t, defined by the Bellman optimality equation

$$J(w_t, I_t) = \max_{c_t,\ p\in\Omega_t}\left[V_0(c_t) + b_t V(\omega_{t+1}, u_t, d_t) + \phi E J(w_{t+1}, I_{t+1})\right] \tag{39}$$

The assumed structure ensures that J is proportional to $w_t^{1-\gamma}$ or to $\ln(w_t)$ if $\gamma = 1$. To verify this, assume

$$J(w_t, I_t) = \frac{[w_t f(I_t)]^{1-\gamma}}{1-\gamma} \tag{40}$$

for some function $f_t \equiv f(I_t)$ and that $\gamma \neq 1$. Substitute (40) into the right-hand-side of (39) and maximizing with respect to c_t leads to the first-order condition

$$\widehat{c}_t^{\,-\gamma} - b_t(w_t - \widehat{c}_t)^{-\gamma}\left[r_{0,t+1}\left((1+u_t)^a - (1+d_t)\right)\right]^{1-\gamma} = \phi(w_t - \widehat{c}_t)^{-\gamma}E_t(r_{t+1}f_{t+1})^{1-\gamma} \tag{41}$$

[4]Except that we do not model preferences for earlier or later resolution of uncertainty. An unverified conjecture is that such preferences could be incorporated in the present model straightforwardly.

Multiplying both sides by $(w_t - \widehat{c}_t)/(1-\gamma)$ and using (40) yields

$$\frac{\widehat{c}_t^{-\gamma}(w_t - c_t)}{1-\gamma} - b_t V(\omega_{t+1}, u_t, d_t) = \phi E_t J(w_{t+1}, I_{t+1})^{1-\gamma}$$

Substituting into (39) and using (40) again yields

$$\begin{aligned} J(w_t, I_t) &= \frac{\widehat{c}_t^{1-\gamma}}{1-\gamma} + \frac{\widehat{c}_t^{-\gamma}(w_t - c_t)}{1-\gamma} \\ &= \frac{\widehat{c}_t^{-\gamma} w_t}{1-\gamma} = \frac{(w_t f_t)^{1-\gamma}}{1-\gamma} \Leftrightarrow f_t^{1-\gamma} = (\frac{\widehat{c}_t}{w_t})^{-\gamma} \end{aligned}$$

Since f_t is independent of w_t, the optimal consumption-wealth ratio $\widehat{c}_t/w_t \equiv \psi_t$ is independent of wealth as well. The ratio, however, may depend on other information at time t. We have thus verified (40).

Removing f_t from (41) we arrive at

$$\begin{aligned} & \widehat{c}_t^{-\gamma} - b_t(w_t - \widehat{c}_t)^{-\gamma} \left[r_{0,t+1} \left((1+u_t)^a - (1+d_t) \right) \right]^{1-\gamma} \\ = \; & \phi (w_t - \widehat{c}_t)^{-\gamma} E_t (r_{t+1}^{1-\gamma} (\frac{\widehat{c}_{t+1}}{w_{t+1}})^{-\gamma}) \\ = \; & \phi E_t (r_{t+1} (\widehat{c}_{t+1}^{-\gamma})) \end{aligned}$$

Let $r_{m,t+1}$ denote the return on an optimal portfolio (market portfolio in equilibrium). Then (41) is equivalent to

$$\phi E_t (r_{m,t+1} (\frac{\widehat{c}_{t+1}}{\widehat{c}_t})^{-\gamma}) + b_t (\frac{1-\psi_t}{\psi_t})^{-\gamma} \left[r_{0,t+1} \left((1+u_t)^a - (1+d_t) \right) \right]^{1-\gamma} = 1 \qquad (42)$$

If $b_t \equiv 0$ then we are back to the familiar consumption-based model.

