Asset Illiquidity and High-water Marks*

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Abstract

In this paper we provide a rationale for the use of high-water mark provisions to adjust the performance fees of investment managers. In our model, illiquid assets exhibit return reversals, where interim losses are followed by larger gains. Liquidation risk thus arises because investors may prematurely withdraw their capital and pursue outside opportunities. A high-water mark precludes existing investors from paying performance fees until past losses have been recovered, thereby increasing the marginal cost of leaving the fund following poor performance. Consequently, a high-water mark allows managers of illiquid assets to retain investors when liquidation is most costly, and induces investors to commit their long-term capital for the fund. Using a large data set on hedge funds, we find that high-water marks are more common among funds investing in illiquid assets, as proxied by self-reported style categories, redemption restrictions, and return reversals.

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1 Introduction

Asset illiquidity and investor flows play a central role in the performance and design of open-ended investment companies. Investors' decision to buy or sell fund shares may lead to costly trades in the fund's assets. For example, Edelen (1999) finds that investor flows can reduce mutual fund performance by as much as $1 - 2\%$ per year. Chordia (1996) and Nanda et al. (2000) develop models and show that these costs can be reduced by imposing explicit restrictions, or "load charges," on investor flows. Therefore, illiquid assets can be managed efficiently by open-ended mutual funds, but only in conjunction with illiquid fund shares.

In this paper, we examine how the design of hedge fund fee structure allows funds to manage different types of assets. Like mutual funds, hedge funds are open-ended investment companies. Unlike mutual funds, however, hedge funds invest in a wider range of assets that have much less liquidity. Therefore, the costs of informationless trading are larger. Not surprisingly, hedge funds impose severe restrictions on investor flows, and these restrictions are greatest for funds holding the most illiquid assets.[1] In addition, the typical hedge fund investor pays a performance-based fee, often equal to 20% of any profits made in the current period. Some hedge funds also include a "high-water mark" (HWM hereafter) provision, which stipulates that the fund manager must recover past losses before collecting a performance fee.

We demonstrate that a HWM-adjusted fee structure provide an efficient mechanism to manage illiquid assets. We first develop a multi-period model of active portfolio management and fund flows in the hedge fund industry. Fund managers investing in illiquid assets face liquidation risk, as investors may withdraw capital prematurely following (temporary) losses, and force a costly liquidation of funds' assets. This liquidation risk may be severe enough for managers to forego illiquid assets (e.g., Pontiff 1996; Shleifer and Vishny 1997). Our model shows that a HWM induces investors to commit their capital to for long-term management, and therefore reduce the liquidation risk of managing illiquid assets. With a large data set on hedge funds, we also find empirical support of our model's predictions.

In our multi-period model, a manager must decide and commit to a compensation (or fee) structure at the fund's inception, and also decide to invest the fund's capital in one of two mutually exclusive assets: A liquid asset exhibits independent and identically distributed returns over each performance

[1] Aragon (2006) finds that share restrictions allow hedge funds to efficiently manage illiquid assets, and that these benefits are captured by investors as a share illiquidity premium.

period, while an illiquid asset exhibits return reversals, where interim losses are ultimately followed by larger gains. The manager faces a more costly liquidation risk when investing in the illiquid asset because, should investors decide to leave the fund following poor performance, this action triggers a liquidation of the fund's assets precisely when the fund's expected returns are the highest. The manager thus has an incentive to adopt a less lucrative fee structure, ex-ante, in order to retain investors, thereby reducing the liquidation risk.

Conditional on the manager's decision to hold the liquid or illiquid asset, he can reduce liquidation risk either by lowering the percentage performance fee (say, from 20% to 18%) or by using a HWM when calculating performance fees. Our main result is that the use of a HWM provides the most efficient way for managers to reduce liquidation risk associated with illiquid assets. The intuition is that, while the manager has an incentive to retain investors at every state of the interim stage when managing either asset, this incentive is greatest when managing the illiquid asset following poor performance.

A fee structure modified by a HWM is the optimal method for such a manager to maximize expected fees, because the impact of the HWM in retaining investors is *state-dependent*. By committing to recover past losses before earning a performance fee, a HWM lowers existing investors' marginal cost of staying with the fund when fund performance has been poor. However, the HWM has no impact on fees or investors' after-fee returns when performance has been good. Therefore, a HWM allows managers to retain investors when liquidation is most costly while charging a relatively high (as compared to the no-HWM scenario) percentage fee and earning higher total fees when the performance has been good.[2]

On the other hand, when returns are independent and identically distributed (i.i.d.), as in the case for the liquid assets, the manager does not have a preference for retaining investors following poor or good performance. Therefore, this manager never finds it optimal to use a HWM to reduce liquidation risk. Instead, the efficient method for these managers to retain investors and maximize fees is to lower the percentage fee, because doing so generates a *state-independent* impact on investors' marginal cost of staying with the fund.

[2] Similar to some hedge funds, private equity funds also face liquidation risk due to their holdings of illiquid assets (e.g., shares of private, start-up firms). These funds typically do not charge performance-based fees until they "exit" the private firms. This infrequent payment of fees serves a similar purpose as how HWMs modify incentive fees in hedge funds (which are paid every six months to one year): Short-term gains and losses of the funds are less important relative to cumulative, long-term returns of the funds.

In the equilibrium, when illiquid asset managers face moderate liquidation risk, they set a HWM to modify performance fees, while managers do not use a HWM when managing liquid assets. However, if liquidation risk is sufficiently high, we show that even with the HWM managers will decide against investing in illiquid assets altogether, despite the fact that this type of assets is expected to deliver a higher *cumulative* return than liquid assets.[3]

With a sample of over $3,500$ hedge funds from the TASS Tremont database, we empirically examine our model's predictions. First, we find that HWMs are more common among funds that hold illiquid assets, as proxied by the funds' self-declared style categories (e.g., convertible arbitrage, short selling, and emerging markets), and the use of share restrictions, such as a lockup provision, and redemption notice periods. Chordia (1996) shows that share restrictions are used by funds to screen for longer-horizon investors, while Aragon (2006) finds that hedge funds use share restrictions tend to invest in illiquid assets. Hence, our empirical results are consistent with our prediction that HWMs are used by illiquid asset managers. Second, we find that the average percentage incentive fee for HWM funds is higher than that for non-HWM funds, consistent with the model prediction that HWM's and (percentage) incentive fees are substitutes.

According to our model, a HWM is valuable for funds that are more likely to illustrate short-term losses *and* long-run mean reversion after losses. We find some support for this prediction at the style-level. Within a specific investment style, HWMs are used more frequently by funds that illustrate *negative* autocorrelations in returns, especially after poor performance in the preceding period, over quarterly or longer horizons.[4] We interpret negative autocorrelations after losses over longer horizons for HWM funds as evidence supporting our model.

Despite growing interests among researchers and regulators on hedge funds, there is to date little work on the optimal fee structure in hedge funds. Goetzmann et al. (2003) evaluate the cost of a HWM-adjusted fee structure to investors, taking as given the fund's fee structure and investment decisions.[5] There is also a strand of literature on the risk-taking behavior of fund managers. For

[3]Brunnermeier and Nagel (2004) provide evidence that liquidation risk led many large hedge funds to avoid short positions in technology stocks during the late 1990s.

[4]Getmansky, Lo, and Makarov (2004) find evidence of positive return autocorrelation using monthly hedge fund returns. We find similar evidence at the monthly horizon, and focus our analysis for longer horizon (quarterly, semi-annual, annual) returns.

[5]Our paper also relates to the strand of literature on agency problems and contracting in the mutual fund industry. For example, Huberman and Kandel (1993), Heinkel and Stoughton (1994), and Das and Sundaram (2002) examine how fee structure based on performance-based fees can signal fund type and resolve the problems of asymmetric information

example, in the models of Hodder and Jackwerth (2005) and Panageas and Westerfield (2004) with exogenous fee structures (with the HWM), the authors demonstrate that the use of HWMs can reduce the risk-taking behavior of risk-averse fund managers. By contrast, in our model of risk neutral managers and investors, we illustrate how HWMs can arise endogenously to solve the problem of long-term capital commitment due to the liquidation risk. In addition, we provide empirical evidence in support for our model.

The rest of the paper is organized as follows. In Section 2, we develop a multi-period model of the hedge fund industry with fund flows, and demonstrate how the addition of a HWM to a performance fee can induce investors to commit their long-term capital. Section 3 presents empirical tests on our model predictions. Section 4 concludes. All the proofs are left to the Appendix.

2 Model of Hedge Fund Industry

Our two-period model is a *partial* equilibrium in that funds' investment and fee structures do not affect interest rates and the aggregate economy. We take as exogenous the general contract structure of a performance fee that is paid out as a fixed percentage (endogenously determined) of profits earned in a given period. We also allow funds to include a HWM provision. A HWM stipulates that the manager is not entitled to receive a performance fee until all previous losses have been recovered. All agents in the model (fund managers and investors) are risk neutral. [6]

2.1 Elements of the Model

Consider a representative fund manager, who is endowed with assets, and a representative investor with an initial wealth of \$1. Both the fund manager and investor are risk neutral and do not discount payoffs. The following list of conditions, maintained throughout the model, along with Figures 1-A and 1-B, describes the assets and payoffs.

A) The fund manager is endowed with two types of mutually exclusive assets: First, an illiquid asset (type I asset) yields a two-period net return of R_I per \$1 investment; and second, a liquid asset (type L assets) yields a two-period net return of R_I; both assets require an investment of \$1 to

on fund manager types.

[6] While risk aversion affects a manager's incentive in risk taking (e.g., Hodder and Jackwerth 2005; Panageas and Westerfield 2004), it is not the focus of our model and does not qualitatively change our results.

start; the manager may start the fund only at Time 0; if the manager chooses against starting the fund, he earns a reservation payoff of $\underline{U}$;

B) At date 0, the investor chooses to invest either in the fund or in an outside opportunity that pays off a net return of r_0 at date 1. The investor's outside opportunity at Time 1 is stochastic. At Time 1, the investor learns the realization of his outside opportunity that will yield a net return of $\widetilde{r}$ in Period 2: With probability θ the return is high, $\widetilde{r} = r \in [0, \underline{U}/\theta]$, and otherwise the return is low, $\widetilde{r} = 0$; the random return $\widetilde{r}$ is *independent* of funds' returns in either period, and its realization is observed at Time 1; the investor's outside opportunity yields a net return of 0 during Period 1.

Insert Figures 1-A and 1-B here.

Figure 1-A describes the timeline and payoffs of a fund that invests in the liquid asset (type L fund). The fund first period return is observed at Time 1, at which point fund flows may occur, while the second period return and final payoffs are realized at Time 2. In each of the two periods, the asset's return is either high (up factor $u > 1$) with probability p_L, or low (down factor $d \equiv 1/u < 1$) with probability $1 - p_L$; given that the returns in the two periods are i.i.d., the net return over the two periods is $R_L = [p_L u + (1 - p_L) d]^2 - 1$.

Figure 1-B depicts the timeline and payoffs of a fund that invests in the illiquid assets (type I fund). In Period 1, the asset's return is either u with probability p_I or d with probability $1 - p_I$. Following good performance in Period 1 ($\$1 \to \$u$), the illiquid assets behave exactly like the liquid assets and with probability p_L $(1 - p_L)$ the fund will have a good (bad) performance in Period 2 and reaches $\$u^2$ ($\$1$). Following poor performance in the first period ($\$1 \to \$d$), with probability q the illiquid asset will "rebound" (or illustrate return reversal) in the second period and reach the highest possible level, $\$u^2$; with probability $1 - q$, the asset again evolves as the liquid asset does in Period 2: with probability p_L it will have a positive return ($\$d \to \$1$) or otherwise it will have a negative return ($\$d \to \$d^2$). To summarize,

$$R_I = p_I \left[p_L u^2 + (1 - p_L)\right] + (1 - p_I) \left[q u^2 + (1 - q) \left(p_L + (1 - p_L) d^2\right)\right] - 1 \tag{1}$$

is the *before-fee*, *net* two period return of investing in the illiquid asset.

