Modelling Non-normality using Multivariate t: Implications to Asset Pricing

RAYMOND KAN and GUOFU ZHOU*

First draft: August 28, 2001
This version: April, 2003

*Kan is from the University of Toronto, Zhou is from Washington University in St. Louis. Kan gratefully acknowledges financial support from the Social Sciences and Humanities Research Council of Canada.

Modelling Non-normality using Multivariate t: Implications to Asset Pricing

ABSTRACT

In this paper, we propose to replace the widely used and firmly rejected normality assumption by a multivariate t distribution for asset returns data.

Ever since Fama (1965), Affleck-Graves and McDonald (1989), and Richardson and Smith (1993), among others, it is well known that asset returns do not follow a normal distribution. Despite of this, the normality assumption is still the working assumption of mainstream finance. For example, Fama and French (1992, 1993) and hundreds of related papers make this assumption that the asset returns are normally distributed. The reason for the wide use of the normality assumption is not because it models financial data well, but due to its tractability so that interesting economic questions can be asked without technical impediments.

The purpose of this paper is to advocate the use of a t distribution in placement of the normal for two reasons.[1] First, it models financial data well in many circumstances. Second, it is almost as tractable as the normal distribution with recent develop of the EM algorithm. Indeed, as we demonstrate later, the t distribution can explain observed large skewness and kurtosis of the data. For example, the multivariate normality assumption of the joint distribution of Fama and French's (1993) asset returns is unequivocally rejected by a kurtosis test with a P-value of less than 0.00001. On the other hand, such a test for a multivariate t distribution has a P-value of 0.55555! However, before the adaption of the multivariate t distribution as a working assumption, we have to address two questions: the importance and tractability. The answer to tractability is a short one. Based on recently developed EM algorithms and the associated asymptotic theory, statistical analysis under t can be done almost as easily as under t without none or little technical difficulties.

Now let us address the importance for the use of the t distribution. Assuming that asset returns are t distributed rather than the normal, we find that this changes drastically our understanding of some of the major issues. First, there is a substantial and economically important difference in estimating expected returns. For example, the expected returns or excess returns on the market index is commonly estimated by using the sample average of the returns. But this is only efficient if the data is normally distributed. Using the CSRP index as a proxy for the market, the skewness and kurtosis tests both strongly reject the normality assumption, but retains a t distribution with a degrees of freedom 8. Under this more realistic t distribution, the exact maximum likelihood estimates of the expected returns or excess returns can be easily obtained. For the last ten years of data before 2002, the differences in the monthly expected return and expected excess are 0.12%

[1]Various finance applications of t and generalized t in the univariate case can be found in Theodossiou (1998) and references therein. Although MacKinlay and Richardson (1991), Zhou (1993) and Geczy (2001) use multivariate t distributions, but their analysis focuses on how results under normality vary when under t without providing the results obtained based on the t assumption.

and 0.14%. When annualized, they are 1.Yof economically significant sizes. This points out that the t distribution plays an importance the estimation of the cost of capital.

Second, the asset beta estimation relies upon the difference between the normal and the t assumptions. Theoretically, under the usual normal assumption, the estimated the asset beta is the slope of the regress of the asset return or excess returns on that of the market. The joint distribution of this asset with others has nothing to do with the same estimates. In contrast, under the more likely t distributional assumption, the returns on other assets do provide statistically important information to better estimate the beta. For example, the beta of the first asset of the 25 of Fama and French (1993) is estimated as 0.98 under the normal, but 1.20 under the t.

Third, the t distribution makes a great difference in testing asset pricing models. At the start, we note that MacKinlay and Richardson (1991), Zhou (1994) and Geczy (2001), among others, do test the CAPM allowing t distributed returns. But the tests are normality based in the sense that the parameter estimates are identically to those of the normal. What the GMM method of MacKinlay and Richardson is to make the standard error of the parameter estimates greater than the normal case to account for the t distributed returns. Zhou and Geczy do the same with the distribution of the test statistics. In contrast, what we do here is to provide the most efficient parameter estimates under the t assumption and use these new estimates to form the appropriate test statistics. As our estimates are potentially much more precise than the usual ones, the associated tests should be more powerful. Indeed, previous tests are unable to reject the Fama-French (1993) three factors, but the new tests do so unequivocally.

Fourth, the Hansen-Jagannathan volatility bound can be much tighter with the recognition of t distributed asset returns. As is well shown now, most asset pricing models can be cast into the form of $E(Rm) = 1_N$ where m is the by stochastic discount factor. Hansen and Jagannathan (1991) show that the lowest volatility bound on m depends on the second order moments of the asset returns alone and independent of the stochastic discount factors. Hence, this bound can serve as useful diagnostic tools for various models. This bound is commonly estimated based on the sample moments. We show that, if the more realistic t assumption is used, the estimated bound is much larger than the sample analogue.

utility gain of incorporating t.

I. Why Multivariate t?

A. Data

B. Normality test

Simple introductions, skewness and kurtosis tests that reject the normal.

II. A comparison of Estimation Accuracy

A. Univariate Case

Intuitive and simple comparisons: estimate the mean and variance for market indices, size portfolios and FF assets under the t versus under normal, also compares with GMM which just enlarges the standard errors of the mean estimates, but the variance estimator may be OK (small sample may be bad).

B. Multivariate Case

III. Exact ML

technical section

A. EM Algorithm

B. Asymptotic Theory

IV. Asset Pricing Tests

V. Hansen-Jagannathan Bound

VI. Extensions to Elliptical Distributions

VII. Conclusions

In this paper, we

References

Geczy, Chris, 2001, Some generalized tests of mean-variance efficiency and multifactor model performance, Working Paper, University of Pennsylvania.

Theodossiou, Panayiotis, 1998, Financial Data and the Skewed Generalized t Distribution, Management Science, Vol. 44, No: 12-1, December 1998, 1650-1661.