Now consider the portfolio choice. Consider a deviation $r_{m,t+1} + \theta(r_{i,t+1} - r_{0,t+1})$. Substituting for r_{t+1} in (39) and differentiating with respect to θ yields the first-order condition

$$\frac{b_t}{\widehat{c}_t^{-\gamma}} \left(\frac{\partial V}{\partial u} E_t U'(x_m)(r_i - r_0) - \frac{\partial V}{\partial d} E_t D'(-x_m)(r_i - r_0) \right) + \phi E_t ((\frac{\widehat{c}_{t+1}^{-\gamma}}{\widehat{c}_t^{-\gamma}}(r_i - r_0)) = 0$$

Notice that

$$E_t(\frac{\widehat{c}_{t+1}^{-\gamma}}{\widehat{c}_t^{-\gamma}}x_i) = E_t(\frac{\widehat{c}_{t+1}^{-\gamma}}{\widehat{c}_t^{-\gamma}}\overline{x}_i) - E_t(\frac{\widehat{c}_{t+1}^{-\gamma}}{\widehat{c}_t^{-\gamma}}\underline{x}_i).$$

The above equation can be combined into

$$E_t\left[\left(\eta_t U'(x_{m,t+1}) + \phi\frac{\widehat{c}_{t+1}^{-\gamma}}{\widehat{c}_t^{-\gamma}}\right)\overline{x}_{i,t+1}\right] - \rho_t E_t\left[\left(\eta_t D'(-x_m) + \phi\frac{\widehat{c}_{t+1}^{-\gamma}}{\widehat{c}_t^{-\gamma}}\right)\underline{x}_{i,t+1}\right] = 0$$

where

$$\begin{aligned}\eta_t &= b_t\widehat{c}_t\left((\frac{1-\psi_t}{\psi_t})r_{0,t+1}\right)^{1-\gamma}((1+u_t)^a - (1+d_t))^{-\gamma}\\ \rho_t &= (1+u_t)^{1-a}/a\end{aligned}$$

The SDF can now be written as

$$\begin{aligned}\delta_{t+1} &= \begin{cases} B_t\left(\eta_t U'(x_{m,t+1}) + \phi\frac{\widehat{c}_{t+1}^{-\gamma}}{\widehat{c}_t^{-\gamma}}\right) & \text{if } G_{t+1}\\ B_t\rho_t E_t\left(\eta_t D'(-x_m) + \phi\frac{\widehat{c}_{t+1}^{-\gamma}}{\widehat{c}_t^{-\gamma}}\right) & \text{if } L_{t+1}\end{cases}\\ B_t &= \frac{1}{B_t\left(\eta_t U'(x_{m,t+1}) + \phi\frac{\widehat{c}_{t+1}^{-\gamma}}{\widehat{c}_t^{-\gamma}}\right) + \rho_t E_t\left[\left(\eta_t D'(-x_m) + \phi\frac{\widehat{c}_{t+1}^{-\gamma}}{\widehat{c}_t^{-\gamma}}\right)\right]}\end{aligned}$$

5 Conclusion

(in progress...)

APPENDIX

Proof of Proposition 1: Assume that $w_m \in (0, \overline{w})$ and $m \in \Omega$ are optimal for (**??**). Fixing w_m and let any arbitrary $p_i \in \Omega$ be given. Let $\pi(\theta) = r_m + \theta(r_i - r_m)$ denote the return on a portfolio $\theta p_i + (1-\theta)m \in \Omega$. The first-order condition requires $\frac{\partial \overline{V}}{\partial \theta}|_{\theta=0} = 0$, which implies

$$\frac{\partial V}{\partial u}E[U'(w_m(r_m - r_0))(r_i - r_m)] + \frac{\partial V}{\partial d}E[D'(w_m(r_0 - r_m))(r_m - r_i)] = 0. \qquad (43)$$

Substituting r_0 for r_i in (43) yields

$$\frac{\partial V}{\partial u}E[U'(w_m(r_m - r_0))(r_0 - r_m)] + \frac{\partial V}{\partial d}E[D'(w_m(r_0 - r_m))(r_m - r_0)] = 0. \tag{44}$$

Subtracting the above equation from (43) and rearranging terms using (2) yields (4).

Next, the first-order condition for w_m is

$$\frac{\partial \overline{V}}{\partial w} = -V_0'(c) + \frac{\partial V}{\partial \omega}r_0 + \left(\frac{\partial V}{\partial u}E[U'(w_m(r_m - r_0))(r_m - r_0)] + \frac{\partial V}{\partial d}E[D'(w_m(r_0 - r_m))(r_0 - r_m)]\right) = 0. \tag{45}$$

By equation (44) the last term in the bracket of (45) drops to zero, yielding (3). These conditions are also sufficient by the assumed quasi-concavity of compound utility V. $\square$

