Divergence of Opinion, Speculative Trading and Asset Pricing: Theory and Evidence

Meijin Wang and Jieyu Li[*]

Abstract: This paper attempts to understand the role of turnover in the cross-section of the expected stock returns by theory analysis and empirical test. A three-period model with short-sales constraints and heterogeneous beliefs shows there is a speculative bubble in equilibrium price; the bubble depends on the float (tradable shares of an asset), the investors' risk-aversion and the trading cost; turnover is negatively correlated with expected return due to the speculative bubble in stock price. We then test the impact of turnover in monthly Chinese stock returns, after controlling for the usual factors (firm size, book-to-market ratio and momentum) and for illiquidity costs. The empirical method is Fama-MacBeth type regressions using risk adjusted returns on individual securities similar to the approach of Brennan, Chordia and Subrahmanyam (1998). We find turnover is one of highly robust determinants of the expected returns, and its effect on returns is stronger among the smaller float stocks. These empirical evidences are consistent with the predictions of the model.

Keywords: divergence of opinion, asset pricing, turnover

1 Introduction

If price and quantity are the fundamental building blocks of any theory of market interactions, the importance of trading volume in understanding the behavior of financial markets is clear (Lo and Wang, 2001). Although trading activity does not play a role in the classic asset pricing theory, trading activity indicators are often used in practice to predict future variation in asset returns, and several recent papers have articulated theories that establish a connection between speculative bubbles and trading activity. Following the basic insights of Miller (1977), Harrison and Kreps (1978), Chen, Hong and Stein (2002), Hong, Scheinkman and Xiong (2004), we propose a discrete-time, three-period model to analyze the joint effects of short-sales constraints and heterogeneous beliefs on stock prices and speculative trading. We introduce several assumptions including a continuum of investors, whose population is normalized to one, investors' risk aversion, the limited supply of tradable shares and transaction cost, and analyze how these assumptions affect the speculative bubble and securities' trading. More specifically, at t=1, investors have heterogeneous beliefs about stock's terminal payoff. In the presence of short-sales constraints, the more pessimistic investors will exit the market, which not only makes the stock price at time 1 higher than average valuation of all investors, but also causes the speculative behavior at time 0 based on future heterogeneous beliefs, that is in order to have the opportunity to resell one's shares to other more optimistic investors in the future for a profit, the investors are willing to pay a "speculative premium"[1]. As a result there is a

[*] Meijin Wang: *Lingnan College, Zhongshan University; Email: wangmj@lingnan.net*. Jieyu Li: *Lingnan College, Zhongshan University; Email:jieyuli@126.com. Phone: 13533663728.*

[1] Namely, the price investor willing to pay for the asset exceeds his evaluation for the future dividend flow of the

bubble component as a resale option in the equilibrium price of time 0. We find a larger dispersion of heterogeneous beliefs, a lower risk absorption capacity or a smaller float will produce a bigger speculative bubble companied by higher turnover and price volatility. Increasing trading cost may be able to reduce the bubble component and turnover.

A testable hypothesis can be obtained from the above theory: the turnover rate is negatively correlated with the expected return. In the empirical analysis, we explore the relationship between turnover and cross-sectional returns in Chinese stock market, and see whether the predictions of the model accord with the empirical evidences. From the theoretical model, we know that strong speculation of Chinese A-share market is possibly due to its particular institutes and surroundings, e.g. short-sales prohibition, personal investors dominating, small tradable shares. Traditional liquidity theory[1] cannot alone explain extremely high turnover and its strong effect on returns. Considering lots of literature (e.g. Datar, Naik and Radcliffe (1998), Su and Mai (2004)) have explained the correlation between return and turnover by traditional liquidity theory, we test the effects of turnover on returns after controlling for the illiquidity measure. In this way, we provide a test of the effects of trading activity not for liquidity reasons but for speculative reasons. In addiction, we investigate the coefficients' stability of turnover and other pricing factors by studying sub-samples.

There is not much empirical analysis about divergence of opinion (especially for the emerging market). Chen, Hong and Stein (2001), Diether, Malloy and Scherbina (2001) separately use "breadth of ownership" and "dispersion in analysts' earnings forecasts" as proxies of divergence of opinion. Both of them find the divergence of opinion is positively correlated with turnover rate, and negatively correlated with expected return. Piqueira (2004) reports evidence from U.S. stock market that higher turnover rates predict lower future returns after controlling for the illiquidity factor. Mei, Scheinkman and Xiong (2004) shows that trading activity caused by investors' speculative motive can help explain a significant fraction of the price difference between the A-class and B-class shares in Chinese stock markets.

2 Model

Consider a pricing model of a single risk asset with three dates, t=1, 2, 3. Each unit of asset pays off $\tilde{f}$ at the final date t=2, where $\tilde{f}$ is distributed normally with mean F_0 and variance f^2. Total supply of the asset is Q. The investors trade the asset at t=0 and t=1, whose per-period objective functions are constant-absolute–risk-aversion (CARA) utility. All the Investors are with short-sales constraints of the risk asset, but can borrow or lend at risk-free interest rate of zero. The population of investors is normalized to 1.

At t=0, each investor has the same prior belief. But at t=1, there is divergence of opinion among the investors[2]. Investor i updates his prior belief of $\tilde{f}$ as V_i, and

asset, and the excess part is called as " speculative premium".

[1] A more liquid asset commands higher market price and lower future returns since investors anticipate the future payment of lower trading costs when selling the asset in the future. In other word, the illiquid cost is positively correlated with expected return. Kyle (1985) considers the cross-sectional difference of turnover can reflect illiquid cost discretion among the stocks, thus turnover is negatively correlated with return. " Illiquid cost" include the difference between the a direct transaction cost, bid-ask spread, market-impact and delay and search costs (Amihud and Mendelson, 1991).

