True Spreads Censored by Tick Size *

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Abstract

A model of true spreads is developed using spread data from the Australian Stock Exchange. This spread model separates the true or intrinsic spread of a stock from the component of spread that is due to tick size. This enables the identification of excessive market spreads due to a large minimum tick size (large minimum price increment).

The true or uncensored spreads are modelled as a LogNormal distribution where the scale (mean) parameter of the distribution is a function of stock Turnover and Volatility, stock Price is not a determinant of true spreads. This is a powerful and intuitive result; true spread is a measure of the true cost of liquidity and this cost is a function of the scarcity of liquidity (Turnover) and the risk of supplying liquidity (Volatility), nominal stock price is irrelevant to the cost of market liquidity.

Observed or censored spreads are a result of partitioning the underlying continuous true spread distribution into discrete tick size intervals. Stock price is important in this partitioning and observed spreads are a function of stock price.

Keywords: True Spread, Censored Spread, Observed Spread, Tick Size, Exchange Policy

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Introduction

Stock exchanges have minimum price increments (often price dependent) on which stocks can be quoted and traded, this is known as the 'tick size'. The tick size on the Australian Stock Exchange for stocks priced \$0.50 (all currency in this paper is in Australian dollars) and above is 1 cent (July 2001). The 'spread' is the difference between the 'actual' price of the stock and the price paid by a liquidity demander (market order trader). Spread is measured as the difference between the mid-point of the bid and ask immediately preceding a trade and the price of the subsequent trade. Therefore spreads can only be observed on half-tick (0.5 cent) size discrete intervals.

It is a reasonable hypothesis that liquidity as supplied by limit order traders has a continuous monotonically increasing supply curve as a function of increasing spread and liquidity as demanded by market order traders has a monotonically decreasing demand curve as a function of increasing spread. Therefore, there is an underlying (unobserved) spread distribution which is a continuous distribution on the positive half-line.

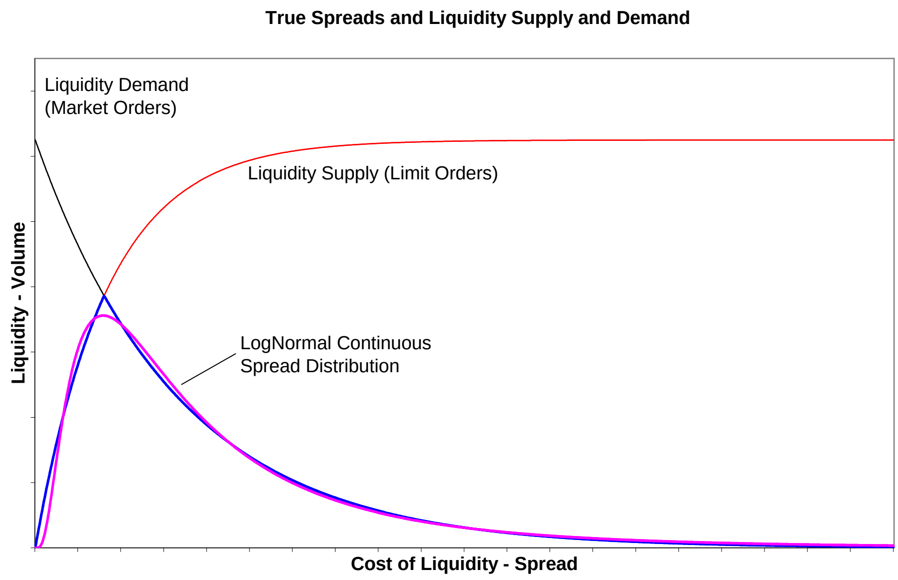

Figure 1: The existence of liquidity supply and demand curves imply an underlying continuous spread distribution.

Harris [5] suggested a likelihood regression for finding a parametric distribution that matches the continuous spread distribution and this technique is further developed in this paper. The unobserved continuous spread distribution is assumed to have a constant shape for all stocks and the the scale of the distribution (the distribution mean) is regressed as a function of stock Turnover, Volatility and Price. Price is found to be not significant in determining the scale (mean) of the unobserved continuous spread. This is a powerful and intuitive result. Turnover is a determinant in the pricing of spread because it is a proxy for the scarcity of stock liquidity. Volatility is a determinant of spread because of the extra risk taken by a limit order trader when providing liquidity for a volatile stock. But it is intuitive that Price is not a determinant of true unobserved spread because liquidity providers and demanders are neutral about the nominal stock price at which liquidity is supplied. For example, in negotiating the cost of supplying \$1m of stock, liquidity providers and demanders are neutral as whether that stock liquidity is supplied as $\$1 \times 1,000,000$ stock units or $\$100 \times 10,000$ stock units.

The unobserved continuous spread distribution is mapped onto the observed discrete spreads by integrating the continuous spreads between the midpoints of the observed spreads which partition the continuous spreads. For example, in figure 2 the midpoint between an observed spread of 0.5 cents and 1.0 cent is 0.75 cents, if the LogNormal continuous spread is integrated (the cumulative LogNormal) between 0 and 0.75 cents the censored spread at 0.5 cents is estimated at 57% of all spreads. The actual observed spread was 55% of all spreads.

Thus observed spreads are estimated as a definite integral of the underlying continuous spreads as partitioned by tick size. Observed spreads can be split into a component which is intrinsic to the stock attributes of Turnover and Volatility - the underlying continuous spread and a component due to the censoring effect of tick size, the Excess spread. Being able to separate these components enables excessive spreads through too large a tick size to be identified.

To clarify terminology for the reader the following definitions of spreads used in the paper are defined.

- The True or Uncensored spread is the mean of the underlying, unobserved continuous spread.