Proof of Proposition 2: Rearranging terms in (4) yields

$$E[U'(w_m\overline{x}_m)r_i] + \rho_m E[D'(w_m\underline{x}_m)r_i)] = E[U'(w_m\overline{x}_m)r_0] + \rho_m E[D'(w_m\underline{x}_m)r_0)]$$

Since right hand side is strictly greater than zero, dividing yields

$$\frac{E[(U'(w_m\overline{x}_m) + \rho_m D'(w_m\underline{x}_m))r_i]}{[E(U'(w_m\overline{x}_m) + \rho_m D'(w_m\underline{x}_m))]r_0} = 1$$

Since the above equation holds for all r_i, it implies a valid SDF given by

$$\delta = \frac{U'(w_m\overline{x}_m) + \rho_m D'(w_m\underline{x}_m)}{[E(U'(w_m\overline{x}_m) + \rho_m D'(w_m\underline{x}_m))]r_0}$$

Defining B as in (9) and using $E(\delta x_m) = 0$ leads to (**??**). Thus (8) follows. $\square$

Proof of Proposition 3: Substituting (13) and (14) into (**??**) yields

$$E(\overline{x}) - E[\lambda^+\overline{x}_m\overline{x}] = \rho_m[E(\underline{x}) + E(\lambda^-\underline{x}_m\underline{x})] \tag{46}$$

$$E(\overline{x}_m) - E(\lambda^+\overline{x}_m^2) = \rho_m[E(\underline{x}_m) + E(\lambda^-\underline{x}_m^2)]. \tag{47}$$

Rearranging terms in (46) and (47) yields

$$E(\overline{x}) - \rho_m E(\underline{x}) = E(\lambda^+ \overline{x}_m \overline{x}) + \rho_m E(\lambda^- \underline{x}_m \underline{x}) \tag{48}$$

$$E(\overline{x}_m) - \rho_m E(\underline{x}_m) = E(\lambda^+ \overline{x}_m^2) + \rho_m E(\lambda^- \underline{x}_m^2). \tag{49}$$

The ratio of (48) over (49) implies

$$\beta = \frac{E(\overline{x}) - \rho_m E(\underline{x})}{E(\overline{x}_m) - \rho_m E(\underline{x}_m)} \tag{50}$$

$$= \frac{E(\lambda^+ \overline{x}_m \overline{x}) + \rho_m E(\lambda^- \underline{x}_m \underline{x})}{E(\lambda^+ \overline{x}_m^2) + \rho_m E(\lambda^- \underline{x}_m^2)}. \tag{51}$$

We choose (51) for the definition of β, which measures how the asset returns move with the market. That (50) is equivalent to the pricing relation in (20) can be readily verified. □

REFERENCES

Abdellaoui, Mohammed (2000): "Parameter-Free Elicitation of Utilities and Probability Weighting Functions," *Management Science* 46, 1497-1512.

—— (2002): "A Genuine Rank-dependent Generalization of the von Neumann - Morgenstern Expected Utility Theorem," *Econometrica*, 70, 717-736.

Allais, M. (1953): "Le Comportement de L'homme Rationel devant Le Risque, Critique des Postulates et Axiomes de L'ecole Americaine," *Econometrica*, 21, 503-46.

—— (1979): "The So-Called Allais Paradox and Rational Decisions Under Uncertainty." Part V of *Expected Utility Hypothesis and the Allais Paradox*, ed. by M. Allais and O. Hagen. Dordrecht, Holland: D Reidel Publishing Company.

Ang, A., and Chen, J. (2001). ‘Asymmetric correlations of equity returns’, forthcoming in *Journal of Financial Economics*.

Ang, A., Bekaert, G., and Liu, J. (2001). ‘Why stocks may disappoint’, Working Paper, Columbia University.

Arrow, K.J. (1964). ‘The role of securities in the optimal allocation of risk-bearing’, *The Review of Economic Studies*, vol. 31, pp. 91-96.

Arrow, K. J. (1971). *Essays in the Theory of Risk Bearing.* Markham Publishing Company, Chicago.

Barberis, N., Huang, M., and Santos, T. (2001). ‘Prospect theory and asset prices’, *Quarterly Journal of Economics*, vol. CXVI, pp. 1-53.

Bawa, V.S. and Lindenberg, E.B. (1977). ‘Capital market equilibrium in a mean-Lower partial moment framework’, *Journal of Financial Economics*, 5, 1977, pp. 189-200.