Assumption 1 a) $p_L \geq \underline{p}_L \equiv \frac{u\sqrt{1+\underline{U}}-1}{u^2-1}$; b) $p_I < p_L$;

and c) $q \geq \underline{q} \equiv \max \left\{ \frac{p_L - p_I}{1-p_I}, \frac{[p_I(u^2+1)-1][p_L(u^2-1)+1]}{(1-p_I)[u^4-(p_L(u^2-1)+1)]} \right\}$.

Assumption 1a) implies that $R_L \geq \underline{U}$, so that the fund manager prefers to participate in the fund industry (and managing the liquid asset) and earn fees over and above their reservation payoffs. We further assume that the manager has all the bargaining power in setting up their fee structure at Time 0, so that the investor will earn his reservation payoff, as defined by the expected return on the outside opportunity.[7]

Since $p_I < p_L$ (Assumption 1b), investing in the illiquid asset is more likely to generate a loss in Period 1 than investing in the liquid assets. However, Assumption 1c), in particular, $q \geq \frac{p_L - p_I}{1-p_I}$, implies that the cumulative return of the illiquid asset over two periods is higher than that of the liquid asset (see Appendix A.1). The fact that the illiquid asset has a lower expected return than the liquid asset in Period 1 can be regarded as the cost in investing illiquid assets. Taken together these assumptions highlight the problem of managing illiquid assets: The fund faces higher liquidation risk in the short run.

Assumption 1c) also implies that the (ex ante) expected return for investing in the illiquid asset in Period 2 is higher following a loss than following a gain in Period 1 (see Appendix A.1). Following poor performance in Period 1 ($\$1 \rightarrow \$d$), the return reversal of the illiquid asset (from $\$d$ to $\$u^2$ with probability q) means the fund can realize a very high return in Period 2. As long as q is high enough, the likelihood of going from $\$d$ to $\$u^2$ more than compensates the possible losses in Period 1. In what follows we show that this assumption allows the HWM to arise endogenously to help the manager to retain investors after a first period loss.

The manager charges a performance-based fee, as a *percentage* (f) of the fund's net profits made in the past period, and can choose to set a HWM at \$1, the initial assets under management at $t = 0$. In Period 2, if the fund has generated a positive return in Period 1 so that the value of assets is $\$u$, as is done in practice the HWM will be automatically reset to $\$u$, so that the fund will not receive any fees in the second period unless the end-of-period assets level is above $\$u$. However, in our two-period model, only the Time 0 HWM has an impact on funds' actions and investors' payoffs. On the other hand, if the fund has incurred a loss during Period 1, the HWM (\$1) prohibits the fund to earn any fees in Period 2 unless the total value of assets of the fund exceeds \$1, the initial assets value at $t = 0$.

[7]This assumption holds if, for example, the amount of available investment capital is more than the total funding needs of the hedge fund industry.

Assumption 2 a) The manager's fee structure is announced at Time 0 and is publicly available; while the *choice of assets* is private information; b) the costs of attracting new investors at Time 1 is prohibitively high; c) funds cannot renegotiate fee structure with investors at Time 1.

Assumption 2a) conforms with industry practice of outlining the management contract in the limited partnership agreement. Assumption 2b) indicates it is prohibitively costly for a fund to attract new investors after Period 1. On the other hand, there can be an outflow of capital at Time 1 as investors withdraw their capital from the fund to pursue outside investment opportunities, which reduces fees earned in Period 2 and can potentially force the fund to shut down.[8] In our model, investors withdraw their capital from a fund by, for example, redeeming shares at Time 1, and pursue (random) alternative investment opportunities whenever the latter yields a higher return. Risk-neutral investors will leave 100% of the remaining capital in the fund if the expected return of the fund is the same or higher than that of the outside opportunity, otherwise they will withdraw all of the capital. Allowing for partial withdrawal will not change our main results.[9]

Assumption 2c) implies that there will be no renegotiation of fee structure announced at Time 0, as commonly observed in practice. Renegotiation is costly for two reasons. First, information on fund's past and future (expected) returns as well as that of investor's outside opportunity may be observable to both parties, but not *verifiable* by a third party (e.g., a court). This problem should be more important if the fund invests in illiquid assets (e.g., private equity or foreign securities). Second, opportunistic behaviors are likely to occur during renegotiation: For example, investors have an incentive to demand a lower fee whenever the realization of their outside opportunity is high. This type of behavior will either delay or cause a breakdown of the renegotiation process, and will affect the manager's ex ante incentive to exert effort (in evaluation of assets and trading strategies prior to the start of the fund).[10] The root of the problem is the lack of credible commitment device on the investors part, so that fund managers are ensured that their effort will be paid off in the second period.

[8] This assumption also rules out the possibility that investors can wait till Time 1 to invest in a fund. In practice, most funds use a "share equalization method," where funds will reset the HWMs for investors arriving *after* the inception of the funds.

[9] Fund inflow can be introduced to our model, for example, by assuming that funds can find new investors at a cost, who learn about the funds' performance in Period 1 before investing at Time 1. In addition, assuming decreasing returns to scale for investing in the funds and outside opportunies yields interior solution for fund size in our risk neutral model. See Berk and Green (2004) for more details on a model of mutual fund flows.

[10] See, for example, Hart and Moore (1988, 1990) for models of renegotiation.

To summarize, given that there will be no fund inflow or renegotiation of a fund's fee structure at Time 1, a fund manager, chooses the fee structure (the percentage incentive fee and the use of HWM) at Time 0; while investors first decide at $t = 0$ whether to invest in a (randomly matched) fund, followed by their withdrawal decision at $t = 1$.

2.2 The Problem of Managing the Illiquid Asset

2.2.1 Investors' After-fee Returns and Investment Decisions

To facilitate exposition, we use backward induction and start at $t = 1$, after fund performance and the realization of $\widetilde{r}$ have become publicly available. We begin with the case when the manager has chosen the illiquid asset and set a percentage fee f without a HWM, and the fund had poor performance in Period 1 (at node $1d$ in Figure 1-B).[11] Due to the bad performance in Period 1, no fees are paid leading to node $1d$, and the fund begins Period 2 with total value of assets $\$d$. Investors' expected, after-fee, net payoff of remaining in the fund for Period 2 is:

$$\pi_{I2}(f|\, 1d) = q\left(u^2 - d\right)(1-f) + (1-q)\left[p_L(1-d)(1-f) - (1-p_L)\left(d - d^2\right)\right], \tag{2}$$

where *second* period fees are paid along the path $d \to u^2$ and $d \to 1$, but not along the path $d \to d^2$. If the fund's first period performance is good (node $1u$ in Figure 1-B), a fee in the amount of $(u-1)f$ is paid out at Time 1, and the fund begins Period 2 with total assets value $\$\,(u-(u-1)f)$. Investors' expected, after-fee net payoff in Period 2 at this node is then:

$$\begin{aligned}\pi_{I2}(f|\, 1u) &= p_L(u-1)(u-(u-1)f)(1-f) - (1-p_L)(u-1)(u-(u-1)f) \\ &= (u-1)(u-(u-1)f)\left[p_L(1-f) - (1-p_L)\right].\end{aligned} \tag{3}$$

As mentioned before, in our two-period model, the HWM (set at \$1) has no impact on fees or after-fee returns unless the fund has incurred a loss in the first period. This implies that, with the HWM the investors' after-fee payoff is the same at node $1u$ as compared to the no HWM case, or $\pi^H_{L2}(f|\, 1u) = \pi_{I2}(f|\, 1u)$. However, compared to (2), investors' after-fee payoff is higher at node $1d$ with the HWM:

$$\pi^H_{I2}(f|\, 1d) = q\left[\left(u^2 - d\right) - \left(u^2 - 1\right)f\right] + (1-q)\left[p_L(1-d) - (1-p_L)\left(d - d^2\right)\right], \tag{4}$$

[11] In terms of notations, expressions for fees and other payoffs with a HWM are indicated by a superscript "H," while expressions without such a superscript denote fees and payoffs with only the IFs.

where fees are paid only along the path of $d \rightarrow u^2$, where the fees are calculated based on the total gains of the fund, *net of losses*, or $\$\left(u^2-1\right)$. By contrast, in the case of no HWM (equation 3 above), fees are paid based on the total gains of $\$\left(u^2-d\right)$, and hence the larger the size of the losses in Period 1, the higher the fees earned in Period 2. The HWM increases investor's after-fee return in Period 2 for a given f along the path of $\$d \rightarrow \$1$, where the fund recovers losses from Period 1 without generating any profits, and the HWM dictates that no fees can be paid out of the fund.

Second, when the liquid asset is chosen at $t=0$, the after-fee payoffs for the investors in Period 2 following poor performance in Period 1 (node $1d$ in Figure 1-A, fund begins Period 2 with $\$d$ under management) is:

$$\pi_{L2}\left(f|\,1d\right)=p_L\left(1-f\right)\left(1-d\right)-\left(1-p_L\right)\left(d-d^2\right), \tag{5}$$

where, once again, fees are paid on the path $d \rightarrow 1$ but not on the path $d \rightarrow d^2$. The after-fee return in Period 2 at node $1u$ (fund begins Period 2 with $\$\left(u-\left(u-1\right)f\right)$ under management), $\pi_{L2}\left(f|\,1u\right)=\pi_{I2}\left(f|\,1u\right)$. In terms of the impact of the HWM, we again have $\pi^H_{L2}\left(f|\,1u\right)=\pi_{L2}\left(f|\,1u\right)$ at node $1u$. Following a loss in Period 1, compared to (5), the after-fee return at node $1d$ is again higher:

$$\pi^H_{L2}\left(f|\,1d\right)=p_L\left(1-d\right)-\left(1-p_L\right)\left(d-d^2\right), \tag{6}$$

where the fees along the path $d \rightarrow 1$ is again waived due to the HWM.

Investors' decision at Time 1 is to compare expected returns of staying in the fund and with that of pursuing the outside opportunity. If a HWM is not used, at node $1u$ investors will be comparing $\pi_{I2}\left(f|\,1u\right)/\left[u-\left(u-1\right)f\right]$ with the realization of $\widetilde{r}$ if illiquid assets have been chosen, or $\pi_{L2}\left(f|\,1u\right)/\left[u-\left(u-1\right)f\right]$ with $\widetilde{r}$ if liquid assets have been chosen; at node $1d$, they will be comparing $\pi_{I2}\left(f|\,1u\right)/d$ or $\pi_{I2}\left(f|\,1d\right)/d$ with $\widetilde{r}$. Similar comparisons can be defined when a HWM is included in the fee structure.

Working backwards, investors' expected, after-fee net payoff investing in a fund that invests in the illiquid asset in Period 1 is:

$$\pi_{I1}\left(f\right)=p_I\left(u-1\right)\left(1-f\right)-\left(1-p_I\right)\left(1-d\right),$$

and payoff investing in a fund with the liquid asset, $\pi_{L1}\left(f\right)$, can be similarly defined. There are two issues regarding investors' decision at Time 0. First, investors are willing to invest in a fund at Time 0 if the *two-period* expected return, including the (valuable) option to withdraw at $t=1$, is higher than that of the cumulative return of investing outside the fund (earning a gross amount of $\$E(1+\widetilde{r})$). Second, since funds do not allow investors to invest at $t=1$ (i.e., funds are closed after $t=0$), by not investing in the fund at $t=0$, investors forego the option to invest in any fund in the second period and receiving potentially much higher return than $(1+\widetilde{r})$. The following lemma summarizes investor's expected payoffs at different time periods.