- The Censored spread is the mean of the definite integrals of the underlying continuous spreads as partitioned by tick size. This is an estimate of Observed spread.
- The Observed spread is the mean of all observed spreads.
- The Excess spread is the component of Observed (or Censored) spread that is due to tick size effects and is simply the Observed spread minus the True spread.

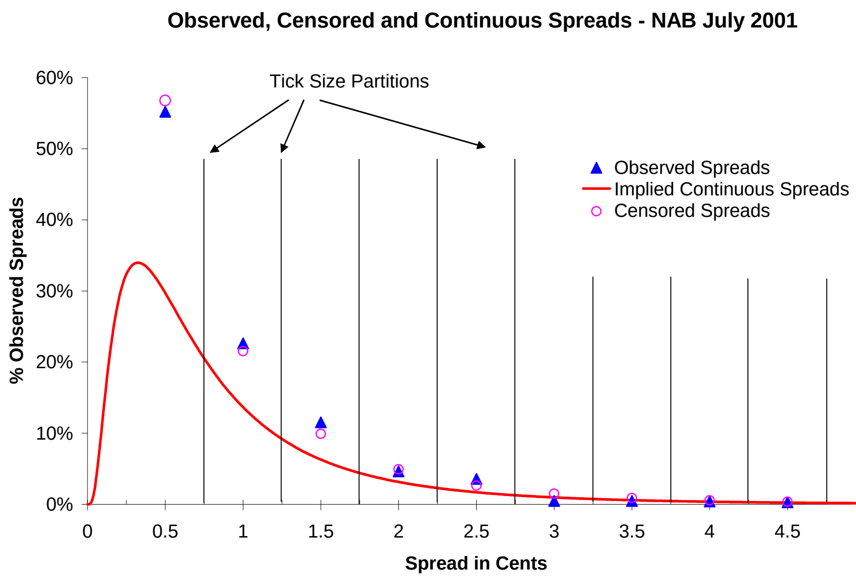

Figure 2: Observed spreads for stock NAB (National Australia Bank, Banking and Finance, July 2001 VWAP Price $32.63, Av Daily Turnover $78.9 million, Annualized Volatility 30.5%) are shown as blue triangles. Tick size censoring means that spreads can only be observed on half-tick (0.5 cent) intervals; at 0.5 cents, 1.0 cent, 1.5 cents etc. The implied LogNormal continuous spread is partitioned by the midpoints between the observed spreads; 0.75 cents, 1.25 cents etc. The shape of the continuous spread distribution (0.834) is constant across all stocks, the mean (scale) of the distribution is a likelihood regression of stock Turnover and Volatility. Stock Price is not a determinant of the mean (scale) of the continuous spread. The Censored spread is the value of the integral of the continuous spread distribution (the cumulative spread distribution) between the spread mid-point partitions and is shown as magenta circles. The Censored spread is an estimate of Observed spreads.

Spreads and Tick Size

Measuring Spread

When measuring spreads, the spread cost paid by market order traders to initiate a trade and execute against limit order traders is of primary interest. It is intuitive to measure the spread as the the absolute difference between the transaction price and the actual stock price.

$$\text{Spread} = |\,\text{TransactionPrice} - \text{ActualPrice}\,| \tag{1}$$

The actual stock price cannot, in general, be observed but lies somewhere between the current quoted bid and ask price. If it is assumed that actual stock prices are symmetrically or uniformly distributed between the bid and ask price, then the expectation of the actual price will be the mid-point of the bid and ask price. Thus a practical spread cost measurement is the absolute difference between the current mid-point of the bid and ask price and the transaction price. Effective spread is defined as the difference between the trade price (P^{trade}) and the mid-point quoted price ($\frac{bid+ask}{2}$) and the average effective spread is calculated by averaging over the number of trades. Effective spread has been chosen as the spread measure in recent investigations into market spread on the Toronto stock exchange by Bacidore [1] and the NASDAQ exchange by Barclay et al [2].

$$S^{\text{effective}} = \frac{\sum_{i=1}^{N} \left| P_i^{\text{bid-ask}} - P_i^{\text{trade}} \right|}{N}, \qquad P_i^{\text{bid-ask}} = \frac{\text{Ask}_i + \text{Bid}_i}{2} \tag{2}$$

Relative effective spread (R_s) is effective spread normalized by dividing by the mid point bid-ask price and, following market convention, is scaled by 10,000 so that the relative spread is expressed in ‘basis points’ (1/100ths of a percent).

$$R_s = \frac{10000}{N} \sum_{i=1}^{N} \left| \frac{P_i^{\text{bid-ask}} - P_i^{\text{trade}}}{P_i^{\text{bid-ask}}} \right| \tag{3}$$

Minimum Relative Spread Cost

The minimum measured spread cost is half the tick size (see figure 3). Since the ASX has a 1 cent tick size for all stock priced $0.50 and above, the

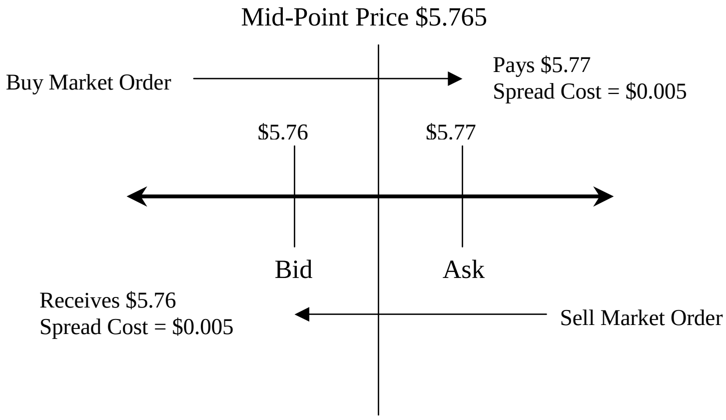

Figure 3: A Tick Size of $0.01 Implies a Minimum Spread Cost of $0.005

minimum spread is $0.005. The minimum relative spread cost in basis points, R_{min}, is given by the following relationship to stock price, P_s in dollars.

$$R_{min} = \frac{50}{P_s} \tag{4}$$

Figure 4 shows the minimum cost of crossing the spread and executing a market order rapidly increases for lower priced stocks.