Benartzi, S. and Thaler, R.H. (1995). ‘Myopic loss aversion and the equity premium puzzle’, *Quarterly Journal of Economics*, (February), pp. 73-92.

Bernardo, A.E., and Ledoit, O. (2000). ‘Gain, loss, and asset pricing’, *Journal of Political Economy*, vol. 108, no. 1, 2000, pp. 144-172.

Breeden, D. (1979). ‘An intertemporal asset pricing model with stochastic consumption and investment opportunities’, *Journal of Financial Economics*, vol. 7, pp. 265-96.

Campbell, J.Y. (1999). ‘Asset prices, consumption, and the business cycle’, Chapter 19 in

John Taylor and Michael Woodford eds. *Handbook of Macroeconomics Vol. 1*, North-Holland, Amsterdam.

Campbell, J.Y. (2000). 'Asset pricing at the millennium', *The Journal of Finance*, vol. LV, pp. 1515-67.

Campbell, J.T., and Cochrane, J.H. (2000). 'Explaining the poor performance of consumption-based asset pricing models', *Journal of Finance*, vol. LV, pp. 2863-78.

Campbell, J.Y., and Viceira, L.M. (2001). *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors*. Unpublished book, Harford University.

Campbell, R., Koedijk, K., and Kofman, P. (2002). 'Increased correlation in bear markets: a downside risk perspective', forthcoming in *Financial Analyst Journal*.

Cass, D. and Stiglitz, J.E. (1970). 'The structure of investor preferences and asset returns, and separability in portfolio allocation: a contribution to the pure theory of mutual funds', *Journal of Economic Theory*, vol. 2, pp. 122-60.

Cochrane, J. H., 1999, "New Facts in Finance," *Economic Perspectives*, Federal Reserve Bank of Chicago 23, 59-78.

Cochrane, J. H., 2001, *Asset Pricing*, Princeton University Press, Princeton, New Jersey.

Cochrane, J.H., and Saa-Requejo, J. (2000). 'Beyond arbitrage: good-deal asset price bounds in incomplete markets', *Journal of Political Economy*, vol. 108, 79-119.

Debreu, G. (1959). *Theory of Value: An Axiomatic Analysis of General Equilibrium.* New Haven: Yale University Press.

Domar, E.V., and Musgrave, R.A. (1944). 'Proportional income taxation and risk-taking', *Quarterly Journal of Economics*, vol. 58, pp. 389-422.

Dybvig, P.H. and Ross, S.A. (1984). 'Short sales restrictions and kinks on the mean variance frontier', *The Journal of Finance*, vol. 39, pp. 239-244.

Dybvig, P.H. and Huang, C.-F. (1988). 'Nonnegative wealth, absence of arbitrage, and feasible consumption plans', *Review of Financial Studies*, vol. 1, pp. 377-401.

Fama, E.F. (1970). 'Multiperiod consumption-investment decisions', *The American Economic Review*, vol. 60, pp. 163-74.

Fama, E.F., and French, K.R. (1993). 'Common risk factors in the returns on stocks and bonds', *Journal of Financial Economics*, vol. 33, pp. 3-56.

Fama, E.F., and French, K.R. (1996). 'Multifactor explanations of asset pricing anomalies', *Journal of Finance*, vol. 51, pp. 55-84.

Fishburn, P.C. (1976). 'Optimal portfolios with one safe and one risky asset: effects of changes in rate of return and risk', *Management Science*, vol. 22, pp. 1064-73.

Harlow, W.V., and Rao, R.K.S. (1989). 'Asset pricing in a generalized mean-lower partial moment framework: theory and evidence." *Journal of Financial and Quantitative Analysis*, (September), pp. 285-311.

Harrison, J.M., and Kreps, D.M. (1979). 'Martingales and arbitrage in multiperiod securities markets', *Journal of Economic Theory*, vol. 20, pp. 381-408.

Hicks, J.R. (1939). *Value and Capital*. Oxford University Press, Fair Lawn, N.J.

Kahneman, D., and Twersky, A. (1979). ‘Prospect theory: an analysis of decision under risk.” *Econometrica*, vol. 47, pp. 263-291.

Knight, F.H. (1921). *Risk, Uncertainty and Profit.* New York: Houghton Mifflin (*Reprints of Economic Classics*, New York, 1964).