[2] Why the beliefs of the investors will diverse? Harris and Raviv (1993) consider it is an common phenomenon that people get different conclusion based on the same facts, which exists not only in the evaluation of the risk assets, but also in the political selection, or horse racing, etc. Lots of economists find the psychological

$V_i \sim N[f^i, s^2]$. That is the investors share the same views concerning the variance of $\tilde{f}$, but have diverse opinions about the mean of $\tilde{f}$. Suppose f^i is uniformly distributed on $[f-h, f+h]$, where f is average estimate of all investors, and h measures the heterogeneity of beliefs. At t=1, investor i chooses X_1^i to maximize his expected utility:

$$MaxE_1^i[-e^{-a(W_1^i + X_1^i(V^i - P_1))}], \tag{1}$$

where a is risk aversion parameter of the investor, W_1^i is the initial wealth at t=1, P_1 is the market price of the asset at t=1. If the market is allowed short-sales, the demand of investor i is $X_1^i = \frac{f^i - P_1}{as^2}$, and the aggregate demand of the market is $D^U = \frac{1}{2h}\int_{f-h}^{f+h}\frac{f^i - P_1}{as^2}df^i$. Let D^U equals the aggregate supply Q, then the equilibrium price in the absence of short-sales constraints is obtained: $P_1^U = f - as^2Q$. However, we assume the short-sales are prohibited in this model, a restriction $X_1^i \ge 0$ must be added to the maximization problem (1). Then the demand of investor i is $X_1^i = Max[\frac{f^i - P_1}{as^2}, 0]$, and aggregate demand of the market is $D^C = \frac{1}{2h}\int_{P_1}^{f+h}\frac{f^i - P_1}{as^2}df^i$. Let D^C equals Q, we get $P_1^C = f + h - 2\sqrt{as^2Qh}$. Note that the restriction is not binding[1] when P_1^C less than the evaluation of the most pessimist, $f-h$. All the investors will enter the market to share the risk of holding asset, as a result the equilibrium price P_1 equals P_1^U, i.e. the average belief of investors f minus the risk premium as^2Q; Only if P_1^C is larger than or equal to $f-h$, relative pessimistic investors exit the market due to short-sales constraints, then P_1 equals P_1^C. So the equilibrium price with short-sales constraints is:

$$P_1 = \begin{cases} f - as^2Q & 0 \le h < as^2Q \\ f + h - 2\sqrt{as^2hQ} & h \ge as^2Q \end{cases}, \tag{2}$$

P_1 satisfies two properties: *firstly*, $P_1 \ge P^U$. This means the equilibrium price with short-sales constraints will not less than the price with short-sales allowed, and P_1 will be strictly larger than P_1^U once the divergence of opinion reaches some extent. *Secondly*, P_1 is a increasing function of h. This implies P_1 will deviate the average evaluation of the investors more and more as the beliefs diverse.

Assume the investors have correct beliefs about $\tilde{f}$ at t=0[2], i.e. $\tilde{f} \sim N(F_0, f^2)$, and they have the same prior beliefs about the distribution of future divergence of opinion as well: the average belief f obeys normal distribution $N(F_0, f^2)$. In other word, although the people expect the future beliefs will diverse, the change of the average valuation at t=0 still surrounds the current expectation F_0; The heterogeneity

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foundation of divergence of opinion from behavior finance. Scheinkman and Xiong(2003) use overconfident as one of the reasons.

[1] "Restriction not biding" means the equal sign of the restriction, $X_i^i \ge 0$, is not come into existence. " biding" means the equal sign is come into existence。 It is immediate that $P_1^C < f - h$ is equivalent to $h < as^2Q$, here restriction not biding; $P_1^C \ge f - h$ equivalent to $h \ge as^2Q$, here restriction biding. Figure 2 and 3 in the following text give the demand of the investors corresponding to the restriction biding or not biding.

[2] Note that the investors are unaware of their evaluation correct. Such assumptions here is to figure out the bubble component is caused by the divergence of opinion, not by the deviation of the average valuation of the people.

of beliefs h is distributed with a density function $g(h)$:

$$g(h)=\begin{cases} h/g^2 & 0\le h<g \\ (2g-h)/g^2 & g\le h\le 2g \\ 0 & ortherwise \end{cases}, \tag{3}$$

where g is the parameter of divergence of opinion. It is immediate that the mean of h, $E[h]=g$, and the variance of h, $Var[h]=g^2/6$, are the monotone increasing functions of g.[1] At t=0, each investor maximize his CARA utility, and the demand of investor i is $X_0^i = Max[\frac{E_0^i[P_1]-P_0}{a\Omega_i},0]$. Since all investors share the same belief, then $E_0^i[P_1]=E_0^j[P_1]$, $\Omega_i=\Omega_j\equiv\Omega$ for arbitrary investor i,j. The market clear condition $\int_i X_0^i = Q$ and $X_0^i=Q,\forall i$ due to the population normalize to one implies the equilibrium price at t=1 as follow:

$$P_0 = E_0[P_1]-a\Omega Q, \tag{4}$$

in order to get the expectation of P_1 at t=0, (2) can be written as:

$$P_1 = f - as^2Q + \begin{cases} 0 & 0\le h< as^2Q \\ (\sqrt{h}-\sqrt{as^2Q})^2 & h\ge as^2Q \end{cases}. \tag{5}$$