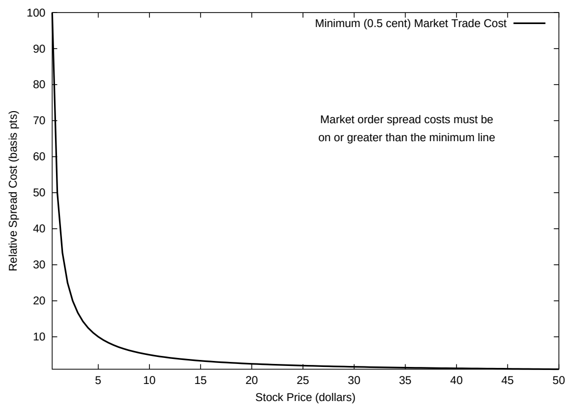

Figure 4: Minimum Relative Spread Cost as a Function of Price

Equation 4 can be transformed with logarithms (in this paper, 'log' symbolizes a logarithm to the base 10, 'ln' symbolizes a natural logarithm) so that the log of the minimum relative spread is linear with respect to the log of price (figure 5).

$$\log R_{min} \;=\; \log 50 \;-\; \log P \tag{5}$$

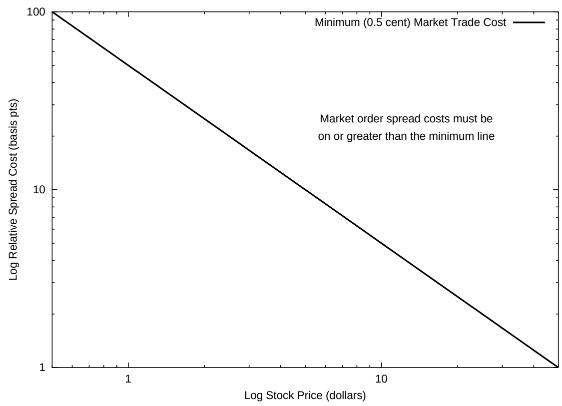

Figure 5: Log Minimum Relative Cost as a Function of Log Price

The cost of executing a market order (for stocks priced above $0.50) on the ASX must be on or above the minimum relative cost line in figure 5. The price of a stock determines the minimum relative cost of executing a market order.

Ordinary Least Squares Models of Spreads

It is a reasonable proposition that market order spread costs are inversely correlated to turnover (market traders are willing to pay for low stock liquidity by paying higher spread costs when executing market orders). The idea that spread may be partially driven by turnover was first suggested by Demsetz [3] and has been further developed by other researchers such as Harris [5] and Stoll [11]. Both authors developed cross sectional regression models where relative spreads are regressed against price, market activity (trades per day), turnover, market capitalization and volatility. These OLS regression models have strong explanatory power when describing spreads on the NYSE and NASDAQ stock markets with Stoll stating (Stoll [11], page 1481) 'few empirical relations in finance are this strong'.

The Parsimonious ASX Spread Regression Model

Following Stoll [11] and Harris [5], ASX relative effective spread data were combined for each stock to produce a cross-sectional regression over the entire data period (July 2001) for each stock (see the subsection titled 'The Spread Data' for a description of the regression data). A modified parsimonious version of the regression equation specified by Stoll [11] was estimated for the aggregated ASX spread data. The regression estimates spread using Turnover, Price and Volatility, adding additional independent variables such as Trade Count and Market Capitalization did not add significant information to the regression. The fitted coefficients are given in table 1.

The regression was checked for heteroscedasticity using the test devised by White [12] for regressions where there is a large number of observations relative to the number of variables. This test is labelled 'Hetero-X'. Testing the regression of equation 6 confirms heteroscedasticity ($F(9, 235) = 32.55$). Where heteroscedasticity is confirmed, heteroscedastic consistent t-statistics for the explanatory variables are generated using the Jack-Knife estimator described by White and MacKinnon [6]. These are labelled 't-JHCSE'. The Partial R^2 statistics for explanatory variables are calculated from the incremental increase in sum of squares by including an explanatory variable given the sum of squares of all other explanatory variables are already included. For further details see Nachtshiem et al [9].

$$\log R_s \;=\; A_0 \;+\; A_1 \log T_s \;+\; A_2 \log P_s \;+\; A_3 \log \sigma_s \;+\; \varepsilon_s \qquad (6)$$

	Intercept A_0	*log* Turnover A_1	*log* Price A_2	*log* Volatility A_3
Coefficients	3.499	-0.268	-0.354	0.233
t-JHCSE	23.663	-9.878	-7.160	7.832
Partial R^2		0.666	0.547	0.179
R^2	0.911			
Hetero-X $F(9,235) \;=\; 32.55$ (p =0.000)				

Table 1: The ASX Parsimonious Relative Spread Regression Model (n=249)

ASX Spreads are Censored by Tick Size

It is important to note that the parsimonious relative spread regression model (eqn 6), although apparently successful in describing observed spreads, is incorrectly specified since relative spreads are censored by minimum tick size (see figure 6). The true regression model must account for this tick size censoring.