Kocherlakota, N.R. (1996). ‘The equity premium: it’s still a puzzle’, *Journal of Economic Literature*, vol. 34, pp. 42-71.

Lindner, J. (1965). ‘The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets’, *Review of Economics and Statistics*, vol. 47, pp. 13-37.

Lucas, R.E. (1978). ‘Asset prices in an exchange economy’, *Econometrica*, vol. 46, pp. 1429-45.

Markowitz, H. (1952). ‘Portfolio selection’, *Journal of Finance* vol. 7, pp. 77-91.

Markowitz, H. (1991). ‘Foundations of portfolio theory’, *Journal of Finance*, vol. 46, pp. 469-77.

Mehra, R., and Prescott, E.C. (1985). ‘The equity premium – a puzzle’, *Journal of Monetary Economics*, vol. 5, pp. 145-61.

Merton, R.C. (1969). ‘Lifetime portfolio selection under uncertainty: the continuous-time case’, *The Review of Economics and Statistics*, vol. 51, pp. 247-57.

Merton, R.C. (1971). ‘Optimum consumption and portfolio rules in a continuous-time model’, *Journal of Economic Theory*, vol. 3, pp. 373-413.

Merton, R.C. (1973). 'An intertemporal capital asset pricing model', *Econometrica*, vol. 41, 867-87.

Mossin, J. (1966). 'Equilibrium in a capital asset market', *Econometrica*, vol. 34, pp. 768-83.

Pratt, J.W. (1964). 'Risk aversion in the small and in the large', *Econometrica*, vol. 32, pp. 122-36.

Roell, A. (1987). 'Risk aversion in Quiggin and Yaari's rank-order model of choice under uncertainty', The ECONOMIC JOURNAL, vol. 97, pp. 143-59.

Ross, S.A. (1976). 'The arbitrage theory of capital asset pricing', *Journal of Economic Theory*, vol. 13, pp. 341-60.

Ross, S.A. (1978a). 'A simple approach to the valuation of risky streams', *Journal of Business*, vol. 3, pp. 453-76.

Ross, S. A. (1978b). 'Mutual fund separation in financial theory – the separating distributions', *Journal of Economic Theory*, vol. 17, pp. 254-86.

Rubinstein, M. (1974). 'An aggregation theorem for securities markets', *Journal of Financial Economics*, vol. 1, pp. 225-44.

Samuelson, P. A. (1969). 'Lifetime portfolio selection by dynamic stochastic programming', *The Review of Economics and Statistics*, vol. 51, pp. 239-46.

Samuelson, P. A. (1970). 'The fundamental approximation theorem of portfolio analysis in terms of means, variances and higher moments', *Review of Economics Studies*, vol. 37, pp. 537-42.

Sharpe, W.F. (1964). 'Capital asset prices: a theory of market equilibrium under conditions of risk', *Journal of Finance*, vol. 19, pp. 423-439.

Shefrin, H., and Statman, M. (2000). 'Behavioral portfolio theory', *Journal of Financial and Quantitative Analysis*, vol. 35, pp. 127-51.

Tobin, J. (1958). 'Liquidity preference as behavior toward risk', *Review of Economic Studies*, vol. 25, pp. 65-85.

Van de Kuilen, G., P. P. Wakker, and L. Zou (2006). 'A midpoint technique for easily measuring prospect theory's probability weighting', Working Paper, CREED, University of Amsterdam.

Von Neumann, J. and Morgenstern, O. (1944). *Theory of games and Economic Behavior*, Princeton: Princeton University Press. Second Edition, 1947. Third Edition, 1953.

Yaari, M.E. (1987). 'The dual theory of choice under risk', *Econometrica*, vol. 55, pp. 95-115.

Zou, L. (2000). 'Inherent Efficiency, Security Markets, and the Pricing of Investment Strategies', Tinbergen Institute Discussion Paper 00-108/2.

http://www.tinbergen.nl/home.html (Publications/Discussion Papers).

Zou, L. (2005). 'Dichotomous Asset Pricing Model', *Annals of Economics and Finance* 6, May 2005, 185-207.

Zou, L. (2006a). 'An Alternative to Prospect Theory', *Annals of Economics and Finance* May 2006, 1-27.

Zou, L. (2006b). 'The best-beta CAPM', *Applied Financial Economics Letters* 2, 131-137.