Lemma 1 a) *The investor's Time* 1 *after-fee payoff, given fee percentage* f *and no HWM, is:*

$$\Pi_{j2}(f) \equiv p_j \cdot \max\{\pi_{j2}(f|1u), \widetilde{r}[u-(u-1)f]\} + (1-p_j)\cdot \max\{\pi_{j2}(f|1d), \widetilde{r}d\}, \tag{7}$$

where $j \in \{I, L\}$ *denotes the choice of assets;*

b) *the Time* 1, *after-fee payoff given* f *and the HWM,* $\Pi^H_{j2}(f)$, *is:*

$$\Pi^H_{j2}(f) \equiv p_j \cdot Max\left\{\pi^H_{j2}(f|1u), \widetilde{r}[u-(u-1)f]\right\} + (1-p_j)\cdot Max\left\{\pi^H_{j2}(f|1d), \widetilde{r}d\right\}, \tag{8}$$

where $j \in \{I, L\}$ *again denotes the choice of assets;*

c) *the investor's time* 0 *expected, after-fee payoff given* f *and without the HWM is:*

$$\Pi_{i0}(f) = Max\{\pi_{i1}(f) + \Pi_{i2}(f), \quad E(1+\widetilde{r})-1\},\ i \in \{I, L\}, \tag{9}$$

and $\Pi^H_{i0}(f)$ *denotes the time* 0 *payoff given* f *and the HWM.*

Notice the 'max' operator in (7) indicates the investor's choice of withdrawal of capital at $t=1$, while the 'Max' in (9) describes the investor's choice of investing in a fund (and subsequent choice of withdrawal at $t=1$) at $t=0$.

2.2.2 Fund Manager's Expected Fees

With the knowledge on the investor's decisions at various points, we now examine the manager's problem. Incorporating the results and expressions from Lemma 1, we first derive the fund manager's expected fees at Time 0, $E(F_I|f,\widetilde{r})$, conditional on the choice of the illiquid asset, fee percentage f and no HWM.

$$\begin{aligned} E(F_I|f,\widetilde{r}) &= p_I[(u-1)f + p_L(u-1)(u-(u-1)f)f \cdot E[I_{1u,I}(f,\widetilde{r})]] \\ &\quad + (1-p_I)\left[q\left(u^2-d\right)f + p_L(1-q)(1-d)f\right]\cdot E[I_{1d,I}(f,\widetilde{r})], \end{aligned} \tag{10}$$

where $I_{1u,I}$ and $I_{1d,I}$ are *indicator* functions on fund flows at nodes $1u$ and $1d$, satisfying:

$$I_{1u,I}(f,\widetilde{r}) \equiv \begin{cases} 1 \text{ (no outflow)}, & \pi_{I2}(f|\,1u)/[u-(u-1)f] \geq \widetilde{r}; \\ 0 \text{ (outflow)}, & \pi_{I2}(f|\,1u)/[u-(u-1)f] < \widetilde{r}; \end{cases}; \tag{11}$$

and

$$I_{1d,I}(f,\widetilde{r}) \equiv \begin{cases} 1 \text{ (no outflow)}, & \pi_{I2}(f|\,1d)/d \geq \widetilde{r}; \\ 0 \text{ (outflow)}, & \pi_{I2}(f|\,1d)/d < \widetilde{r}; \end{cases}; \tag{12}$$

When a HWM is used to modify the performance fee, the manager's expected fees from investing in the illiquid asset are:

$$\begin{aligned} E\left(F_I^H\middle|\,f,\widetilde{r}\right) &= p_I\left[(u-1)f + p_L(u-1)(u-(u-1)f)f\cdot E\left[I_{1u,I}^H(f,\widetilde{r})\right]\right] \\ &\quad + (1-p_I)\,q\left(u^2-1\right)f\cdot E\left[I_{1d,I}^H(f,\widetilde{r})\right], \end{aligned} \tag{13}$$

where $I_{1u,I}^H$ is another indicator function on fund flows at node $1u$, satisfying:

$$I_{1u,I}^H(f,\widetilde{r}) \equiv \begin{cases} 1 \text{ (no outflow)}, & \pi_{I2}^H(f|\,1u)/[u-(u-1)f] \geq \widetilde{r}; \\ 0 \text{ (outflow)}, & \pi_{I2}^H(f|\,1u)/[u-(u-1)f] < \widetilde{r}; \end{cases}, \tag{14}$$

while $I_{1d,I}^H$ is the indicator function at node $1d$, and can be similarly defined.

A comparison of (10) and (13) again confirms the fact that the HWM lowers the expected fees of the fund in Period 2, as no fees will be earned along the path $d \to 1$ and fees will be paid based on u^2-1 rather than u^2-d along the path $d \to u^2$.

With the choice of the liquid asset, the manager's expected fees, without the HWM, are:

$$\begin{aligned} E\left(F_L\middle|\,f,\widetilde{r}\right) &= p_L\left[(u-1)f + p_L(u-1)(u-(u-1)f)f\cdot E\left[I_{1u,L}(f,\widetilde{r})\right]\right] \\ &\quad + (1-p_L)\,p_L(1-d)f\cdot E\left[I_{1d,L}(f,\widetilde{r})\right], \end{aligned} \tag{15}$$

where $I_{1u,L}$ and $I_{1d,L}$ are again indicator functions on fund flows (similarly defined as $I_{1u,I}$ and $I_{1d,I}$ above). With the presence of the HWM, the manager's time 0 expected fees with the choice of the liquid asset, are:

$$E\left(F_L^H\middle|\,f,\widetilde{r}\right) = p_L\left[(u-1)f + p_L(u-1)(u-(u-1)f)f\cdot E\left[I_{1u,L}^H(f,\widetilde{r})\right]\right]. \tag{16}$$

Notice that there will be no fees earned at node $1d$ with the payoff structure of the liquid asset (can only come back from d to \$1) and the HWM.

To summarize, a manager's problem at time 0, (P), is as follows:

$$\underset{\{i\in[I,L],f,H\}}{Max} E\left[F_i^{(H)} \middle| f, \widetilde{r}, C \right] \tag{IC-L}$$

$$\text{s.t. } E\left(F_i \middle| f,\widetilde{r}\right) \text{ and } E\left(F_i^H \middle| f,\widetilde{r}\right), i\in\{I,L\} \text{ are defined in (10) through (16);} \tag{Fees-Flow}$$

$$\Pi_{i0}(f) \text{ and } \Pi_{i2}(f),\ i\in\{I,L\} \text{ are defined in Lemma 1;} \tag{IC-Investor}$$

$$E\left(F \middle| f,\widetilde{r}\right) \geq \underline{U}. \tag{IR-L}$$

In general, the type G manager's optimal choice of assets and the fee structure (percentage fee, f, *and* the use of HWM) depends on payoffs of the assets and those of investors' outside opportunities. There are three sets of constraints in (P). First, given a fee structure (f and the HWM), expected fees earned by the fund depends on whether there is fund flow in either of the two states (nodes 1u and 1d) at $t = 1$. Second, the incentive compatibility constraints of the investors (IC-Investor) include their decisions to invest in a fund at $t = 0$, and whether to stay with the same fund at $t = 1$. These decisions depend on fund returns, fee structure, and the realizations of investors' outside opportunities. Finally, the third constraint is the participation constraint for the fund manager, in that expected fees earned by the fund must cover his reservation payoff.

2.3 Equilibrium: Optimal Fee Structure and Choice of Assets

Before deriving the equilibrium, we first provide some general discussions on the optimal fee structure. First, since a manager cannot revise the fee structure at Time 1 based on the realization of $\widetilde{r}$ (Assumption 2), the total (expected) fees are not always increasing in f , the percentage fee (set at Time 0). This is because while a higher f generates higher total fees if the investor stays with a fund in Period 2, it can lead to an outflow at Time 1 and reduces the second period fees to 0. An outflow is possible when the realization of $\widetilde{r}$ is r (with probability θ); as the value of r rises, it becomes more costly for the fund to retain the investor at both nodes ($1u$ and $1d$). On the other hand, with probability $(1-\theta)$ the realization of $\widetilde{r}$ is 0 and the investor will stay with certainty. Thus, lowering f too much reduces total fees as the fund 'overpays' for the investor to stay in the states where it is costless to retain investors.

Second, given the above arguments, and the linearity of fee structures and risk neutrality of agents, the optimal f that maximizes the manager's expected fees will be corner solutions, either at a level that makes the investor indifferent between withdrawing capital and staying with the fund, or at a level that makes the manager indifferent between shutting down or operating the fund. Therefore, the manager decides, at Time 0, whether it is worthwhile to retain the investor at both nodes $1u$ and $1d$ at Time 1, or at one of the two nodes, or at neither node; and the associated, fee-maximizing combination of f and HWM under a particular investor-retaining plan.

Third, by modifying the performance fees only after losses in the first period, a HWM provides a *state-dependent* fee waiving scheme to retain the investor. As shown in (2) through (6), the HWM provides a discrete fee transfer, in the amount of $f(1-d)$, from the fund to the investor at node $1d$ (so long as the fund's end-of-Period 2 value exceeds \$1). Hence, with the HWM and the fee transfer, the fund does not need to lower f as much to retain the investor at node $1d$ as it does without the HWM. On the other hand, the HWM has no direct impact on fees earned at node $1u$, though it does affect total expected fees earned from this node through f. Therefore, a HWM can increase the total fees because with a higher f the fund earns higher fees during Period 1 and in Period 2 whenever the realization of $\widetilde{r}$ is 0.

Finally, the state-dependent role of the HWM is more valuable for the manager whose liquidation risk is more costly at node $1d$. This is true when the manager invests in the illiquid asset: With probability $1-p_I$ the fund will incur a loss during Period 1 and will not earn any fees; but the fund's expected return in Period 2 (from node $1d$) is the highest because the possibility of going from $\$d$ to $\$u^2$. Hence, liquidation due to the investor's withdrawal of capital at this node would be very costly and the role of HWM is significant. On the contrary, at node $1u$ the fund has already earned fees in Period 1 and the fund's expected return and associated expected fees in Period 2 are lower than those from node $1d$.

When investing in the liquid asset with i.i.d. returns in each period, the fund's expected return at node $1u$ is the same as that at node $1d$, and the manager does not have a stronger preference for retaining investors following either poor or good performance in Period 1. Therefore, the efficient method for the manager to maximize total expected fees is to simply lower the percentage incentive fee (f), because doing so generates a symmetric, or *state-independent* impact on the investor's marginal cost of staying with the fund.

We need the following definitions in deriving the optimal fee structures and the equilibrium.

Definition 1 *Given the choice of the illiquid asset and the value of* r, *let* $F_I^* \equiv \arg\max E\left(F_I \mid f, r\right)$ *and* $F_I^{H*} \equiv \arg\max E\left(F_I^H \mid f, r\right)$ *be the optimal fee percentages for non-HWM funds and HWM funds,* *respectively; and let* $V_I(r) \equiv E\left(F_I \middle| F_I^*, r\right)$ *and* $V_I^H(r) \equiv E\left(F_I^H \middle| F_I^{*H}, r\right)$ *be the corresponding value functions of expected fees.*

We first examine the order of fund flows and the relative importance of retaining the investor in different states when managing the illiquid asset. Since with probability $1-\theta$ the realization of investors' outside investment opportunity $(\widetilde{r})$ is 0 and there is no flow, we focus on the parameter r ($\widetilde{r} = r$ with probability θ), which, along with fee structure set at Time 0, determines the outcome of fund flow at Time 1.

Lemma 2 *Given the choice of the illiquid asset,*

a) $V_I(r)$ *and* $V_I^H(r)$ *are both decreasing functions in* r;

b) *for both HWM funds and non-HWM funds, as* r *increases from* 0 *to* $\underline{U}/\theta$, *there is no flow for low values of* r, *flow occurring at node* $1u$ *but* not *at node* $1d$ *for moderate values of* r, *and flow occurring at both nodes* $1u$ *and* $1d$ *for high values of* r;

c) *for any* $r \in [0, \underline{U}/\theta]$, $F_I^{H*}(r) \geq F_I^*(r)$.