Figure 1 insert here

In the view of the investors at t=0, (5) includes two uncertainty terms, that is the f and the subsection function about h. The expectation of f is F_0, shows the investors' valuation of terminal dividend at the moment. The subsection function represents the value of the market reopen at t=1. As depicted in figure 1, this value is like an option, which embodies the right that the owner at t=0 can sell asset to other investors at t=1. When other investors have enough optimistic expectations about the future dividends, namely the heterogeneity of beliefs is large enough (larger than as^2Q), this option can be in the money. Take expectation of (5), and put it into (4), the equilibrium price at t=0 can be obtained:

$$P_0 = F_0 - a\Omega Q - as^2Q + E_0[z(h)], \tag{6}$$

where $z(h)=(\sqrt{h}-\sqrt{as^2Q})^2 I\{h\ge as^2Q\}$, $I(\bullet)$ is the indicator function. The first term in (6) is F_0, the fundamental of the asset. The second term is $a\Omega Q$, risk premium for holding the asset from t=0 to t=1. The last term is option value caused by the investors' speculation on the future heterogeneous beliefs. This option is called as speculative bubble, denoted by B:

$$B = E_0[(\sqrt{h}-\sqrt{as^2Q})^2 I\{h\ge as^2Q\}]. \tag{7}$$

Proposition 1: The size of the speculative bubble increases as the parameter of divergence of opinion g increases, and decreases as the risk aversion coefficient a, the float Q increase.

Proof is given in the appendixes.

[1] The parameter, h, can be assumed to obey natural logarithm distribution, but this will complicate the problem without making any essential differences to the following analysis.

Intuitively, higher is the parameter g, larger is the heterogeneity of the investors' beliefs at t=1. As a result, the option of the asset owner at t=0 will have bigger probability to be in the money, and be more likely to sell the asset to more optimistic investors, acquiring higher profit. Look in this way, the value of the option increases naturally. The role of higher float and higher risk aversion happens to opposition, it takes a greater divergence in opinion in the future for an asset buyer to resell the shares, which means a less valuable resale option today.

Studying the asset pricing based on the divergence of opinion has many merits, one of which is being capable of connecting the trading activity, the price and the price volatility together. Volatility of the price is defined as $\Omega = Var[P_1 - P_0] = Var[P_1]$, replace P_1 with (5), it is immediate that the volatility is composed of the variance of two uncertainty term in (5), i.e. $\Omega = Var[f] + Var[z(h)] = f^2 + Var[z(h)]$. Obviously, the speculative component of the price augments the price volatility. Underside we are going to consider the expected turnover from t=0 to t=1.

Figure 2 and figure 3 insert here

Figure 2 and figure 3 separately describe the demand of investors at t=1 when the short-sales constraints is biding and not biding. X-axis f^i stands for investors' valuation about the terminal dividend, range of which is $[f-h, f+h]$. Y-axis $X_1(f^i)$ denotes the demand of the investors. f_{NT} is valuation of the investor whose demand does not change from t=0 to t=1, the corresponding demand is $X_1^{NT} = X_0^i = Q$. f_{0T} is valuation of the investor whose demand is just zero at t=1, thus f_{0T} only appears in figure 2. Define turnover as $TR = \int_i \left| X_1^i - X_1^{NT} \right| d_i / (2Q)$, i.e. the float divided by trading volume. Calculate the turnover by taking integral according to different intervals of f^i:

$$TR = \begin{cases} \frac{1}{2Q}\left(\int_{f_{NT}}^{f+h} (X_1(f^i) - X_1^{NT})df^i + \int_{f_{0T}}^{f_{NT}} (X_1^{NT} - X_1(f^i))df^i + \int_{f-h}^{f_{0T}} X_1^{NT} df^i\right) & 0 \le h < as^2Q \quad (8.1) \\ \frac{1}{2Q}\left(\int_{f_{NT}}^{f+h} (X_1(f^i) - X_1^{NT})df^i + \int_{f-h}^{f_{NT}} (X_1^{NT} - X_1(f^i))df^i\right) & h \ge as^2Q \quad (8.2) \end{cases}$$

the first integral in (8.1) denotes the trading volume of the asset buyers from t=0 to t=1; the second integral denotes the trading volume of the seller but not quitting the market; the last integral is trading volume of the investors quitting the market. No one quit the market at t=1 When $h \ge as^2Q$, so (8.2) only includes the first two integral of (8.1). Simplify (8), we have:

$$TR = \begin{cases} h^2/(2as^2Q) & 0 \le h < as^2Q \\ (2\sqrt{h} - \sqrt{as^2Q})^2/2 & h \ge as^2Q \end{cases}. \qquad (9)$$

Proposition 2: The expected turnover $E_0[TR]$ and volatility Ω increase as the parameter of divergence of opinion g increases, and decrease as the risk aversion coefficient a, the float Q increase.

Proof is given in appendix.

3 Empirical Analysis

From the combination of proposition 1 and 2, it can be deduced that the turnover will go up as the speculative bubble augments, which implies the stock with higher turnover rate will have lower expected future returns in cross-sectional view. Note that the traditional liquidity theory predicts above relationship as well, so we proof the forecasting ability of the speculative trading on the cross-sectional returns by testing the hypothesis 1.

Hypothesis 1 *Expected cross-sectional returns are still correlated with turnover negatively after controlling for illiquidity cost.*

Specifically, we introduce the incidence of observed zero daily returns, called as "Zero-return Ratio"[1], as a more accurate proxy for illiquidity costs than turnover, testing the effects of turnover on returns after controlling for the illiquidity measure. In this way, we provide a test of the effects of trading activity not attributable to liquidity reasons. Proposition 1 and 2 also show that the speculative bubble and the turnover are both the decreasing functions of the float Q, from which we can deduce that the effect of turnover on return should rise as the float falls. By analyzing the sub-samples partitioned by the float size, we test the following hypothesis:

Hypothesis 2 *The effect of turnover on cross-sectional return is stronger among the smaller float stocks.*

3.1 DATA

The data consist of monthly returns, turnover rate and other firm characteristics for a sample of China's A-share market for the period August 1995 to December 2004. Data source is the China Stock Market & Accounting Research Database (CSMAR). In order to calculate some characteristics, a stock to be included in the sample must satisfy certain criteria: there are at least one year trading and financial data before the date on which the stock enter the sample; and at least twenty-four months' sample points in the sample period. For each stock the following variables are calculated at t month as follows:

TRVR: Average daily turnover at t-1 month.