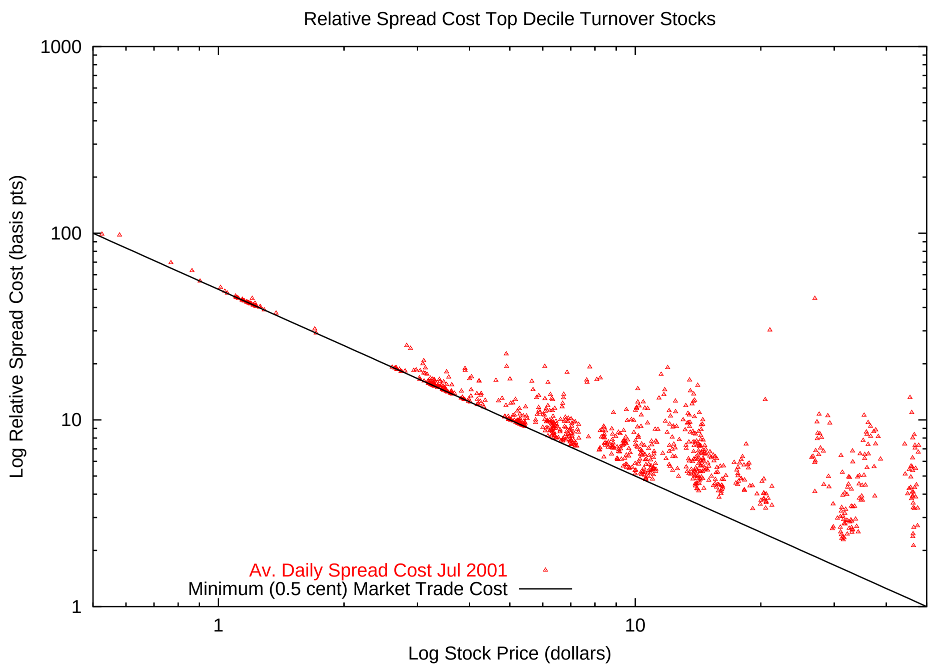

Figure 6: Daily relative spreads against Stock Price for Top Decile (highest) Turnover Stocks. Observed spreads are censored against the minimum spread line (black line) for lower priced stocks (higher minimum spread).

A Model of True Spreads

The previous section showed that OLS models of spreads, although highly predictive, were an incorrect description of spread behaviour because of data censoring. In this section the spread model first proposed by Harris [5] is generalized and developed as a model of spreads on the ASX. This model assumes that relative spreads form a continuous distribution and that the observed discrete half-tick size relative spreads are a result of partitioning this continuous distribution into discrete half-tick size increments. Harris used maximum log-likelihood techniques to estimate the underlying parametric relative spread distribution and this technique is further developed by extending the size of the discrete half-tick size distribution observed to 50 observations and examining different candidate parametric relative spread distributions (Harris only examined the Gamma distribution).

Developing a model of the underlying continuous parametric relative spread distribution allows the observation of the true spread schedule of stocks and enables the partition of observed spreads into true spreads intrinsic to the stock attributes of Turnover, Volatility and Price and 'excess' spreads imposed by the tick size schedule. These excess spreads are generated when the continuous parametric relative spread distribution is mapped onto the observed discrete half-tick size spread distribution (see figure 2).

When the true spread regression model is examined, the log Price explanatory variable is shown to be not significant and this is explained as a powerful and intuitive result. The parametric continuous relative spread distribution, the true spread, is a function of stock liquidity (turnover) and stock risk (volatility), whereas the nominal stock price is not an important determinant of true spreads. *Price is only introduced as an important explanatory variable in relative spreads by the 'tick-size censoring function' that maps the underlying parametric continuous relative spread distribution on to the discrete half-tick size spread distribution.*

The Spread Data

The ASX market data used in this paper is very rich and allows the ASX market to be replayed with complete accuracy. In particular, the complete depth of the limit order queues are known before each trade and traded spread calculations are unambiguous and accurate. For all stocks (non ordinary stock securities such as warrants, hybrid debt-equity, etc. were excluded) trading on the ASX in July 2001 with a stock price equal to or greater than \$0.50 (all stocks with a minimum 1 cent tick size) the actual spread relative to the bid-ask mid-point price paid by pure market orders (buyers and sellers) was recorded in 50 x 0.5 cent spread bins. Each spread bin corresponding to a particular spread size recorded the total volume of the market order trades at that spread size. The first bin corresponded to the minimum observable tick size of 0.5 cents, the next bin 1.0 cent and so on. The final bin, 50×0.5 cents $= 25$ cents, contained the sum of the volume of all spreads ≥ 25 cents. Only stocks that had at least 1 spread observation per day (1 pure market trade per day) for the 20 business days of market data during July 2001 were analyzed, 249 stocks had spread data for the 20 business days. For each of the 249 stocks, Turnover, Price and Volatility were calculated using the following definitions:

- *Price* was the VWAP (Volume Weighted Average Price) price of the stock for all on-market trades in July 2001.
- *Turnover* was the turnover of all on-market trades in July 2001 divided by the 20 business days to give an average daily turnover.
- *Volatility* was calculated as a weekly standard deviation from the previous 52 weekly prices. Weekly prices were used rather than daily prices to minimize serial price correlations in low turnover stocks which would (downward) bias the volatility estimates.

A Parametric Continuous Spread Distribution

Assume a cumulative probability for relative true spread, $\Phi(r;\Lambda)$, where Λ is a vector of distributional parameters (possibly scalar).