Proof. See Appendix A.2. ∎

Given that at Time 0 the fund manager sets the fee structure (f and the HWM) to make investors earn exactly their reservation payoff ($E[\widetilde{r}] = \theta r$), the parameter r captures the fund's cost of retaining the investor at Time 1. An increase in r forces the fund to pass a larger fraction of its total expected return to the investor in order to attract him to the fund, and hence the expected fees decrease with r (Lemma 2a).

To understand Lemma 2b), first notice that conditional on a given fee structure expected fees are higher if there is no fund flow at Time 1; conditional on a particular outcome of the fund flow at Time 1 expected fees increase with f. Combining these results with that of Lemma 2a), it is easy to see that for low values of r there will be no flow at either node, and expected fees earned are the highest in this region.

As r rises, it becomes too expensive to retain the investor at both nodes $1u$ and $1d$. While retaining the investor at node $1d$ can be done by either lowering f or by the combination of f and a HWM, the only way to retain the investor at node $1u$ is to lower f. When managing the illiquid asset, a fund's expected return from node $1d$ and associated fees earned in Period 2 are higher than those from node $1u$. Accordingly, retaining the investor at node $1d$ is more valuable than at node $1u$. Another way to see why funds will let the investor leave at node $1u$ first is that, the opportunity cost of lowering f (for both HWM and non-HWM funds) to retain the investor is high, because the fund loses fees in Period 1 and in other states of Period 2 (i.e., when $\widetilde{r} = 0$).

As r further rises, retaining the investor at node $1d$ also becomes expensive, in that the benefits of retaining the investor (when $\widetilde{r} = r$ and earning a fee in Period 2) are outweighed by the costs of losing fees in Period 1 and when $\widetilde{r} = 0$. As mentioned before, the HWM funds have an advantage in retaining the investor over non-HWM funds. As a result, for a range of intermediate r, there will be fund flow at node $1d$ for non-HWM funds but HWM funds are able to retain the investor . But for higher values of r, even the HWM funds find it too costly to retain the investor and there will be fund flows for all funds whenever $\widetilde{r} = r$ at Time 1.

To summarize, Lemma 2b) illustrates that there are *three* regions of fund flows for a fund holding the illiquid asset. (Given that the addition of HWM changes the optimal percentage fee f, these regions will be different for HWM funds vs. no-HWM funds.) First, for a range of low r, there is no fund flow at either node; second, for a range of intermediate r, fund flow occurs at node $1u$ and when $\widetilde{r} = r$, or State $(1u, r)$ only; and finally, for high r, fund flow occurs at both States $(1u, r)$ and $(1d, r)$.

For any given r that induces fund flows and the associated $F_I^*(r)$, i.e., the optimal percentage fee of a non-HWM fund, adding a HWM to modify this percentage fee results in a discrete wealth transfer from the fund to the investor. Thus, a small increase from $F_I^*(r)$ allows a HWM fund to retain the investor in the same states as the non-HWM, but this increase in fee percentage clearly raises the total expected fees of the HWM fund (Lemma 2c).

The next result illustrates that, since the HWM provides a state-dependent, or *asymmetric* fee waiving scheme to the investor, it is valuable for managing the illiquid asset only when there are *asymmetric flows.*

Lemma 3 *A HWM is used at Time* 0 *for managing the illiquid asset* only if *there is fund flow at State* $(1u, r)$ *but* not *at State* $(1d, r)$ *at Time* 1.

Proof. See Appendix A.3. ■

A HWM allows the fund to charge a higher f and still retain investors at node $1d$ *and* earn higher total fees. However, the value of the HWM disappears under two circumstances. First, when there is no fund flow (low r), even without the HWM the fund can charge a high f and still retain the investor at all states. Compared to lowering f, which provides a state-independent fee waiving scheme, adding a HWM is less effective for the fund since the HWM is only effective when the fund reaches node $1d$.

Second, when r is high, even with the HWM the fund finds it too costly to retain the investor at node $1d$, and by Lemma 2b) there will be fund flow whenever $\widetilde{r} = r$. But in these cases the HWM again loses its benefits as a state-dependent fee waiving device, and the fund is better off by simply adjusting f, a state-independent device, to maximize fees earned from other states.

The next proposition summarizes results from the previous two lemmas.

Proposition 1 *When a fund invests in the illiquid asset, there exist* $\underline{r}$ *and* $\overline{r}$, $0 < \underline{r} < \overline{r} < \underline{U}/\theta$, *such that*

a) *for* $r \in [0, \underline{r})$, $V_I(r) > V_I^H(r)$;

b) *for* $r \in [\underline{r}, \overline{r}]$, $V_I(r) < V_I^H(r)$;

c) *for* $r > \overline{r}$, $V_I(r) > V_I^H(r)$.

Proof. See Appendix A.4. ■

Proposition 1b) presents the central result of the model. Given that r is moderately high, so that it is feasible to keep the investor in the fund in all states except for State $(1u, r)$, it is important that a type L fund keeps the investor at node $1d$. A HWM, by waiving fees between \$1 (where the HWM is set) and \$$d$ (value of fund's assets after first period losses), lowers the investor's marginal cost of staying with the fund at node $1d$. This enables the fund to charge a higher f and still retains the investor at node $1d$ while earning higher fees elsewhere, so that the overall effect of the HWM is that it increases the total expected fees. Therefore, when the cost of retaining the investor falls in the region specified in Proposition 1b), a type L fund will optimally choose to use a HWM to modify its percentage fee.

When r is low so that it is inexpensive to retain the investor at either node (Proposition 1a), the optimal percentage fee f is set to retain the investor at State $(1u, r)$; but this optimal f is the same for both non-HWM and HWM funds, because the HWM does not have any impact on fees or returns at node $1u$. Since the HWM lowers the fees earned at node $1d$ (even though investors will always stay at this node) but does not have any impact at node $1u$, it is not the efficient way to retain the investor in both states. Instead, adjusting percentage fee f provides a symmetric and optimal device to retain the investor and maximize total fees.

With very high r, it becomes too expensive to keep the investor at node $1d$ even with the HWM, and there will be fund flows whenever $\widetilde{r} = r$. The comparative advantage of the HWM as a state-dependent, fee waiving device is again disappeared. In fact, the investor prefers a lower f (recall $F_I^{H*} \geq F_I^*$ for a given r) that allows him to invest more dollars at Time 1 in their outside option r. Therefore, the optimal fee structure is to adjust the percentage fee f to maximize fees earned in Period 1 and in the non-flow states of Period 2. This corresponds to Proposition 1c).

The next proposition illustrates the optimal fee structure when investing in the liquid asset.

Definition 2 *For a given r, $F_L^* \equiv \arg\max E(F_L | f, r)$ and $F_L^{H*} \equiv \arg\max E\left(F_L^H \middle| f, r\right)$ are the optimal percentage fees for a fund investing in* liquid *assets, and $V_L(r) \equiv E(F_L | F_L^*, r)$ and $V_L^H(r) \equiv E\left(F_L^H \middle| F_L^{*H}, r\right)$ are the corresponding value functions.*

Proposition 2 *When a fund invests in the liquid asset, for all $r \in (0, \underline{U}/\theta)$, $V_L(r)$ and $V_L^H(r)$ are decreasing functions in r; moreover, $V_L(r) > V_L^H(r)$.*

Proof. See Appendix A.5. ■

As discussed earlier, the liquid asset has i.i.d. returns, and thus it is equally important for the fund to retain the investor at nodes $1d$ and $1u$. For a given percentage fee f, the HWM raises the investor's payoff only at node $1d$ but does not affect the investor's payoff at node $1u$. This asymmetric fee waiving device is clearly not well suited for managing assets illustrating symmetric payoffs following gains and losses. The efficient device is to adjust f alone, because it retains the investor and maximizes fees along all paths symmetrically. This general argument works for all cases of fund flows, as specified in Proposition 2.

With optimal fee structures in place for investing in different types of assets, we can now derive the equilibrium of asset selection and payoffs.

Proposition 3 *For any $r \in (0, \underline{U}/\theta)$, the investor receives reservation payoffs at Time 0; at Time 1, the investor withdraws their capital from the fund if $\widetilde{r} = r$* and *the fund's expected return is less than r. Moreover,*

a) *When $r \in (0, \underline{r})$, the manager invests in the illiquid asset and sets $f = F_I^*$ without a HWM;*

b) *There exists an $r^S \in (\underline{r}, \underline{U}/\theta)$ such that when $r \in \left[\underline{r}, r^S\right]$, the manager sets $f = F_I^{*H}$ with the HWM (\$1) and invests in the illiquid asset;*

c) *When $r \in \left(r^S, \underline{U}/\theta\right)$, the manager invests in the liquid asset and no HWM will be used.*

The equilibrium indicated in Proposition 3 follows directly from Propositions 2 and 1. First, the manager's choice of assets and fee structure depends on the cost of retaining the investor at Time 1. For low r, they invest in the illiquid asset but is better off without using the HWM (Part a). When r is in the intermediate range, the manager will invest in the illiquid asset and add a HWM to the performance fee; HWM funds (investing in the illiquid asset) charge higher incentive fees than non-HWM funds (investing in the liquid asset; Part b). When r is high, the manager finds it too costly to invest in the illiquid asset due to the liquidation risk at Time 1 (with or without the HWM), and hence the liquid asset is selected (Part c). This case is consistent with empirical evidence in, for example, Brunnermeier and Nagel (2004). They find that many large hedge funds avoided taking short positions in technology stocks during the late 1990s, a strategy that would lead to high, long-term profits but face significant short-term liquidation risk.

Insert Figure 2 here.

Figure 2 presents a numerical example of Proposition 3. The parameters are given as follows:

$$p_L = 0.75,\ p_I = 0.5,\ q = 0.65;\ u = 1.2;\ \theta = 0.5.$$

We plot value functions, or total expected fees $\left(V_L, V_L^H;\ V_I, V_I^H\right)$ from managing the liquid vs. the illiquid asset as functions of the gross return of the investor's outside opportunity, or $1 + r$ (occurring with probability θ). The two lines at the bottom of the graph (starting from the vertical axis) represent total fees from investing in the liquid asset. Consistent with Proposition 2, the manager is better off *not* using the HWM for all values of r. The two lines at the top of the graph represent total fees from investing in the illiquid asset. Consistent with Proposition 1, there are three regions of r in which the impact of HWM differs. First, for $1 + r \in (1.0, 1.22)$, the manager is better off not using the HWM because it is inexpensive to the investor at either node (and there is no flow in any state), and hence charging the highest possible f to retain the investor at node $1u$ is the profit-maximizing strategy. Second, for $1 + r \in [1.22, 1.34]$, there is fund flow at node $1u$ but not at node $1d$ due to the use of HWM, and the manager is better off with the HWM. Third, when $1 + r > 1.34$, it becomes too expensive to retain the investor at node $1d$ even for HWM funds, and with fund flows occurring whenever $\widetilde{r} = r$, the HWM loses its value in providing asymmetric rebates of fees; thus, the manager is again better off not using the HWM.

Comparing the payoffs from investing illiquid and liquid assets, we observe, consistent with Proposition 3, that, there are three possible equilibrium outcomes in terms of asset choice and the use of HWMs. First, for low r $[1 + r \in (1.0, 1.22)]$, the manager invests in the illiquid asset but does not use the HWM. Second, for medium r $[1 + r \in (1.22, 1.33)]$, the manager invests in the illiquid asset and uses the HWM. Interestingly, when $1 + r \in (1.28, 1.31)$, the manager invests in the illiquid asset if and only if they use a HWM (the line "Y-Z" of "L-HWM" lies above "S-nHWM", which lies above "L-nHWM"). Finally, for high r $[1 + r > 1.33]$, the liquid asset is selected, because the manager's fees (with or without a HWM) from investing in the illiquid asset are below that from investing liquid assets.