ILLIQ: Zero-returns Ratio at t-1 month, where Zero-returns ratio is no-price-change days divided by total trading days in a month.

LNSZ: The natural logarithm of the market value of the tradable A-shares at the end of month t-2, it denotes the firm size effect.

BM: Ratio of the book value of equity to the market value of equity. The BM values of July L year to June L+1 year take the end of the previous year L-1market value and book value. Following Fama and French (1992), book-to-market ratio values greater than the 0.995 fractile or less than the 0.005 fractile are set to equal the 0.995 and 0.005 fractile values, respectively.

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[1] Lesmond, Ogden and Trzcinka (1999) argue that if the value of an information signal is insufficient to outweigh the costs associated with transacting, then market participants will elect not to trade, resulting in an observed zero return. A security with high transaction costs will have less frequent price movements and more zero returns than a security with low transaction costs. Lesmond, Ogden, and Trzcinka (1999) and Bekaert, Harvey, and Lundblad (2003) has found "Zero-return Ratio" to be an effective measure of market liquidity in U.S. stock markets and several emerging markets.

RET4-6: The cumulative return over the three months from t-4 to t-6, it proxy for momentum.

FLTRT: Float ratio at month t-1, i.e. tradable share divided by total share.

RSK: Monthly return volatility, it is defined as $\sum_{m=t-14}^{t-2} r_{im}^2 + 2\sum_{m=t-13}^{t-2} r_{im} r_{im-1}$.

All variables involving the price level are lagged by two months in order to avoid biased estimation caused by thin trading. Table 1 reports the grand time-series and cross-sectional means, standard deviations, maximum, medians and minimum of the security characteristics. Comparing the statistics with other emerging markets' in the table 1 of Bekaert, Harvey and Lundblad (2003), it can be seen that the average daily turnover rate (the mean is 1.51375%) of Chinese A-share market is much larger than those of the others emerging markets; and the Zero-returns Ratio is lower than the other emerging markets as well.

Table 2 reports the averages of the month by month cross-sectional correlations of the variables that we use in our analysis. Turnover rate is negatively correlated with ILLIQ, and the degree of correlation with the liquidity measures is not particularly high to suggest that turnover is an accurate proxy for liquidity. The largest correlation with turnover is RSK (positive), which is consistent with the prediction of proposition 2 that "the price volatility and trading activity will increase at the same time when the divergence of opinion becomes larger". Similar to the finding of Lee and Swaminathan (2000), turnover rate is positive correlated with the cumulative return which is proxy of momentum. Note that the firm size indicator LNSZ is negatively correlated with turnover rate and positive correlated with illiquid cost, which is unlikely to consistent with ordinary argument that "larger firm size companies have more liquid".

3.2 METHODOLOGY

We follow the method of Brennan, Chordia and Subrahmanyam (1998) (BCS). By contract with the two-steps method of Fama-Macbeth (1973) (FM), the innovation of BCS is using individual security risk-adjusted excess returns as dependent variables in the second-pass regressions, which can not only get rid of the effect of the common risk factors when estimating the security characteristics, but also avoid the errors-in-variables and data snooping problem associated with forming portfolios as is done in standard Fama and MacBeth (1973) applications.

Whereas Wu and Xu (2004) found the three factors model of Fama and French (1993) is able to give better description of the change of Chinese stocks' cross-sectional returns than CAPM, we use Fama-French factors to risk adjust individual security excess returns. The specifics of the method are as follows.

First, estimating the time series regression using each stock's monthly excess return and the Fama and French (1993) factors:

$$R_{jt} - R_{ft} = a + \sum_{k=1}^{3} b_{jk} F_{kt} + \sum_{k=1}^{3} b'_{jk} F_{kt-1} + e_{jt}, \qquad (10)$$

where R_{jt} is return on security j at month t, R_{ft} is riskless rate, and F_{kt} (k=1,2,3) are the returns on the kth factor at month t.[1] Unlike the rolling regressions in BCS, we estimate the betas in sample for short historical data of Chinese stock markets.[2]

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[1] Riskless interest rate R_{ft} is monthly return of three-month deposit's annual interest rate. F_{kt} (k=1,2,3) includes mkt, smb and hml, where mkt is the excess market return; smb is the factor returns are the returns on a portfolio which is long in small stocks and short in large; hml is the returns on a portfolio which is long in high book-to-market stocks and short in low book-to-market stocks.

[2] As Cochrane (2001) points out the factor betas can be estimated using either rolling regressions or

We adopt the Dimson (1979) procedure with one lag to adjust the estimated factor loadings to help with estimation problems related to thin trading. The risk adjusted excess returns can be calculated from equation (10) as:

$$R_{jt}^{*} = R_{jt} - R_{ft} - \sum_{k=1}^{3}(\hat{b}_{jk} + \hat{b}_{jk}^{'})F_{kt} .$$