The actual effective spreads paid by market order traders for stock S are observed and summed on a $K \times 0.5$ cent grid (in this paper, $K = 50$). The K spread step probabilities can be written recursively in terms of the cumulative probability distribution of the relative true spread.

$$\begin{aligned} Pr(n=1) &= \Phi(\pi(1,P_s);\Lambda) \\ Pr(n=2) &= \Phi(\pi(2,P_s);\Lambda) - Pr(n=1) \\ \vdots \quad &\vdots \\ Pr(n=K-1) &= \Phi(\pi(n-1,P_s);\Lambda) - Pr(n=K-2) \\ Pr(n \geq K) &= 1 - Pr(n=K-1) \end{aligned} \tag{7}$$

The function $\pi(n, P_s)$ is a mapping function from the continuous (but unobserved) spread distribution to the observed discrete half-tick sized spreads. The mapping function does this by mapping the observed nth 0.5 cent spread step onto a continuous spread that represents the mid-point of n spread steps and $n+1$ spread steps. An unobserved continuous spread, r, will be observed as an actual spread of $n \times 0.5$ cents if the unobserved continuous spread is in the following interval.

$$\pi(n-1, P_s) \leq \; r \; < \pi(n, P_s) \tag{8}$$

The arithmetic mid-point between two observed spreads is used to determine the boundary for a continuous unobserved spread to move from an observed spread step size of $n \times 0.5$ cent steps to $(n+1) \times 0.5$ cent steps. Therefore $\pi(n)$ is defined as follows (in basis points):

$$\pi(n, P_s) = \frac{50\,(n+0.5)}{P_s}, \quad \pi(0, P_s) = 0 \tag{9}$$

The Loglikelihood Function

A multinomial likelihood function (constant omitted) can be generated for M stocks with K actual spread step observations (each observation is $m_{j,k}$) for each stock.

$$L(\Lambda) \; = \prod_{j=1}^{M} \prod_{k=1}^{K} \left[Pr(n=k) \right]^{m_{j,k}} \tag{10}$$

The consequent log-likelihood function is defined as:

$$\ell(\Lambda) \; = \; \ln L(\Lambda) \; = \sum_{j=1}^{M} \sum_{k=1}^{K} m_{j,k} \ln Pr(n=k) \tag{11}$$

If $f_j(k)$ is introduced as the observed relative frequency of actual spread step k for the jth stock and substituting equations 7 into the log-likelihood equation 11.

$$M_j \; = \sum_{k=1}^{K} m_{j,k} \qquad f_j(k) \; = \; \frac{m_{j,k}}{M_j}$$

$$
\begin{aligned}
\ell(\Lambda) \;=\; \sum_{j=1}^{M} M_j \Bigg[& f_j(1)\ \ln[\, \Phi(\pi(1,P_j);\Lambda)\,] + \\
& f_j(2)\ \ln[\, \Phi(\pi(2,P_j);\Lambda) \;-\; \Phi(\pi(1,P_j);\Lambda)\,] + \\
& \vdots \\
& f_j(K-1)\ \ln[\, \Phi(\pi(K-1,P_j);\Lambda) \;-\; \Phi(\pi(K-2,P_j);\Lambda)\,] + \\
& f_j(K)\ \ln[\, 1 - \Phi(\pi(K-1,P_j);\Lambda)\,] \Bigg]
\end{aligned} \tag{12}
$$

In order to solve for the relative spread distribution, a distributional class, $\Phi(r;\Lambda)$, needs to be selected with a mean of R_s for different values of the shape parameter vector Λ and supported on the non-negative numbers, $r \in [0,\infty)$. Then Λ can be solved by maximizing the log-likelihood equation 12.

$$
\Lambda^0 \;=\; \arg\max_{\Lambda}\, [\, \ell(\Lambda)\,] \tag{13}
$$

For purely practical reasons associated with the optimization software, the optimization is actually performed by minimizing the negative value of $\ell(\Lambda)$.

$$
\Lambda^0 \;=\; \arg\min_{\Lambda}\, [\, -\,\ell(\Lambda)\,] \tag{14}
$$

Equation 14 was solved numerically by software written in the specialist econometrics programming language **OX** developed by Jurgen A. Doornik [4]. In all optimizations, convergence to optimal values was prompt and consistent with no evidence of local minima.

Parameter Estimation of Continuous Spread Distributions

Harris [5] uses the Gamma distribution to model the spread distribution by solving equations 12 and 14. The Gamma distribution can be parameterized by shape (λ) and scale (β) parameters:

$$
\Phi(r;\lambda,\beta) \;=\; \frac{1}{\beta^{\lambda}\,\Gamma(\lambda)} \int_0^r x^{\lambda-1}\, e^{-x/\beta}\, dx \qquad r \in [0,\infty) \tag{15}
$$

$$E[\Phi(r;\lambda,\beta)] \;= \lambda\beta \tag{16}$$

If the true spread, R_s^{true} (abbreviated to R_s^t), is defined as the mean of the continuous spread distribution.

$$R_s^t \;=\; E[\Phi(r;\lambda,\beta)] \;=\; \lambda\beta \tag{17}$$

The scale parameter (β) can be modified to be a function of the shape parameter (λ) and the mean uncensored spread (R_s^t). With this specification, the shape parameter (λ) can be constant and the scale parameter (β) varied so that the distributional mean is always R_s^t.

$$\beta \;=\; \frac{R_s^t}{\lambda} \tag{18}$$

The true spread (R_s^t) can be modelled explicitly as a function of the Turnover, Price and Volatility of stock S. The same functional form as the parsimonious regression (eqn 6) is chosen.

$$\log R_s^t \;=\; B_0 \;+\; B_1 \log T_s \;+\; B_2 \log P_s \;+\; B_3 \log \sigma_s \tag{19}$$

Therefore the scale parameter of the Gamma distribution is written as a function of the shape parameter and the parameters of the regression equation (eqn 19).

$$\beta \;=\; 10^{-(B_0 \,+\, B_1 \log T_s \,+\, B_2 \log P_s \,+\, B_3 \log \sigma_s)}\,\frac{1}{\lambda} \tag{20}$$