2.4 Empirical Predictions

Our model links a hedge fund's investment strategy with its compensation structure. In particular, we predict that HWM's are associated with funds that invest in illiquid assets. As proxies for illiquid assets, we first consider the fund's self-reported style categories. Our model predicts that HWMs are more likely to be found among funds adopting styles (such as short selling), and less likely to be used among fund adopting more liquid styles, such as managed futures. Second, we consider the use of share restrictions as proxies for funds investing in illiquid assets. According to Chordia (1996) and Lerner and Schoar (2004), funds use restriction to screen for longer-horizon investors.

Third, by our own definition, illiquid assets exhibit return reversals, as short-term losses are likely to be followed by higher gains. Therefore, compared to non-HWM funds, HWM funds are more likely to exhibit negative autocorrelation in returns after periods with negative returns, but HWM funds and non-HWM funds' return autocorrelation patterns do not differ after periods with positive returns. Finally, our model also predicts that HWM and performance fee (percentage) are substitutes, in that, HWM funds tend to charge higher performance fees.

3 Empirical Evidence

3.1 Description of Data

The main database used in our study consists of share characteristics and historical returns of individual hedge funds. These data were provided by TASS Tremont Ltd., a leading hedge fund data vendor.

The raw sample includes 3, 501 funds, of which approximately two-thirds are 'live' as of January, 2002. The remaining funds are considered 'defunct.' Defunct funds have ceased reporting to TASS, but may not have ceased operations. Each fund reports a monthly time series of returns, calculated net of fees. Each fund also reports a single, updated snapshot of it's organizational characteristics, including the fund's liquidity and fee structure. Characteristics of a defunct fund are those disclosed in the fund's annual report to TASS. The sample period covers January 1994 through December 2002.

3.2 Summary Statistics

Table 1 reports summary statistics for the hedge fund sample. The first three variables - hwm, mfee, and ifee - represent the funds HWM, fixed management fee percentage, and performance fee percentage. Observe that 37% of funds use a HWM. The variables lockind and notice correspond to a fund's redemption restrictions. Specifically, *lockind* is a dummy variable that equals one if the fund has a lockup provision; and zero otherwise. A fund's lockup period is the minimum holding period for any investor in the fund. The variable notice is the number of days an investor must provide the fund before the investor can redeem his/her share. The variable *age* equals the number of months between the fund's initial observation date and the earliest initial date across all funds within the same fund family. The variables *size* is the fund's initial reported estimated asset size (in $millions). $offshore$ is an indicator variable equal to 1 if the fund is domiciled offshore. *perscap* equals 1 if the fund manager invests his own wealth in the fund's portfolio; and zero otherwise.

Pairwise correlations between fund characteristics are reported in Table 2. The results reveal a strong positive correlation between HWM's and the percentage incentive fee ($ifee$). Consistent with our model, these two components of a fund's fee structure appear to be substitutes. In addition, HWM's are positively related to funds' use of redemption restrictions. According to Chordia (1996) and Lerner and Schoar (2004), funds use restrictions to screen for long-horizon investors, while Aragon (2006) finds that hedge funds use share restrictions tend to invest in illiquid assets. Therefore, we interpret this as support for our prediction that HWM's are more common among funds holding illiquid assets.

3.3 Empirical Results

Table 3 reports the results from a probit analysis of funds' use of HWM's. Consistent with our model and the univariate results, HWMs and (percentage) performance fees appear to be substitutes. In fact, the probability of a HWM increases by 10% per 10% increase in a fund's incentive fee. We also find the use of HWMs is positively correlated with a fund's decision to impose share restrictions. Funds with a lockup provision are 43% more likely to have a HWM than funds without a lockup. In addition, funds with a 30 day redemption notice period are 30% more likely to have a HWM. To the extent that share restrictions are proxies for funds facing high liquidation risk, these results support our predictions.

Along with the use of redemption restrictions, we also use hedge funds' self-reported style categories as proxies for the extent to which they hold illiquid assets. Table 4 reports, for each self-reported style category, the proportion of funds that use a HWM to calculate performance fees. Compared to the average frequency across all funds (37%), HWM's are more frequently used by strategies that are considered "illiquid," or, long-term in nature, including Convertible Arbitrage (45%), Short Bias (52%), Event Driven (44%), and Fixed Income Arbitrage (48%). In contrast, HWM's are least frequently used among strategies that are considered "liquid," including Global Macro (24%) and Managed Futures (14%).

The main prediction of our model is that the returns on HWM funds exhibit autocorrelation. In particular, fund returns exhibit negative autocorrelation following a negative return. Previous research (e.g., Asness, Krail, and Liew 2001; Getmansky, Lo, and Makarov 2004) find positive autocorrelation in monthly hedge fund returns. However, our model does not necessarily imply negative autocorrelation at a monthly frequency. Moreover, long-horizon returns may exhibit negative autocorrelation while short-horizon returns are positively autocorrelated. In the analysis we consider monthly, quarterly, 6-month, and annual return frequencies.

To test this prediction we estimate style-level autocorrelation coefficients using pooled regressions. First, we estimated the autocorrelation coefficient (ρ) from the regression,

$$r_{i,t} = \alpha + \rho r_{i,t-1} + \epsilon_{i,t} \tag{17}$$

where $r_{i,t}$ is the return on fund i in period t, for all funds i in a given style category. We also allow the autocorrelation coefficient to depend on the sign of the previous period's return. That is,

$$r_{i,t} = \alpha + \rho^{+}[r_{i,t-1}]^{+} + \rho^{-}[r_{i,t-1}]^{-} + \epsilon_{i,t} \tag{18}$$

Results are reported for monthly (mth), quarterly (qtr), semi-annual (sann), and annual (ann) return horizons. Funds are split into HWM mark (hwm = 1) and no HWM (hwm = 0) groups. Standard errors are calculated assuming clustering at each observation date.

The results are reported in Table 4. First observe the estimated autocorrelation coefficient from (17). The monthly return frequency reveals significant positive autocorrelation. For example, at the aggregate-level, the autocorrelation coefficients are statistically significant .09 and .10, for non-HWM and HWM funds, respectively. Positive autocorrelation is also evident in nearly all of the style-categories (except Short Bias and Managed Futures). This is consistent with Getmansky, Lo, and Makarov (2004), who also find positive autocorrelation. Also notice that, as the return horizon increases, the degree of autocorrelation diminishes to zero. Getmansky, Lo, and Makarov (2004) argue that positive autocorrelation may reflect illiquidity in the underlying asset held by the fund. Lo and Mackinlay (1990) show that long-horizon returns have a mitigating effect on autocorrelation due to illiquidity.

Table 4 also reports results from estimating (18). This specification may provide a more powerful test of our model. That is, if HWMs are indeed a mechanism for managing illiquid assets, then we would expect negative autocorrelation to appear following a negative return. At the aggregate-level, there does not appear to be a greater negative autocorrelation for HWM funds. For example, using an annual return horizon, the point estimate of ρ^{-} equals $-.25$ and $-.16$ for non-HWM and HWM funds, respectively. Moreover, each of these estimates is not statistically different from zero.

Support for our theory is provided at the style-level. For some horizon larger than monthly, the Convertible Arbitrage (qtr and ann), Emerging Markets (ann), Event Driven (qtr), Fund of Funds (ann), and Managed Futures (qtr) have a statistically significant negative point estimate of ρ^{-}. For nearly all of these style categories (except Emerging Markets), the point estimate of ρ^{-} for non-HWM funds is either positive or greater than that for the HWM group.

4 Summary and Conclusion

This paper shows how a high-water mark can arise endogenously to overcome the capital commitment problem inherent to the hedge fund industry. When funds invest in illiquid assets, investors may withdraw capital prematurely and force a costly liquidation of funds' assets. A high-water mark lowers existing investors' marginal cost of staying with the fund after poor performance, while it has no impact on investors' after-fee returns after good performance. This *state-dependent* impact of the high-water mark in retaining investors provides an efficient way for managers to avoid liquidation risk *and* maximize expected fees when investing in illiquid assets. With a high-water mark in place to modify performance fees, investors are willing to commit their capital for the long term.

We also find empirical support for our model using a large data set on hedge funds. First, we find that high-water marks are more common among funds in self-reported style categories that are associated with more illiquid assets, impose share restrictions. We also find some evidence at the style-level that high-water marks are associated with *negative* return autocorrelations over quarterly or longer horizons. Second, we find that the average percentage performance fee for funds using high-water marks is higher than that for non-high-water mark funds, consistent with the model prediction that high-water marks and performance fees are substitutes.

Appendix Proofs

A.1 Proof of corollaries given Assumption 1:

First, since $R_L = [p_L u + (1-p_L)d]^2 - 1$, Assumption 1a), or $p_L \geq \frac{u\sqrt{1+\underline{U}}-1}{u^2-1}$ implies $R_L \geq \underline{U}$ (manager's reservation payoff). Second, based on (1), we have

$$\begin{aligned} R_I - R_L &= \left[p_I p_L + (1-p_I) q - p_L^2\right] u^2 + \left[p_I (1-p_L) + (1-p_I)(1-q) p_L - 2p_L (1-p_L)\right] \\ &\quad + \left[(1-p_I)(1-q)(1-p_L) - (1-p_L)^2\right] d^2. \end{aligned}$$

It suffices to show that if the coefficient of u^2 is positive while the coefficient of d^2 is negative then the above expression is positive. First, $p_I p_L + (1-p_I) q - p_L^2 > 0 \iff q > \frac{p_L(p_L-p_I)}{1-p_I}$; on the other hand, $(1-p_I)(1-q)(1-p_L) - (1-p_L)^2 < 0 \iff q > \frac{p_L-p_I}{1-p_I}$; combining both inequalities we have $q > \frac{p_L-p_I}{1-p_I} \iff R_I > R_L$.

Finally, given the choice of the illiquid asset, the (ex ante) expected return in Period 2 following a loss in Period 1 is $(1-p_I) R_I(1d)$, where $R_I(1d) = \left[qu^2 + (1-q)\left(p_L + (1-p_L)d^2\right)\right]$ is the return of the fund in Period 2 after reaching Node $1d$; the (ex ante) expected return in Period 2 following a gain is $p_I R_I(1u)$, where $R_I(1u) = \left[p_L u^2 + (1-p_L)\right]$ is the return of the fund in Period 2 after reaching Node $1u$. $(1-p_I) R_I(1d) \geq p_I R_I(1u) \iff q > \frac{\left[p_I\left(u^2+1\right)-1\right]\left[p_L\left(u^2-1\right)+1\right]}{(1-p_I)\left[u^4-\left(p_L\left(u^2-1\right)+1\right)\right]}$. Hence, Assumption 1c) implies that $R_I > R_L$ and $(1-p_I) R_I(1d) \geq p_I R_I(1u)$. ■

A.2 Proof of Lemma 2 (order of fund flow):

First, as discussed in Section 2.1, given that the manager has all the bargaining power in negotiating with the investor, the optimal f is set at Time 0 such that at Time 1 the investor is indifferent between withdrawing his capital and earning r ($\widetilde{r} = r$) versus staying with the fund and earn π_{I2}, *and* the investor earns a non-negative return in Period 1. Since $r > 0$, an optimal f that makes the investor indifferent about staying with the fund at node $1u$ or $1d$ must yield a positive return in Period 1. Hence, f_I^* $\left(f_I^{*H}\right)$ is set such that the investor earns exactly a net return of r in Period 2 without the HWM (with the HWM). Since the sum of the fund's fees and the investor's after-fee return is a constant (R_I over two periods), expected fees of the fund (with and without the HWM) are inversely related to the investor's after-fee return, which increases as r increases from 0 to $\underline{U}/\theta$. This proves Part a).