The risk adjusted excess returns R_{jt}^{*} are then used as dependent variables in the following cross-sectional pricing regression, which is estimated monthly:

$$R_{jt}^{*} = b_0 + \sum_{m=1}^{M} b_m Z_{mjt} + h_{jt} , \tag{11}$$

where Z_{mjt} is firm characteristic m on security j at month t. The time-series of monthly estimated coefficients and the corresponding variances for the characteristic m, denoted as $\overline{b}_m$ and $\overline{s}_m$, can be obtained from the cross-sectional regression by monthly. We estimate the mean of $\overline{b}_m$ by two ways. The first, in which we call the *raw estimate*, is given by $b_{mr} = (i^{'}\Omega_m^{-1}i)^{-1}i^{'}\Omega_m^{-1}\overline{b}_m$, where i is the unit vector, Ω_m is diagonal matrix, the diagonal elements are $\overline{s}_m$, the cross-sectional estimated variance of characteristic m. Note that although the factor loadings, betas, are estimated with error, this error affects only the dependent variable, R_{jt}^{*}, in the cross-sectional ordinary least squares regression. As long as the errors in the estimated loadings are not correlated with the security characteristics, the raw estimate should be unbiased.[1] However this correlation maybe exists, for the robustness of our results, the monthly regression coefficients on each of the characteristics are regressed in time series on the three common factors to yield what we call the *purged estimate*, which allows for the possibility that the estimation errors in the monthly estimates depend on the factor realizations. This estimator is given by: $b_{mp} = \overline{e}^{'}(\overline{F}^{'}\Omega_m^{-1}\overline{F})^{-1}\overline{F}^{'}\Omega_m^{-1}\overline{b}_m$, where vector $\overline{e} = (1,0,0,0)^{'}$, $\overline{F}$ is the matrix of common risk factors returns augmented by a vector of units. In other word, regressing $\overline{b}_m$ of each characteristic on the three common factors of Fama-French, the constant term from this OLS regression is the purged estimator.

The final equation need to be estimated is (11), characteristics set Z includes TRVR, ILLIQ, LNSZ, BM, RET4-6, RSK and FLTRT. We will test the hypothesis 1 by significant level and direction of the coefficient of turnover rate. We also estimate the equations not including ILLIQ or not including TRVR.

3.3 RESULTS

3.3.1 Full sample results

Table 3 presets estimate results of three models which contain different characteristics as independent variables. When not controlling for illiquidity cost, turnover rate in model 1 has significant and negative effect on return whatever raw estimator or the purged estimator. The coefficient on turnover remains strongly significant and negative after controlling for the measures of illiquidity. Therefore, the empirical evidences are in line with Hypothesis 1, showing the coexistence of

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simply, in sample.

[1] We can see BCS avoided the errors-in-variables problem just by incorporating the estimated errors of beta in the dependent variable R_{jt}^{*}. BCS emphasized there is no *a prior* reason to believe that *errors* in the estimated loadings will be correlated with the security characteristics.

divergence of opinion and short-sales constraints in the stock market, which causes speculative behavior among investors and bubbles in stock prices, is one of reasons for the expected returns being negatively correlated with turnover rates. There is only slightly change in the coefficient of turnover from model 1 to model 3, purged estimator just falls 0.0089, which further explains the majority effect of turnover on return is not because of “liquidity premium”, but of “speculative premium”; the turnover rate contains more information about speculative activity; As the speculative trading increases 1%, the expected return will decrease 0.6143% (See the Purged estimator of TRVR in model 3).

In the model 2, the purged estimator of RSK is significant at 5% level, the coefficient is -.0064, which shows the expected return will decrease instead when the monthly volatility increases. Obviously, it is not consistent with risk compensation theory provided by Merton (1987), but with speculative bubble theory based on divergence of opinion. Proposition 1 and 2 indicate the volatility will increase as bubbles boost up, thus expected return may be negatively correlated with return volatility. When including the turnover rate, RSK becomes insignificant. This suggests both of turnover and volatility contain the information of speculative activity, but turnover should be more efficient as the proxy of speculative bubble.

Other results can be observed in table 3: ILLIQ is significant at least 5% level whatever turnover is included or not, which means there exists illiquid compensation in China’s A-share market; expected return increase .0166% as illiquid cost increase 1% (See Purged Estimator of ILLIQ in model 3). The market value of tradable shares, the book-to-market ratio and the cumulative return still have significant explainable ability on expected return after the excess return has been risk adjusted by three Fama-French factors. Therefore the size effect (LNSZ), the value effect (BM) and the momentum effect (RET4-6) affect the stock return as non-risk characteristics: the return of small size higher than the big size firm, the return of value stock higher than the growth stock, the return of large median momentum larger than the small momentum, which are consistent with known theories and empirical works. But in three models, the coefficients of float ratio are not significant at all. [1]

3.3.2 Results across float size groups

It is deduced from proposition 1 and 2 that the increasing of the float will lessen the speculative bubble and turnover rate. We can see this is the case in table 4. The means of the turnover rates in three sub-samples are .016, .0154 and .014 as the float rise from Q1 to Q3. Compare the results of Q1 with Q3: the coefficient of turnover rate is larger in Q1, monthly volatility and float ratio are also significant at 5% level in Q1; This suggests the speculative trading is more active and has stronger effect on future return among the small float size sample. The pricing factors of stocks with large float size appear to be close to those of the mature markets: liquidity effect, tradable market value, book-to-market ratio and cumulative return are all significant at 1%, monthly volatility and float ratio are insignificant. Thus the hypothesis 2 gets the support of the empirical evidences to some extent.

3.3.3 Results across cumulative return groups

Subrahmanyam (2005) explores whether the cross-section of expected stock

[1] If using the Fama-Matbeth estimator with unit weighting matrix, FLTRT can be significant.

returns is robust across stock groups sorted by past monthly return. Results show that the book-to-market ratio and momentum effects are remarkably robust to sorting on past returns. However, turnover is negatively related to future returns for stocks with abnormally low stock price performance in the recent past, but positively related to returns for well-performing stocks. He consider these evidences support the theory notion that "momentum investing by institutions affects prices, so that trading activity predicts future returns particularly strongly for those stocks that have experienced extreme moves in either direction". Table 5 gives the estimate results in the case of Chinese Stock Market. Contract to the findings of Subrahmanyam (2005), we find that turnover is negatively significant in all three sub-samples. This means Chinese Stock market as an emerging market is different from the maturity markets. Momentum investing by institutions (if exists) doesn't affect prices too much, and the speculative behavior caused by divergence of opinion is dominant in market. From Q1 to Q3, the significant levels of turnover increase, which probably because the investors behavior more overconfident on recent past well-performing stocks, and greater extent of overconfidence will be with larger heterogeneous beliefs and higher speculative sentiment (Scheinkman and Xiong (2003)).