Substituting the re-parameterized Gamma distribution into the log-likelihood equation 12 produces a model with 5 parameters to solve, the shape of the continuous distribution (λ) and the scale of the continuous distribution expressed as a regression on the stock Turnover (T_s), Price (P_s) and Volatility (σ_s).

$$\Lambda^0 \;=\; \{\lambda^0, B_0^0, B_1^0, B_2^0, B_3^0\} \;=\; \arg\min\,[\,-\ell(\lambda, B_0, B_1, B_2, B_3)\,] \tag{21}$$

The Gamma distribution suggested by Harris [5] has a rich range of distributional shapes but there are other shape and scale distributions defined on the positive half-line and suggested by inspection of the binned spread data that can be used in the general log-likelihood model developed in equation

12. Alternatives are the Weibull distribution and the Log-Normal distribution. The Exponential distribution is also modelled because the results of modelling the Gamma and Weibull distributions (table 2) produced shape parameters close to this distribution.

The Exponential distribution is dependent on a single scale parameter β.

$$\Phi(r;\beta) \;=\; 1 \;-\; \exp\big[-\beta\, r\big] \tag{22}$$

$$E[\Phi(r;\beta)] \;=\frac{1}{\beta} \;=\; R_s^t \tag{23}$$

Therefore the scale parameter of the exponential distribution is written as a function of the parameters of the regression equation only (there is no shape parameter).

$$\beta = \; 10^{-(B_0 \,+\, B_1 \log T_s \,+\, B_2 \log P_s \,+\, B_3 \log \sigma_s)} \tag{24}$$

The Weibull distribution is parameterized by a shape (λ) and scale (β) parameter.

$$\Phi(r;\lambda,\beta) \;=\; 1 - \exp\big[-(\frac{r}{\beta})^{\lambda}\big] \tag{25}$$

$$E[\Phi(r;\lambda,\beta)] \;=\; \beta\,\Gamma(\frac{\lambda+1}{\lambda}) \;=\; R_s^t \tag{26}$$

$$\beta \;=\; 10^{(B_0 \,+\, B_1 \log T_s \,+\, B_2 \log P_s \,+\, B_3 \log \sigma_s)} \left[\Gamma(\frac{\lambda+1}{\lambda})\right]^{-1} \tag{27}$$

The Log-Normal distribution is parameterized by a shape (λ) and scale (β) parameter.

$$\Phi(r;\lambda,\beta) \;=\; \frac{1}{2}\left[1 \,+\, \mathrm{erf}\left(\frac{\ln r - \beta}{\sqrt{2}\lambda}\right)\right], \qquad \mathrm{erf}(z) \;=\; \frac{2}{\sqrt{\pi}} \int_0^z \; e^{-t^2}\, dt \tag{28}$$

$$E[\Phi(r;\lambda,\beta)] \;=\; \exp\big[\beta + \frac{\lambda^2}{2}\big] \;=\; R_s^t \tag{29}$$

$$\beta \;=\; \ln(10)(B_0 \;+\; B_1 \log T_s \;+\; B_2 \log P_s \;+\; B_3 \log \sigma_s) \;-\; \frac{\lambda^2}{2} \tag{30}$$

The important attribute of this log-likelihood specification is that by the regression model for the true spread (R_s^t) avoids any censorship bias due to the censoring of relative spreads because of tick size. The maximum likelihood parameter estimates from equation 19 will give us estimates of the true spread regression model.

The maximum likelihood estimates were obtained for the spread distributional data of 249 stocks. The minimum number of spread observations for all stocks was 70 (median spread observations 731, max spread observations 39,831) and following Harris [5] this was used as the M_j term in the log-likelihood function (eqn 12) for all stocks ($M_j = 70, \forall j$). This results in conservative parameter interval estimates and avoids high turnover stocks being over-influential . The proportion of total volume for each spread bin, $f_j(k)$, was calculated by dividing the volume summed in the kth spread bin by the total volume executed as pure market orders for the stock.

Log-likelihood Regression Results

The results of fitting the alternative distributions to the binned volume data are tabulated in table 2. The true relative spread regression model is robust and insensitive to the particular distribution used. The Weibull and Gamma distributions both have estimated optimal shape parameters close to 1. A shape parameter of 1 defines the exponential distribution for both these distributions. Therefore the fitted Weibull and Gamma distributions do not provide significant parametric shape information. In contrast, the Log-Normal distribution does provide distributional shape information.

The estimated true relative spread regressions have larger coefficients for log Turnover and log Volatility than the parsimonious OLS spread regression, but the main difference is the negligible coefficient for log Price. The log Price coefficient is not significantly different from zero for any of the log-likelihood regressions. This means that there is no relationship between stock price and true uncensored spreads.

This is a powerful result, true relative spreads are a function of stock turnover (the cost of liquidity) and volatility (the cost of risk); there is no relationship to stock price. The price effect on observed (censored) spreads is introduced by the censoring of true uncensored spreads due to the interaction of tick size and stock price.

$$R_s^{\text{censored}} = \text{Censor}(\, R_s^{\text{true}}; \text{TickSize, Price, }) \tag{31}$$

Distribution	λ Shape	B_1 log Turnover	B_2 log Price	B_3 log Volatility	B_0 Intercept
LogNormal	**0.834**	**-0.420**	**-0.010**	**0.493**	**4.443**
Std Error	0.0106	0.0081	0.0146	0.0225	0.0487
$-\ell(\Lambda^0)$	12268				
Gamma	**1.127**	**-0.415**	**-0.014**	**0.469**	**4.368**
Std Error	0.0286	0.0076	0.0134	0.0209	0.0457
$-\ell(\Lambda^0)$	12289				
Weibull	**1.031**	**-0.418**	**-0.0067**	**0.473**	**4.382**
Std Error	0.0126	0.0079	0.0139	0.0215	0.0472
$-\ell(\Lambda^0)$	12296				
Exponential		**-0.422**	**0.004**	**0.480**	**4.407**
Std Error		0.0080	0.0136	0.0219	0.0475
$-\ell(\Lambda^0)$	12299				
OLS		**-0.268**	**-0.354**	**0.233**	**3.499**

Table 2: True relative spread distribution parameters (50 x 0.5 cent bins; n=249 Stocks) for different parametric distributions. Also, for comparison, the coefficents of the OLS regression (equation 6, table 1).