Second, conditional on a given fee structure, expected fees are higher if there is no fund flow at Time 1; conditional on a particular outcome of the fund flow at Time 1 (e.g., no flow at either node), expected fees increase with f with and without the HWM as long as f is not too high or too low to induce a different outcome (of fund flow) at Time 1. Therefore, when r is low, there will be no flows and $f^*(r)$ and $f^{*H}(r)$ are high. As r rises, it becomes too expensive to retain the investor at both nodes $1u$ and $1d$, but given $(1-p_I) R_I(1d) \geq p_I R_I(1u)$ (Assumption 1c) and Appendix A.1 above), retaining the investor at State $(1d, r)$ is more profitable than at State $(1u, r)$ when the fund sets the optimal f at Time 0. The optimal f is set such that there is fund flow at State $(1u, r)$ but not at State $(1d, r)$, and the f maximizes the Time 0 expected fees with the expectations about what will occur at Time 1. As r further rises, retaining the investor at State $(1d, r)$ also becomes too expensive, and at Time 0 the fund sets f such that there will be fund flow whenever $\widetilde{r} = r$ at Time 1, while the optimal f set at Time 0 maximizes expected fees with the expectations about flows at Time 1. This proves Part b).

Third, for low r and hence low cost of retaining the investor when $\widetilde{r} = r$, the (Time 0) optimal fees are set at:

$$\begin{aligned} \pi_{I2}(f|\,1u) &= \pi^H_{I2}(f|\,1u) = (u-1)(u-(u-1)f)[p_L(1-f)+(1-p_L)] \geq [u-(u-1)f]\cdot r, \\ \iff & \quad f^*_I = f^{*H}_I = \frac{u-1-r}{p_L(u-1)}, \end{aligned} \tag{19}$$

with or without the HWM. As r increases, the optimal fee, without the HWM, is set at:

$$\begin{aligned} \pi_{I2}(f|\,1d) &= q\left(u^2-d\right)(1-f)+(1-q)\left[p_L(1-d)(1-f)-(1-p_L)\left(d-d^2\right)\right] \geq rd, \\ \iff & \quad f^*_I = \frac{q\left(u^2-d\right)+(1-q)(1-d)(p_L-(1-p_L)d)-rd}{q(u^2-d)+(1-q)(1-d)p_L}. \end{aligned} \tag{20}$$

With the HWM, we have

$$\begin{aligned} \pi^H_{I2}(f|\,1d) &= q\left[\left(u^2-d\right)-\left(u^2-1\right)f\right]+(1-q)\left[p_L(1-d)-(1-p_L)\left(d-d^2\right)\right] \geq rd, \\ \iff & \quad f^{*H}_I = \frac{q\left(u^2-d\right)+(1-q)(1-d)(p_L-(1-p_L)d)-rd}{q(u^2-1)}. \end{aligned} \tag{21}$$

Clearly, $f^{*H}_I > f^*_I$. Finally, as r further increases, there will be fund flow whenever $\widetilde{r} = r$. Hence, the optimal fee is set such that the investor will not withdraw when $\widetilde{r} = 0$. Therefore, we have

$$\pi_{I2}(f|\,1u) = \pi^H_{I2}(f|\,1u) \geq 0 \qquad \iff \qquad f^*_I = f^{*H}_I = \frac{1}{p_L}. \tag{22}$$

Therefore, $f^{*H}_I \geq f^*_I$ for all $r \in (0, \underline{U}/\theta)$. ■

A.3 Proof of Lemma 3 (HWM and fund flow):

We need to show that a HWM is not used at Time 0 if there is no fund flow at Time 1 or there is always fund flows whenever $\widetilde{r} = r$ at Time 1. First, based on the proof of Lemma 2 in A.2, we know that for a range of low r such that there is no flows, $f^*_I = f^{*H}_I$, which implies the fees earned in Period 1 and from node $1u$ are the same with and without the HWM. However, with the HWM the fees earned from node $1d$ is lower than without the HWM: with probability q the fund goes from $d \to u^2$ in Period 2 and fees in the amount of $(1-d)f$ is waived due to the HWM; with probability $(1-q)p_L$ the fund goes from $d \to 1$ and again fees $(1-d)f$ is waived; total amount of expected fees lost at node $1d$ due to the HWM is then $(q+(1-q)p_L)(1-d)f$. Therefore, for these low r (and no flows) the fund does better without the HWM.

Second, for high r such that there is always flows whenever $\widetilde{r} = r$, again we have $f^*_I = f^{*H}_I$, and again total expected fees earned will be less with the HWM since there will be a loss of fees at node $1d$ in the amount of $(1-\theta)(q+(1-q)p_L)(1-d)f$. Hence, for these high r the fund does better without the HWM. ■

A.4 Proof of Proposition 1:

We first prove that there is a range of intermediate r such that $V_I(r) < V^H_I(r)$. Based on Lemma 3, in this region HWM funds must have flow at State $(1u, r)$ but not at State $(1d, r)$. There are two cases. First, for relatively low r, fund flow also occurs in non-HWM funds at State $(1u, r)$ but not at State $(1d, r)$. For HWM funds, we have

$$V_I^H(r) = p_I\left[(u-1) f_I^{*H} + (1-\theta)(u-1)\left(u-(u-1) f_I^{*H}\right) f_I^{*H}\right] + (1-p_I) q\left(u^2-1\right) f_I^{*H},$$

where f_I^{*H} is defined in (21) in A.3. For non-HWM funds, we have

$$\begin{aligned} V_I(r) &= p_I\left[(u-1) f_I^* + (1-\theta)(u-1)(u-(u-1) f_I^*) f_I^*\right] \\ &\quad + (1-p_I)\left[q\left(u^2-d\right) f_I^* + p_I(1-q)(1-d) f_I^*\right], \end{aligned}$$

where f_I^* is defined in (20) in A.3.

Let $\Delta V \equiv V_I^H(r) - V_I(r)$, and $\Delta f \equiv f_I^{*H} - f_I^* > 0$, then

$$\begin{aligned} \Delta V &= p_I\left[(u-1)\Delta f + (1-\theta) u(u-1)\Delta f - (1-\theta)(u-1)^2\left(f_I^{*H2} - f_I^{*2}\right)\right] \\ &\quad + (1-p_I)\left\{q\left(u^2-1\right) f_I^{*H} - \left[q\left(u^2-d\right) f_I^* + p_I(1-q)(1-d) f_I^*\right]\right\}, \end{aligned}$$

where the term $(1-\theta)(u-1)^2\left(f_I^{*H2} - f_I^{*2}\right) \approx 0$. Notice that f_I^* $\left(f_I^{*H}\right)$ are set such that the investor is indifferent from staying with the fund without the HWM (with the HWM) at State $(1d, r)$ and leaving the fund. Hence,

$$\begin{aligned} q\left(u^2-1\right) f_I^{*H} &= q\left(u^2-d\right) f_I^* + p_I(1-q)(1-d) f_I^* \\ &= q\left(u^2-d\right) + (1-q)(1-d)(p_L - (1-p_L) d) - rd, \end{aligned}$$

and

$$\Delta V = p_I(u-1)[1 + (1-\theta) u]\Delta f > 0.$$

In the second case, for relatively high r, fund flow occurs in non-HWM funds at State $(1u, r)$ *and* State $(1d, r)$, while HWM funds are able to retain the investor at State $(1d, r)$. For non-HWM funds, we have

$$\begin{aligned} V_I(r) &= p_I\left[(u-1) f_I^* + (1-\theta)(u-1)(u-(u-1) f_I^*) f_I^*\right] \\ &\quad + (1-p_I)(1-\theta)\left[q\left(u^2-d\right) f_I^* + p_I(1-q)(1-d) f_I^*\right], \end{aligned}$$

where f_I^* is defined in (22) in A.3. For HWM funds both the expected fees and optimal f are the same as those in the first case specified above. Then,

$$\begin{aligned} \Delta V &= p_I\left[(u-1)\Delta f + (1-\theta) u(u-1)\Delta f\right] + \\ &\quad (1-p_I)\left\{q\left(u^2-1\right) f_I^{*H} - (1-\theta)\left[q\left(u^2-d\right) f_I^* + p_I(1-q)(1-d) f_I^*\right]\right\}, \end{aligned}$$

where $\Delta f \equiv f_I^{*H} - f_I^*$, which can be negative since it is possible that $f_I^{*H} < f_I^*$. Given that θ is not too high (*need an upper bound on* θ*!*), $\Delta V > 0$.

Finally, based on the above, we know $\underline{r}$ is such that:

$$V_I^H\left[r \mid \text{flow at State } (1u, r) \text{ but not at State } (1u, r)\right] = V_I\left[r \mid \text{no flow}\right],$$

and $\overline{r}$ is such that

$$V_I^H\left[r \mid \text{flow at State } (1u, r) \text{ but not at State } (1u, r)\right] = V_I\left[r \mid \text{flow whenever } \widetilde{r} = r\right]. \quad \blacksquare$$

A.5 Proof of Proposition 2:

First, when liquid assets are chosen, we can show that it is equally important to keep the investor at State $(1u, r)$ and State $(1d, r)$. First, before-fee return at node $1u$ is:

$$R_L(1u) = (u-1)[p_L u - (1-p_L)],$$

in dollar terms, and in percentage terms (off \$$u$),

$$R_L(1u) = \frac{u-1}{u}[p_L u - (1-p_L)].$$

Similarly, before-fee return at node $1d$ is:

$$R_L(1d) = (1-d)[p_L - (1-p_L)d],$$

in dollar terms, and in percentage terms (off \$$d$),

$$\begin{aligned} R_L(1d) &= \frac{1-d}{d}[p_L - (1-p_L)d] = (u-1)[p_L - (1-p_L)d] \cdot \frac{u}{u} \\ &= \frac{u-1}{u}[p_L u - (1-p_L)] = R_L(1u). \end{aligned}$$

Given that it is equally important to keep the investor at both nodes, HWM loses its value in terms of asymmetrically boosting the investor's after-fee return. Further, HWM reduces the expected fees of the fund at node $1d$, and in fact, given a HWM, the fund earns a fee of 0 at node $1d$, while expected fees are positive without the HWM at node $1d$. Hence, we have for all r, $V_I^H(r) < V_I(r)$. ■

References

[1] Aragon, G., 2005. Share Restriction and Asset Pricing: Evidence from the Hedge Fund Industry, forthcoming, *Journal of Financial Economics*.

[2] Berk, J., and R. Green, 2004. Mutual Fund Flows and Performance in Rational Markets, *Journal of Political Economy* 112 (6), 1269-1295.

[3] Brown, K., W.Van Harlow, and L. Starks, 1996. Of Tournaments and Temptations: An Analysis of Managerial Incentives in the Mutual Fund Industry, *Journal of Finance* 51, 85-110.

[4] Brunnermeier, M., and S. Nagel, 2004. Hedge Funds and the Technology Bubble, *Journal of Finance* 59(5), 2013-2040.

[5] Chevalier, J., and G. Ellison, 1997. Risk-taking by Mutual Funds as a Response to Incentives, *Journal of Political Economy.*

[6] Chordia, T., 1996. The structure of mutual fund charges. Journal of Financial Economics 41, 3-39.

[7] Christoffersen, S., 2001. Why Do Money Managers Voluntarily Waive Their Fees? *Journal of Finance* 56 (3), 1117-1140.

[8] Das, S., and R. Sundaram, 2002. Fee Speech: Signaling, Risk-sharing, and the Impact of Fee Structures on Investor Welfare, *Review of Financial Studies* 15 (5), 1465-1497.

[9] Elton, E., M. Gruber, and C. Blake, 2004. Incentive Fees and Mutual Funds, *Journal of Finance* 58 (2), 779-804.

[10] Fung, W., and D. Hsieh, 1999. A Primer on Hedge Funds, *Journal of Empirical Finance* 6, 309-331.