3.3.4 The stability analysis of the characteristics

If the effects of the characteristics on returns are pervasive, they should be evident in reasonable large-sample cross-sections, with little regard to how these cross-sections are constructed. Table 4, table5 and table 6 show that the turnover rate and tradable market value are significant at every sub-sample, which suggests they are remarkably robust to sorting on cross-sections and time periods. For other characteristics, the illiquid cost just keep stability on time periods with 10% significant level; the book-to-market ratio and the momentum effect which are documented as notable factors of return can not retain stability whatever on cross-sections or on time periods.

4 Further Discussion: Introduce Transaction Cost

Assume whatever buying or selling, the investors must pay off $c \geq 0$ per unit during transaction from t=0 to t=1. When choosing his optimal quantity at t=1, the investor will consider the effect of the transaction cost, and his maximum problem becomes as:

$$\underset{X_i \geq 0}{Max} E_1^i[-e^{-a(W_1+X_i(V_i-P_1)-c|X_1^i - X_0^i|)}]. \tag{12}$$

Figure 4 and Figure 5 show the demand of every investor, $X_1(f^i)$, solved from(12). Compare Figure 2 with figure 4, figure 3 with figure5, we see a obvious change on the investors' demand due to transaction cost, i.e. there are c/h people not trading at t=1,and not just unique as in Figure 2 and 3. The valuations of these people are uniformly distributed on $[f_{NT} - c, f_{NT} + c]$.So as the transaction cost becomes higher, there are more people do not participate in trading at t=1.

Similar to solution of section 2, the equilibrium price at t=1 and t=0 are given as:

$$P_1 = \begin{cases} f - as^2Q & 0 \leq h < c + as^2Q \\ f + h - c - 2\sqrt{as^2Q(h-c)} & h \geq c + as^2Q \end{cases}, \tag{13}$$

$$P_0 = \hat{f}_0 - a\Omega Q - as^2Q + E_0[(\sqrt{h-c} - \sqrt{as^2Q})^2 I\{h \geq c + as^2Q\}], \tag{14}$$

then the speculative bubble $B = E_0[(\sqrt{h-c} - \sqrt{as^2Q})^2 I\{h \geq c + as^2Q\}]$; turnover rate and volatility rate are:

$$TR = \begin{cases} 0 & 0 \leq h < c \\ (h-c)^2/(2as^2Q) & c \leq h < as^2Q \\ (2\sqrt{h-c} - \sqrt{as^2Q})^2/2 & h \geq c + as^2Q \end{cases}, \tag{15}$$

$$\Omega = Var[P_1 - P_0] = Var[f] + Var[(\sqrt{h-c} - \sqrt{as^2Q})^2 I\{h \geq c + as^2Q\}]. \tag{16}$$

It is not hard to proof that the proposition 1 and 2 still come into existence. B, $E_0[TR]$ and Ω are all the decreasing function of c, namely improving the transaction cost can reduce the speculative bubble and return volatility, and decrease the turnover as well. However, we argue the increase of the transaction cost maybe has limited effect on the speculative activity, and the decrease of the transaction cost is not always be able to active the market. First, the most popular and feasible tool for the government is modification of the transaction tax, e.g. the stamp duty, but the divergence of opinion which affect the bubble mostly changes dynamically, it is hard for government to do forecast and response. Second, we assume transaction cost, c, is independent with the divergence of opinion similar to the assumption of Scheinkman and Xiong (2003). If the relationship is endogenous, the effects of transaction cost on speculative bubble, volatility and turnover may become weak, or even opposition. For example, Li (2005) finds the transaction tax, the return volatility and the turnover rate have complex relationship among each other in a speculative market. Increase (Decrease) of the transaction tax is not always able to decrease (increase) the volatility and turnover of the stock.

5 Conclusion

Following the basic insights of Harrison and Kreps (1978) etc., we propose a discrete-time, three-period model to analyze the joint effects of short-sales constraints and heterogeneous beliefs on stock prices and speculative trading. We find that there exists speculative bubble component in the equilibrium price; the expanding of bubbles size accompanies the increase of the turnover rate and price volatility; the bubble will enlarge as the parameter of the heterogeneous beliefs goes up, on the contrary, the increase of the risk-aversion coefficient, the float size and the transaction cost will weaken the bubble. The analysis of above theory enlightens us to understand the effect of turnover on cross-sectional return from the view of speculative trading. The empirical evidences are consistent with the prediction of the theory: After controlling the illiquid cost, the turnover rate is still correlated negatively with the cross-sectional expected return, which suggests the speculative activity caused by heterogeneous beliefs and short-sales constraints is an important factor of asset pricing. On the stability analysis, turnover and firm size are robust, but illiquid cost, momentum effect and book-to-market ratio lack of robustness.

The theory and empirical evidences all show that the turnover is one of the feasible proxies for speculative bubble. Relative to other measure indicators, e.g. dispersion in analysts' earnings forecasts, the stock issue ratio and the return volatility, turnover is more convenient, and perhaps more efficient.