The Tick Size Censoring Function

In this section a formal model of the tick size censoring function is developed. The observed actual spreads can be modelled as a half a tick size partitioning of the underlying continuously distributed true spread and the censoring function is simply the probability weighted sum of the underlying continuously distributed true spread partitioned into discrete observed spreads.

Note that only the censoring function is affected when tick size is varied through stock exchange policy change. Therefore a variable tick size, ρ, is introduced when defining the censoring function. This enables the censoring function us to be parameterized for different market tick sizes and the hypothetical effect of varying tick size on observed spreads (true spreads are independent of tick size) can be investigated.

The censored mean spread over the observed discrete half tick size distribution is calculated by summing over the probability that the continuous true spread will be in the kth half spread bin, $Pr(k, \rho, P_s)$, by the relative spread size of the kth bin, $\mu(k, \rho, P_s)$.

$$R_s^{censored} = \sum_{k=1}^{\infty} Pr(k, \rho, P_s)\, \mu(k, \rho, P_s) \tag{32}$$

Formally, the sum in equation 32 is over an infinite number of terms because the uncensored continuous distribution of relative spreads, $\Phi(r; \Lambda)$, is defined over the whole non-negative real interval $[0, \infty)$. In practise, an arbitrarily accurate approximation can be achieved by truncating the summation when the remaining terms of the infinite sum become small.

The following auxiliary function is defined. The half tick size mid-point function (eqn 9) is augmented by an explicit tick size argument, ρ - tick size in dollars. (the mid-point is in basis points).

$$\pi(n, \rho, P_s) = \frac{5000\, \rho\, (n + 0.5)}{P_s}, \quad \pi(0, \rho, P_s) = 0 \tag{33}$$

Let $\Phi(r; \Lambda)$ be the true spread cumulative distribution function for a vector of known spread distribution parameters Λ. The probability of a true relative spread being observed at the kth half-tick size bin, $Pr(k, \rho, P_s)$, is simply the cumulative spread distribution at the kth spread bin mid-point less the cumulative spread distribution at the $(k-1)$th mid-point.

$$Pr(k, \rho, P_s) \; = \; \Phi(\pi(k, \rho, P_s); \Lambda) \; - \Phi(\pi(k-1, \rho, P_s); \Lambda) \tag{34}$$

The relative spread (in basis points) of the kth half tick size, parameterized for market tick size ρ (in dollars) is:

$$\mu(k, \rho, P_s) \; = \; \frac{5000 \, k \, \rho}{P_s} \tag{35}$$

The Censoring Function

Using the functions defined above the complete censoring function is defined:

$$\begin{aligned} R_s^{censored} \; &= \; \sum_{k=1}^{\infty} \; Pr(k, \rho, P_s) \, \mu(k, \rho, P_s) \\ &= \; \sum_{k=1}^{\infty} \; \big[\, \Phi(\pi(k, \rho, P_s); \Lambda) \; - \Phi(\pi(k-1, \rho, P_s); \Lambda) \big] \; \frac{5000 \, k \, \rho}{P_s} \end{aligned} \tag{36}$$

The infinite sum can be truncated to approximate the censored spread such that the remaining (infinite) sum is less than a small value, ϵ:

Analysis of Results

Censored Spreads, True Spreads and Excess Spreads

For the 249 stocks in the data sample, true spreads were calculated and compared to the Censored spreads of each stock. The calculation of true spreads uses the LogNormal regression results of table 2 and is a function of Turnover and Volatility only, stock price (P_s) is not significant. Censored spreads were calculated from the underlying continuous spreads using equation 36 and stock price and tick size are introduced as determinants of censored spread.

$$R_s^{\text{true}} = 10^{\,4.443\,-\,0.420\log T_s\,+\,0.493\log \sigma_s} \tag{37}$$

The comparison is graphed in figure 7, the difference between the true spreads and censored spreads is the excess spread due to tick size. From the graph it is readily seen that where censored spreads are well above the minimum spread line, then there is little or no difference between true spreads and censored spreads (no excess spread). However, for lower priced stock, where the censored spread is near or on the minimum spread line, there is significant difference between true and censored spreads.

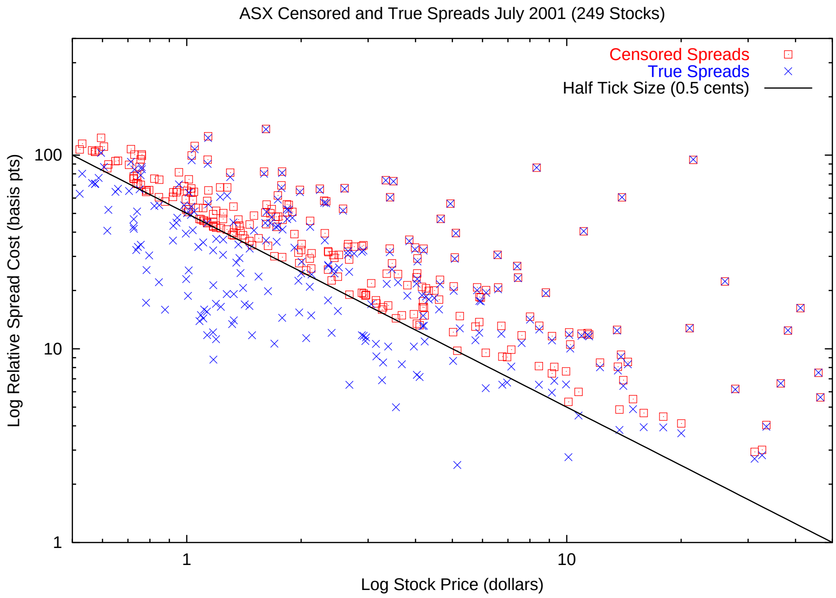

Figure 7: True spreads compared to Censored Spreads, 249 stocks.