[11] Getmansky, M., A. Lo, and I. Makarov, 2004. An Econometric Model of Serial Correlation and Illiquidity in Hedge Fund Returns, *Journal of Financial Economics* 74 (3), 529-610.

[12] Goetzmann, W., J. Ingersoll, and S. Ross, 2003. High-water Marks and Hedge Fund Management Contracts, *Journal of Finance* 58 (4), 1685-1717.

[13] Grinblatt, M., and S. Titman, 1989. Adverse Risk Incentives and the Design of Performance-Based Contracts, *Management Science*, 35, 807-822.

[14] Hart, O., and Moore, J., 1988. Incomplete Contracts and Renegotiation, *Econometrica* 56, 755-85.

[15] Heinkel, R., and N. Stoughton, 1994. The Dynamics of Portfolio Management Contracts, *Review of Financial Studies* 7, 351-387.

[16] Hodder, J., and J. C. Jackwerth, 2004. Incentive Contracts and Hedge Fund Management, working paper, University of Wisconsin, Madison.

[17] Huberman, G., and S. Kandel, 1993. On the Incentives for Money Managers: A Signaling Approach, *European Economic Review* 37, 1065-1081.

[18] Lerner, J., and A. Schoar, 2004. The Illiquidity Puzzle: Theory and Evidence from Private Equity, *Journal of Financial Economics* 72 (2), 3-40.

[19] Lo, A., and C. MacKinlay, 1990.

[20] Lynch, A., and D. Musto, 2003. How Investors Interpret Past Fund Returns, *Journal of Finance* 58 (5), 2033-2058.

[21] Panageas, S., and M. Westerfield, 2004. High Water Marks? High Risk Appetite? Hedge Fund Compensation and Portfolio Choice, working paper, University of Pennsylvania.

[22] Pontiff, J., 1996. Costly Arbitrage: Evidence from Closed-end Funs, *Quarterly Journal of Economics* 111, 1135-1152.

[23] Shleifer, A., and R. Vishny, 1997. The Limits of Arbitrage, *Journal of Finance* 52 (1), 35-55.

[24] Starks, L., 1987. Performance Incentive Fees: An Agency Theoretic Approach, *Journal of Financial and Quantitative Analysis* 22 (1), 17-32.

[25] Stein, J., 2005. Why are Most Funds Open-end? Competition and the Limits of Arbitrage, *Quarterly Journal of Economics* 120, 247-272.

Table 1: Summary Statistics of Hedge Fund Sample

The table reports the summary statistics for 3501 hedge funds. The variable *hwm* equals 1 one if the fund has a highwater mark, and zero otherwise. $mfee$ and $ifee$ are the fixed percentage management fee and percentage incentive fee. *lockind* is an indicator variable equal to 1 if the fund has a lockup provision. *notice* is the fund's redemption notice period in days. The variable *age* equals the number of months between the fund's initial observation date and the earliest initial date across all funds within the same fund family. The variables *size* is the fund's initial reported estimated asset size (in \$millions). $offshore$ is an indicator variable equal to 1 if the fund is domiciled offshore. *perscap* equals 1 if the fund manager invests his own wealth in the fund's portfolio; and zero otherwise. Significance levels for each pairwise correlation is reported below each estimate.

	nobs	mean	median	sd	min	max
hwm	3501	0.37	0.00	0.48	0	1
mfee	3496	1.44	1.20	0.88	0	20
ifee	3497	16.95	20.00	7.09	0	50
lockind	3501	0.18	0.00	0.38	0	1
notice	3501	25.46	30.00	25.17	0	180
age	3501	19.35	1.00	35.24	1	304
size	2601	25.50	3.95	378.56	0	18574
offshore	3501	0.55	1.00	0.50	0	1
perscap	3501	0.45	0.00	0.50	0	1

Table 2: Correlation Matrix of Fund Characteristics

The table reports the pairwise correlations of fund characteristics for 3501 hedge funds. The variable *hwm* equals 1 one if the fund has a highwater mark, and zero otherwise. *mfee* and *ifee* are the fixed percentage management fee and percentage incentive fee. *lockind* is an indicator variable equal to 1 if the fund has a lockup provision. *notice* is the fund's redemption notice period in days. The variable *age* equals the number of months between the fund's initial observation date and the earliest initial date across all funds within the same fund family. The variables *size* is the fund's initial reported estimated asset size (in $millions). *offshore* is an indicator variable equal to 1 if the fund is domiciled offshore. *perscap* equals 1 if the fund manager invests his own wealth in the fund's portfolio; and zero otherwise. Significance levels for each pairwise correlation is reported below each estimate.

	hwm	mfee	ifee	lockind	notice	age	size	offshore	perscap
hwm	1.00								
	-								
mfee	-0.12	1.00							
	0.00	-							
ifee	0.17	0.07	1.00						
	0.00	0.00	-						
lockind	0.40	-0.12	0.09	1.00					
	0.00	0.00	0.00	-					
notice	0.29	-0.18	0.02	0.32	1.00				
	0.00	0.00	0.35	0.00	-				
age	0.00	0.05	-0.04	-0.04	0.00	1.00			
	0.86	0.01	0.02	0.01	0.95	-			
size	-0.01	0.00	-0.01	-0.01	-0.02	0.00	1.00		
	0.72	0.89	0.70	0.56	0.24	0.96	-		
offshore	-0.08	0.10	-0.01	-0.20	-0.07	0.06	-0.02	1.00	
	0.00	0.00	0.66	0.00	0.00	0.00	0.31	-	
perscap	-0.25	-0.02	0.04	-0.10	0.00	-0.07	-0.04	-0.05	1.00
	0.00	0.28	0.01	0.00	0.94	0.00	0.07	0.00	-

Table 3: Probit Analysis of Highwater Mark

The table reports the results from a probit analysis of the *hwm* variable for 3501 funds. The variable *hwm* equals 1 one if the fund has a highwater mark, and zero otherwise. $mfee$ and $ifee$ are the fixed percentage management fee and percentage incentive fee. *lockind* is an indicator variable equal to 1 if the fund has a lockup provision. *notice* is the fund's redemption notice period in days. The variable *age* equals the number of months between the fund's initial observation date and the earliest initial date across all funds within the same fund family. The variables $size$ is the fund's initial reported estimated asset size (in $millions). $offshore$ is an indicator variable equal to 1 if the fund is domiciled offshore. *perscap* equals 1 if the fund manager invests his own wealth in the fund's portfolio; and zero otherwise. Estimated marginal effects for standardized (mean zero, variance 1 across firms) characteristics; heteroscedasticity-consistent t-statistics are reported below each estimate.

mfee	ifee	lockind	notice	age	size	offshore	perscap
-0.07	0.01						
(-3.05)	(10.53)						
-0.04	0.01	0.41	0.00				
(-2.50)	(9.93)	(16.44)	(9.88)				
-0.04	0.01	0.41	0.00	0.00			
(-2.51)	(10.00)	(16.50)	(9.90)	(1.89)			
-0.04	0.01	0.45	0.00	0.00	0.00		
(-2.18)	(8.00)	(14.47)	(9.58)	(2.80)	(0.27)		
-0.04	0.01	0.45	0.00	0.00	0.00	0.00	
(-2.18)	(8.00)	(14.20)	(9.58)	(2.79)	(0.28)	(0.16)	
-0.04	0.01	0.43	0.01	0.00	0.00	-0.02	-0.30
(-1.78)	(8.50)	(12.82)	(9.87)	(1.95)	(-0.46)	(-1.06)	(-13.45)

Table 4: Autocorrelation in Fund Returns (1994-2002)

This table reports the results of pooled estimation of autocorrelation coefficients. The coefficient ρ is estimated using the pooled regression,

$$r_{i,t} = \alpha + \rho r_{i,t-1} + \epsilon_{i,t}$$

for all funds i in a given style category. The coefficients ρ^+ and ρ^- are estimated using the pooled regression,

$$r_{i,t} = \alpha + \rho^+[r_{i,t-1}]^+ + \rho^-[r_{i,t-1}]^- + \epsilon_{i,t}$$

Results are reported for monthly (mth), quarterly (qtr), semi-annual (sann), and annual (ann) return horizons. Funds are split into highwater mark (hwm=1) and no highwater mark (hwm=0) groups. Standard errors are calculated assuming clustering at each observation date. t-statistics are reported in the row following the point estimates.

	Aggregate (hwm=37%)						Convertible Arbitrage (hwm=45%)					
	hwm=0			hwm=1			hwm=0			hwm=1		
	ρ	ρ^+	ρ^-	ρ	ρ^+	ρ^-	ρ	ρ^+	ρ^-	ρ	ρ^+	ρ^-
mth	0.09	0.10	0.08	0.10	0.14	0.04	0.20	0.27	0.12	0.31	0.39	0.21
	3.54	2.76	2.20	2.03	2.30	0.52	4.11	5.79	1.22	3.67	7.84	1.01
qtr	0.05	0.07	0.00	0.06	0.15	-0.17	0.17	0.26	0.00	0.01	0.29	-0.58
	1.22	1.76	0.04	0.81	2.61	-1.04	2.27	3.09	-0.02	0.06	4.09	-3.00
sann	0.08	0.11	-0.02	0.20	0.22	0.11	0.12	0.10	0.18	-0.12	0.03	-0.68
	1.37	1.33	-0.16	3.59	4.02	0.66	1.17	0.81	0.81	-0.95	0.31	-1.48
ann	-0.03	-0.01	-0.25	0.01	0.04	-0.16	0.02	-0.07	0.48	-0.12	0.07	-1.36
	-0.84	-0.11	-1.00	0.24	0.54	-0.57	0.18	-0.63	1.67	-1.28	0.53	-4.01

Table 4: cont.

	Short Bias (hwm=52%)						Emerging Markets (hwm=20%)					
	hwm=0			hwm=1			hwm=0			hwm=1		
	ρ	ρ^+	ρ^-	ρ	ρ^+	ρ^-	ρ	ρ^+	ρ^-	ρ	ρ^+	ρ^-
mth	0.03	-0.09	0.18	0.07	0.23	-0.10	0.19	0.24	0.13	0.18	0.13	0.23
	0.40	-0.80	1.64	0.80	2.02	-0.88	4.63	4.19	1.59	3.08	1.39	2.71
qtr	-0.11	-0.15	-0.06	-0.11	-0.05	-0.18	0.07	0.06	0.10	-0.01	0.05	-0.10
	-0.92	-0.90	-0.25	-0.69	-0.29	-0.46	0.79	0.81	0.42	-0.06	0.55	-0.48
sann	0.01	-0.23	0.30	-0.04	0.09	-0.26	0.06	0.11	-0.07	0.12	0.04	0.28
	0.11	-1.84	1.84	-0.25	0.58	-0.61	0.54	1.54	-0.16	0.97	0.27	0.83
ann	0.07	-0.06	0.21	-0.12	-0.33	0.13	-0.21	0.03	-1.10	-0.31	-0.13	-0.68
	0.31	-0.35	0.41	-1.10	-1.46	0.43	-1.56	0.27	-3.57	-1.99	-0.82	-3.39

	Equity Market Neutral (hwm=49%)						Event Driven (hwm=44%)					
	hwm=0			hwm=1			hwm=0			hwm=1		
	ρ	ρ^+	ρ^-	ρ	ρ^+	ρ^-	ρ	ρ^+	ρ^-	ρ	ρ^+	ρ^-
mth	0.09	0.14	0.00	0.11	0.20	-0.05	0.17	0.14	0.23	0.16	0.17	0.14
	2.15	3.20	0.04	2.03	3.13	-0.53	4.10	2.53	3.53	3.99	2.94	2.27
qtr	0.10	0.21	-0.14	0.15	0.25	-0.14	0.08	0.11	0.03	0.10	0.28	-0.26
	1.04	3.34	-0.67	1.82	2.01	-1.20	1.33	1.53	0.26	1.25	3.06	-2.35
sann	0.24	0.29	0.00	0.19	0.22	0.04	0.07	0.14	-0.10	0.06	0.18	-0.43
	6.18	9.53	0.03	1.47	1.50	0.24	0.82	1.67	-0.55	0.50	2.16	-1.06
ann	0.14	0.14	0.11	0.25	0.24	0.53	-0.07	-0.05	-0.14	0.03	0.10	-0.68
	1.49	1.15	0.31	2.57	2.57	1.61	-0.92	-0.60	-0.59	0.18	0.68	-1.45

Table 4: cont.