Appendix

Proof of proposition 1:

Already know that h obeys a distributed function with density function $g(h)$ (See formula(4)), and $g(h)$ is segmented function. Let $g_1(h)=h/g^2$, $g_2(h)=(2g-h)/g^2$, directly calculating the expectation of the function $z(h)=(\sqrt{h}-\sqrt{as^2Q})^2 I\{h\geq as^2Q\}$:

$$B=E_0(z(h))=\begin{cases}\int_{as^2Q}^{g} z(h)g_1(h)dh+\int_{g}^{2g} z(h)g_2(h)dh & g\geq as^2Q\\ \int_{as^2Q}^{2g} z(h)g_2(h)dh & as^2Q/2\leq g<as^2Q\end{cases} \tag{A.1}$$

where we assume $as^2Q/2\leq g$ to guarantee the probability of the option being in the money doesn't equal zero. Using multi-derivative of (A.1) to judge the monotone of $E_0(z(h))$ on g 、 Q and a, it can be obtained that speculative bubble, B ,is the increasing function of g ,and decreasing function of Q and a .¶

Proof of proposition 2:

The proof process is the same as proposition 1. Directly calculating the variance of the function $z(h)$, we have:

$$\Omega=Var(z(h))=\begin{cases}\int_{as^2Q}^{g}(z(h)-B)^2 g_1(h)dh+\int_{g}^{2g}(z(h)-B)^2 g_2(h)dh & g\geq as^2Q\\ \int_{as^2Q}^{2g}(z(h)-B)^2 g_2(h)dh & as^2Q/2\leq g<as^2Q\end{cases} \tag{A.2}$$

Let $TR_1(h)=h^2/(2as^2Q)$, $TR_2(h)=(2\sqrt{h}-\sqrt{as^2Q})^2/2$, the expectation of the turnover rate, TR ,is:

$$E_0[TR]=$$
$$\begin{cases}\int_{0}^{as^2Q} g_1(h)TR_1(h)dh+\int_{as^2Q}^{g} g_1(h)TR_2(h)dh+\int_{g}^{2g} g_2(h)TR_2(h)dh & g\geq as^2Q\\ \int_{0}^{g} g_1(h)TR_1(h)dh+\int_{g}^{as^2Q} g_2(h)TR_1(h)dh+\int_{as^2Q}^{2g} g_2(h)TR_2(h)dh & as^2Q/2\leq g<as^2Q\end{cases} \tag{A.3}$$

Using multi-derivative of (A.2) and (A.3) to judge the monotone of Ω and $E_0[TR]$ on g 、 Q and a, it can be obtained that price volatility and the turnover rate are both the increasing functions of g ,and decreasing functions of Q and a .¶

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Table 1
Summary Statistics

This table represents the main statistics, including the means, standard errors, minimum values, median values and maximum values, for the total sample stocks over 113 months from Aug. 1995 through Dec. 2004.

	TRVR	ILLIQ	LNSZ	BM	RET4-6	FLTRT	RSK
Mean	.0151375	.0304999	13.49955	.3106235	.0195081	.3490608	.2020412
S.E.	.0172305	.0428009	.7809395	.1720212	.218374	.1438732	.3587492
Minimum	.0000636	0	10.43998	.005	-1.189123	.0147	.000544
Median	.0093195	0	13.50915	.2764	-.008352	.3394	.1130152
Maximum	.3279952	.5	16.76708	.995	4.016094	1	15.49178

Table 2
Correlation Matrix of Firm Characteristics

This table presents time-series of monthly cross-sectional correlations between the firm characteristics (including the risk-adjust return) used in pricing regressions.

	ADJRET	TRVR	ILLIQ	LNSZ	BM	RET4-6	FLTRT	RSK
ADJRET	1	-0.0646	0.024	-0.039	0.0212	0.006	0.0046	-0.0279
TRVR		1	-0.1176	-0.1244	0.0129	0.0646	-0.0317	0.2088
ILLIQ			1	0.0128	0.0548	-0.016	0.004	-0.0636
LNSZ				1	0.2318	0.1345	0.3542	0.0491
BM					1	0.0793	0.1624	-0.0864
RET4-6						1	0.011	0.1409
FLTRT							1	0.0437
RSK								1

Table 3

Cross-sectional Regression Estimates of Equation (11)

The estimates in the column labeled "Raw" are time series averages of the monthly estimates, while those in the columns labeled "Purged" are the intercept terms from regressions of the monthly estimates on the factors. T-statistics are in parentheses. *, **an ***separately denote the significant levels of 10%, 5% and 1%.

	Model 1		Model 2		Model 3	
	Raw	Purged	Raw	Purged	Raw	Purged
Intercept	.0758***	.0866***	.0559***	.0666***	.0748***	.0857***
	(4.53)	(5.39)	(3.58)	(4.55)	(4.49)	(5.36)
TRVR	-.6421***	-.6232***			-.6326***	-.6143***
	(-8.07)	(-7.98)			(-7.99)	(-7.89)
ILLIQ			.0293***	.0262***	.0179***	.0166**
			(4.24)	(3.71)	(2.89)	(2.58)
LNSZ	-.0053***	-.0062***	-.0043***	-.0052***	-.0053***	-.0062***
	(-4.28)	(-5.34)	(-3.61)	(-4.69)	(-4.27)	(-5.34)
BM	.0123***	.0141***	.0118***	.0135***	.0118***	.0136***
	(3.49)	(3.98)	(3.23)	(3.75)	(3.35)	(3.87)
RET4-6	.0149***	.0136***	.0129**	.0121**	.0146***	.0134**
	(2.80)	(2.55)	(2.38)	(2.23)	(2.77)	(2.52)
RSK	-.003	-.0033	-.0056*	-.0064**	-.0028	-.0031
	(-1.09)	(-1.20)	(-1.84)	(-2.05)	(-1.03)	(-1.12)
FLTRT	.0037	.0041	.0047	.005	.0038	.0042
	(1.10)	(1.19)	(1.42)	(1.44)	(1.14)	(1.22)

Table 4

Regression estimates of the sub-samples by float size (FLTSZ) grouping

For each cross-section, divide the stocks into three groups according to FLTSZ, i.e. small(30%), neutral(40%), large(30%). Then Q1 (Q3) denotes Small (Large) group including all cross-sectional small (large) groups, and Q2 denotes Neutral group including the cross-section stocks not in Q1 and Q3. FLTSZ=monthly tradable market value/monthly closing price. From Q1 to Q3, the means of turnover rate are .01596, .01536 and .01401in turn. We do regression for each sub-sample by BSC methodology.