Censored Spreads as an Estimate of Observed Spreads

The model of censored spreads developed in this paper should be able to predict observed spreads accurately and without bias. This is readily tested by modelling observed spreads as a linear regression of censored spreads. So the censored spreads are calculated using the results from the LogNormal continuous spread model and fitted to the following linear regression

$$R_s^{\text{observed}} = k\, R_s^{\text{censored}} + b + \varepsilon_s \tag{38}$$

The intercept term (b) in equation 38 is not significant so the regression is refitted without an intercept term and this regression shows that k (slope) is not significantly different from 1. The R^2 of the regression is 0.86 and the 'Hetero-X' statistic confirms heteroscedasticity. So censored spreads are an accurate and unbiased estimate of observed spreads.

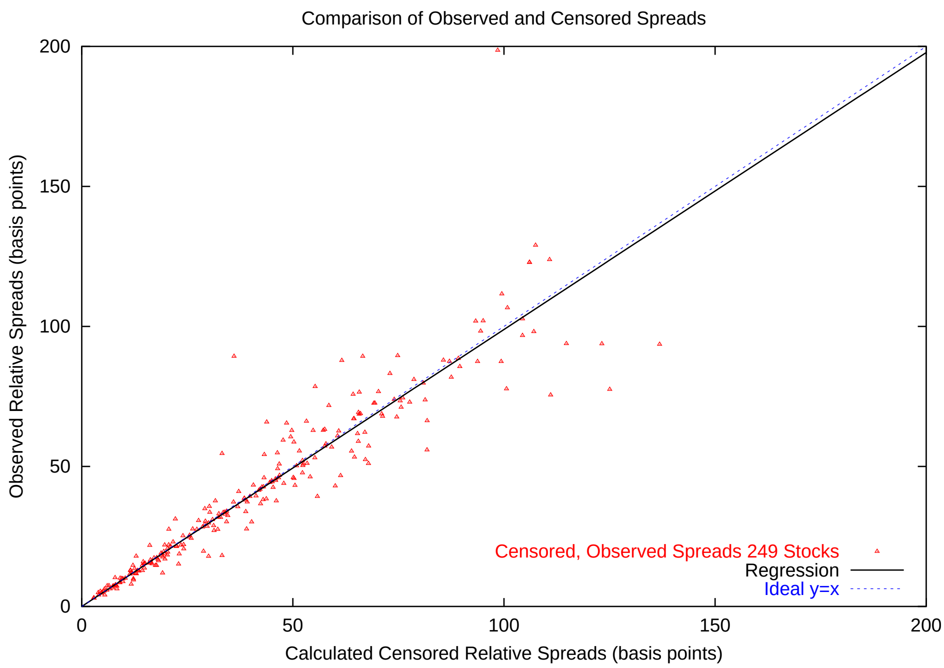

Figure 8: The Relationship between Censored Spreads and Observed Spreads (249 stocks, slope 0.989, slope std error 0.021, no significant intercept)

Summary

The loglikelihood spread regression model suggested by Harris [5] is generalized and applied to effective spread data from the Australian Stock Exchange. This model enables the identification of an unobserved continuous spread distribution related to the cost of the supply of liquidity. This unobserved continuous spread distribution is partitioned ('censored') by market tick size into the observed discrete spreads.

For some higher turnover, low priced stocks traded on the ASX, the 1 cent minimum tick size (stock price $\geq$ \$0.50) is too large and all spreads are observed at the minimum 0.5 cent spread, these spreads are censored by tick size. This implies that ordinary least squares linear regression models of spreads on the ASX are miss-specified because this tick size censoring is not addressed by the regression. The loglikelihood spread regression model overcomes this problem and allows a model of unobserved continuous spreads to be developed.

The continuous regression model is robust. Continuous spread regression models were developed with 4 different parameterized distributions (see table 2) with similar regression results for turnover and volatility, all rejected price as significant. Stock turnover is a proxy for the scarcity of liquidity and increasing turnover results in decreasing spread. Volatility increases the risk of limit order traders (liquidity suppliers) because of the increased likelihood that the stock price will move away from the limit order price before the limit order is executed, so increasing stock volatility increases the unobserved continuous spread. Price is not an important determinant of the unobserved continuous spread and this is intuitively appealing - the spread costs of trading of stock should not be influenced by nominal stock price. For example, in negotiating the cost of supplying \$1m of stock, liquidity providers and demanders are neutral as whether that stock liquidity is supplied as $\$1 \times 1,000,000$ stock units or $\$100 \times 10,000$ stock units.

Not only does the continuous spread regression allow the investigation of the underlying influences on spreads but it also gives us a powerful tool for investigating the effect of tick size on the cost of trading on the ASX. The unobserved continuous uncensored regression can be used as an accurate and unbiased model (equation 38 and figure 8) of observed spreads by explicitly tick-size censoring. Any changes in tick size on the ASX can simply be modelled as a change in the spread censoring function and resultant changes

in stock trading behaviour estimated. The definition of relative spreads as a censored regression and the resultant model of true continuous spreads and censored spreads is new to the microstructure literature.

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