	Fixed Income Arbitrage (hwm=43%)						Funds of Funds (hwm=29%)					
	hwm=0			hwm=1			hwm=0			hwm=1		
	ρ	ρ^{+}	ρ^{-}	ρ	ρ^{+}	ρ^{-}	ρ	ρ^{+}	ρ^{-}	ρ	ρ^{+}	ρ^{-}
mth	0.19	0.16	0.21	0.29	0.39	0.20	0.12	0.12	0.12	0.22	0.19	0.29
	3.25	3.21	2.10	2.77	7.44	1.13	3.21	2.03	2.77	3.33	1.80	3.52
qtr	0.24	0.23	0.26	0.28	0.40	0.09	0.07	0.08	0.06	0.20	0.21	0.18
	2.42	1.46	1.57	2.47	4.72	0.36	1.49	1.31	0.80	2.30	1.98	1.14
sann	0.21	0.41	-0.11	0.31	0.49	-0.11	0.03	0.12	-0.18	0.30	0.32	0.24
	2.38	4.29	-0.98	2.22	3.93	-0.54	0.58	1.18	-0.74	2.79	2.88	0.89
ann	-0.12	0.09	-0.56	-0.10	0.10	-1.22	-0.11	-0.11	-0.09	-0.06	-0.03	-0.20
	-0.72	0.46	-1.60	-0.55	4.32	-1.47	-1.14	-1.04	-0.39	-0.56	-0.21	-1.62

	Global Macro (hwm=24%)						Long Short Equity (hwm=48%)					
	hwm=0			hwm=1			hwm=0			hwm=1		
	ρ	ρ^{+}	ρ^{-}	ρ	ρ^{+}	ρ^{-}	ρ	ρ^{+}	ρ^{-}	ρ	ρ^{+}	ρ^{-}
mth	0.11	0.09	0.14	0.05	0.07	0.01	0.10	0.11	0.09	0.09	0.14	0.01
	3.00	1.31	2.87	0.78	0.82	0.04	1.86	1.43	0.97	1.55	1.83	0.05
qtr	0.07	0.04	0.12	0.09	0.15	-0.10	0.08	0.13	-0.04	0.08	0.17	-0.15
	1.38	0.65	1.14	0.88	1.20	-0.75	1.31	3.01	-0.26	1.02	2.43	-0.69
sann	0.21	0.13	0.43	0.13	0.20	-0.15	0.17	0.16	0.22	0.24	0.24	0.24
	3.69	1.62	3.23	2.17	3.15	-0.74	1.44	0.95	1.35	4.21	3.76	1.60
ann	0.16	0.17	0.08	-0.03	-0.16	1.05	-0.03	-0.04	0.25	0.05	0.01	0.36
	1.34	1.38	0.22	-0.63	-1.28	2.15	-0.71	-1.52	1.18	0.50	0.12	0.99

Table 4: cont.

Managed Futures (hwm=14%)

	hwm=0			hwm=1		
	ρ	ρ^+	ρ^-	ρ	ρ^+	ρ^-
mth	0.00	0.02	-0.03	-0.05	0.04	-0.19
	0.01	0.47	-0.67	-0.84	0.62	-1.91
qtr	-0.03	-0.01	-0.09	-0.04	0.08	-0.39
	-0.99	-0.28	-1.16	-0.63	1.55	-2.05
sann	-0.06	0.00	-0.35	-0.06	-0.07	-0.06
	-1.46	0.01	-1.13	-0.65	-0.98	-0.17
ann	0.04	0.12	-0.28	-0.18	-0.04	-0.96
	0.89	2.37	-1.36	-1.43	-0.21	-1.23

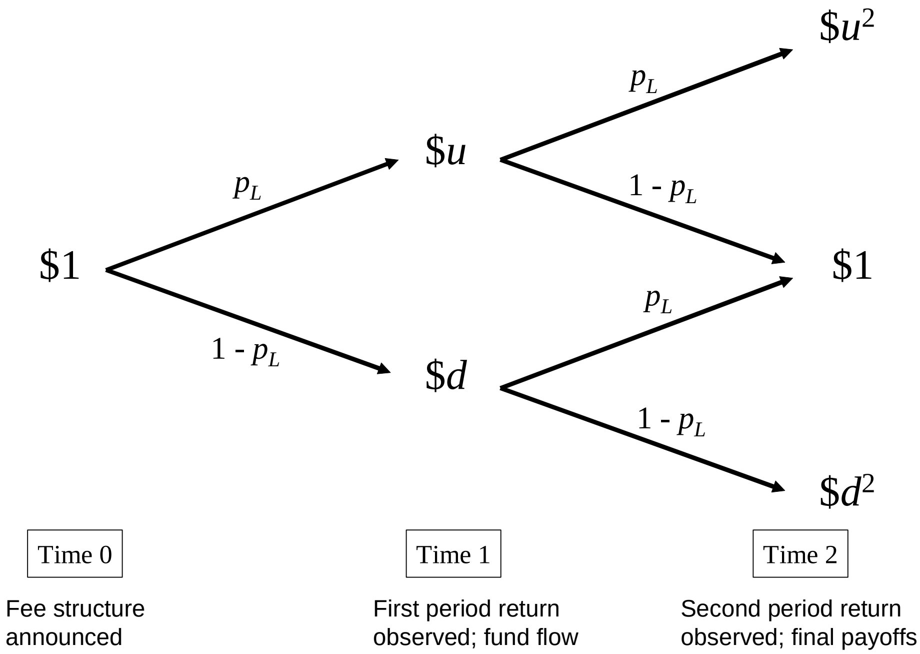

Figure 1-A Timeline/Payoffs of Liquid Assets

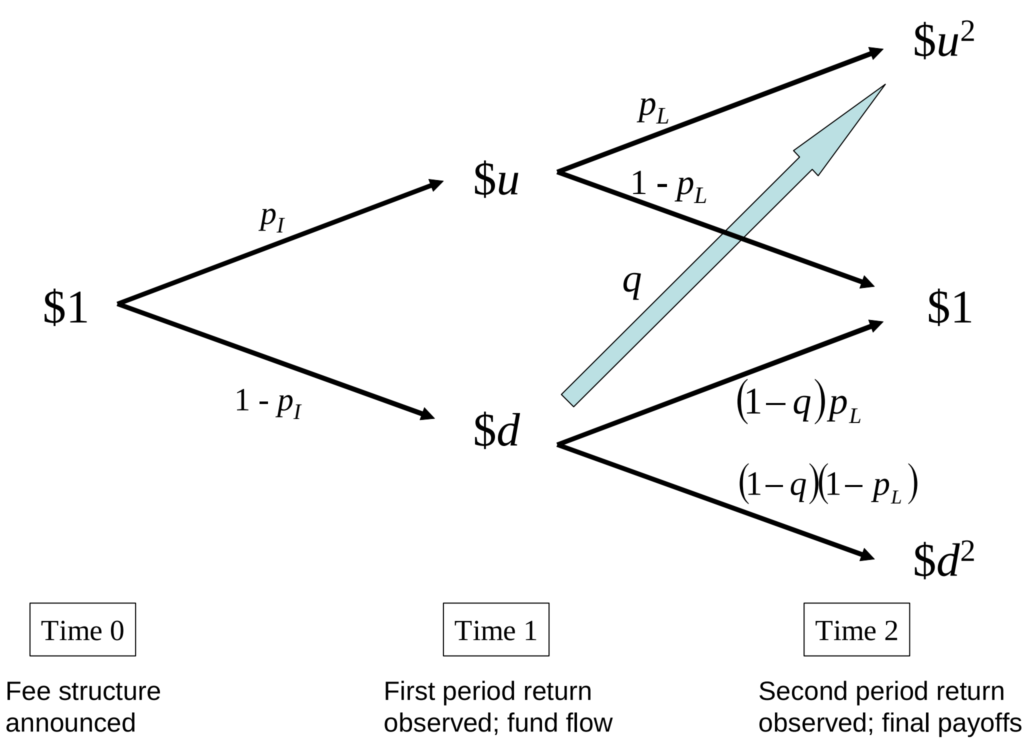

Figure 1-B Timeline/Payoffs of Illiquid Assets

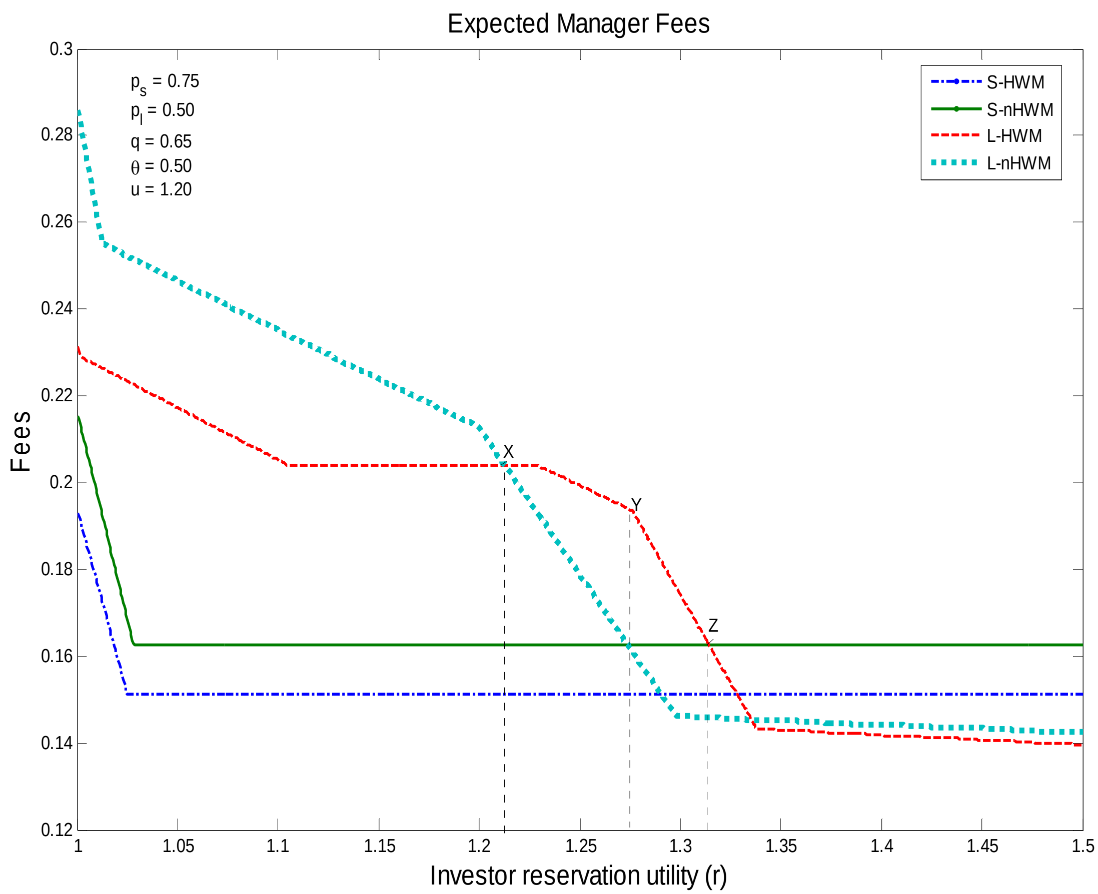

Figure 2 Expected Fees from Managing Liquid and Illiquid Assets