	TRVR	ILLIQ	LNSZ	BM	RET4-6	RSK	FLTRT
Q1	-.6371***	.00002	-.0109***	.0092	.0086	-.0087**	.0174**
Small	(-6.79)	(0.00)	(-4.28)	(1.25)	(1.19)	(-2.30)	(2.55)
Q2	-.6495***	.0198*	-.0104***	.0116***	.0167***	-.0016	.0049
Neutral	(-7.16)	(1.91)	(-4.29)	(2.71)	(3.01)	(-0.60)	(1.10)
Q3	-.5983***	.0247***	-.0042***	.0093***	.0177***	.0005	-.0021
Large	(-6.44)	(2.92)	(-2.65)	(2.65)	(2.90)	(0.12)	(-0.49)

Table 5

Regression estimates of the sub-samples by accumulated return (RET2-12) grouping

The division method is the same as the method of table 4, but here according to RET2-12. From Q1 to Q3, the means of turnover rate are.0133115, .0148592 and .0173267in turn. We do regression for each sub-sample by BSC methodology.

	TRVR	ILLIQ	LNSZ	BM	RET4-6	RSK	FLTRT
Q1	-.5881***	.0276**	-.0077***	.0151***	.0118	.0076	.0086*
Small	(-5.75)	(2.56)	(-5.16)	(2.96)	(1.46)	(1.02)	(1.86)
Q2	-.7093***	.017*	-.0073***	.0074*	.0134**	-.0037	.0049
Neutral	(-7.77)	(1.92)	(-4.91)	(1.66)	(2.28)	(-0.59)	(1.17)
Q3	-.6308***	.0114	-.0042***	.01668***	.0126**	-.0011	-.0005
Large	(-8.06)	(0.99)	(-3.83)	(3.44)	(2.54)	(-0.45)	(-0.11)

Table 6

Regression estimates of the sub-samples by period grouping

	TRVR	ILLIQ	LNSZ	BM	RET4-6	RSK	FLTRT
1995~	-.6769***	.0246*	-.0113***	.0118	.0204***	.0007	.0246***
1999	(-6.25)	(1.84)	(-5.74)	(1.48)	(2.89)	(0.24)	(3.41)
2000~	-.5951***	.0145*	-.0051***	.0152***	.0091	-.0059	-.0028
2004	(-5.41)	(1.95)	(-3.52)	(4.29)	(1.35)	(-1.16)	(-0.80)

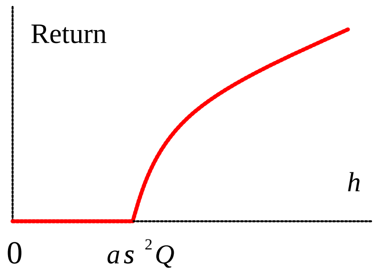

Figure 1 Payoff of the Option

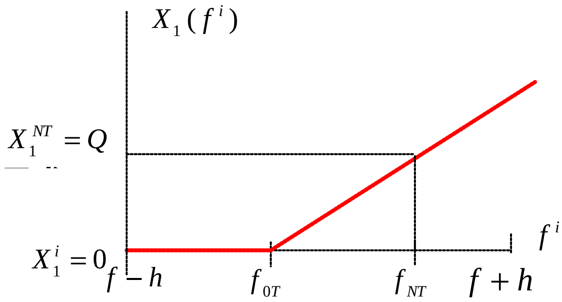

Figure 2 Demand of the investors when the short-sales constraints are binding.

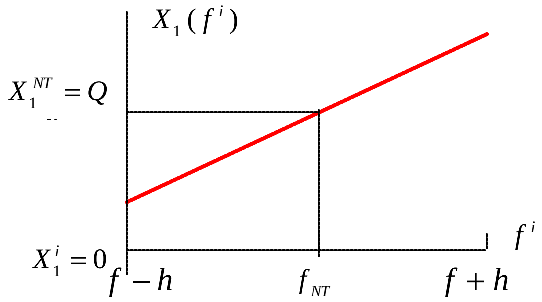

Figure 3 Demand of the investors when the short-sales constraints are not binding.

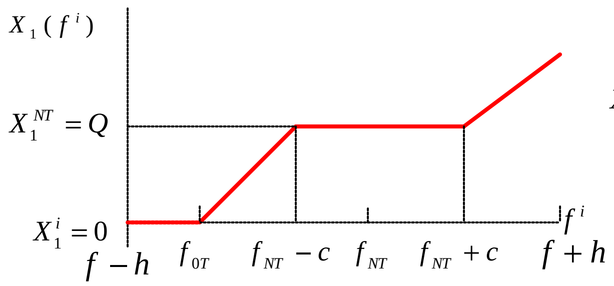

Figure 4 Demand of the investors when the transaction cost exists and short-sales constraints are biding.

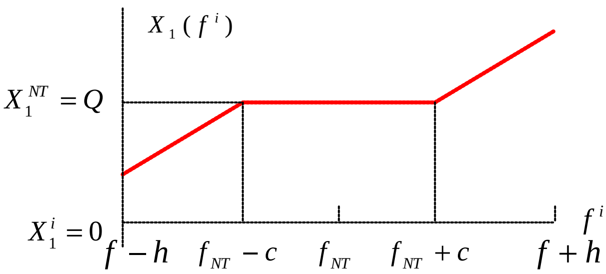

Figure 5 Demand of the investors when the transaction cost exists and short-sales constraints are not biding.