# International Diversification with Large- and Small-Cap Stocks


## Abstract

To the extent that investors diversify internationally, large-cap stocks receive the lion's share of fund allocation. Increasingly, however, large-cap stocks or stock market indices tend to co-move, mitigating the benefits from international diversification. In contrast, stocks of locally oriented, small companies do not exhibit the same tendency. In this paper, we assess the potential of small-cap stocks as a vehicle for international portfolio diversification during the period 1980-1999. To that end, we form and utilize three market cap-based index funds, i.e., large-, mid-, and small-cap funds, from each of our sample countries. The key findings of our paper include: First, small-cap funds cannot be 'spanned' by stock market indices or large-cap funds. Further, international small-cap funds have relatively low correlations not only with large-cap funds, but also with each other. Thus, international diversification would be more effective with a combination of large- and small-cap funds than with large-cap funds alone. This can justify the recent proliferation of small-cap oriented international mutual funds in the U.S. Second, the optimal international portfolio tends to comprise the U.S. market index and foreign small-cap funds; neither foreign market indices nor mid-cap funds receive positive weights during our sample period. The extra gains from the augmented diversification with small-cap funds are statistically significant unless additional transaction costs for small-cap funds become excessive.

## 1. Introduction

Since the classic works of Grubel (1968), Levy and Sarnat (1970), and Solnik (1974), numerous studies have documented the gains from international portfolio diversification. As is well known, the gains from international diversification stem mostly from the relatively low correlation among international securities when compared to domestic securities. Further, previous studies, e.g., Heston and Rouwenhorst (1994) and Griffin and Karolyi (1998), have shown that relatively low international correlations and the associated diversification gains are attributable chiefly to country factors, rather than industry factors. Relatively low international correlations, together with the gradual liberalization of capital markets, are indeed responsible for the rising volume of cross-border investments and proliferation of international mutual funds in the U.S. and abroad.

As international capital markets become more integrated, however, stock market correlations have risen, diminishing the potential gains from international diversification. Longin and Solnik (1995), for example, document that international correlations among stock market indices have indeed increased over the 30-year period 1960-1990. Goetzmann, Li, and Rouwenhorst (2004) also show that international correlations tend to be higher during the periods of higher economic and financial integration. Kasa (1992), on the other hand, argues that the stock market indices of major countries are cointegrated and they are driven by a set of common stochastic trends. He interprets the cointegration as implying that the gains from international diversification can be rather modest in the long run, partially justifying the observed 'home-bias' in portfolio holdings. Higher international correlations observed in recent years clearly cast doubt on the strength and validity of the case for international diversification argued by the classic studies. Increasingly, practitioners also express the same concern. For instance, the *Economist* recently remarked that: "Individual stockmarkets are increasingly driven by global rather than local factors….Globalization and the information-technology boom appear to have increased the importance of worldwide factors in steering share prices at the expense of local country factors."[1]

[1] The quotation is from an article, 'Dancing in Steps', the *Economist,* March 24, 2001, p.90.

To the extent that investors diversify internationally, large-cap stocks have received the lion's share of overseas investments. This 'large-cap bias' is understandable as investors naturally gravitate toward well-known, large foreign companies that are highly visible and often multinational.[2] The large-cap bias is also broadly consistent with the Huberman's (2001) proposition that familiarity breeds investment. Those investors especially institutional investors who track national market indices may also reinforce this bias as market indices are dominated by large-cap stocks. Similarly, in documenting the gains from international diversification, academic studies tend to use large-cap stocks or national stock market indices dominated by the former. The potential role of small-cap stocks in international diversification has not received attention in these studies.

As we show in this paper, the return generating mechanisms for large- and small-cap stocks are quite different. Specifically, returns on large-cap stocks are substantially driven by common global factors. In contrast, returns on small-cap stocks are primarily driven by local and idiosyncratic factors. This difference in return generating mechanism is understandable considering that many large-cap stocks tend to be those of multinational companies with a substantial foreign customer and investor base, whereas small-cap companies are likely to be more locally oriented with a limited international exposure. As a result, the gains from international diversification with large-cap stocks can be modest as their returns are substantially driven by common global factors. However, the same skepticism may not be applicable to small-cap stocks as their returns are substantially generated by local and idiosyncratic factors. Thus, small-cap stocks can potentially be an effective vehicle for international diversification.

It is against this backdrop that investment companies in recent years have introduced small-cap oriented international mutual funds, allowing investors to diversify into foreign small-cap stocks without incurring excessive transaction costs. Many investment companies such as Fidelity, ING, Lezard, Merrill Lynch, Morgan Stanley, Oppenheimer, and Templeton currently offer small-cap oriented international mutual funds in the U.S. The recent advent of international small-cap funds is thus highly instructive and also suggestive of the unique role that small-cap stocks can play in global

[2] In their study of foreigners' equity holdings in Japan, Kang and Stulz (1997) show that foreign investors prefer large, export-oriented, liquid, and U.S. cross-listed firms.

risk diversification.[3] Although there are currently more than 50 small-cap oriented international mutual funds in the U.S., little is known about the potential of small-cap stocks as a vehicle for international diversification.[4] The current paper purports to fill this gap in the literature.

Specifically, the purpose of this paper is to assess the potential benefits from international diversification with small-cap as well as large-cap stocks. We examine the issue from the perspective of a U.S. (or dollar-based) investor who has diversified internationally with MSCI country indices or large-cap stocks but desires to augment her investment with small-cap funds from major foreign countries. Our paper thus addresses the following question: Are there 'additional gains' from international diversification with small-cap stocks? In this study, we consider ten developed countries with relatively open capital markets – Australia, Canada, France, Germany, Hong Kong, Italy, Japan, the Netherlands, the United Kingdom, and the United States. Our sample comprises two countries from North America, three from Asia/Pacific, and five from Europe. It is noted that international investors do not face formal barriers to investing in stocks of these countries. For the sake of analytical tractability and also consistency with industry practices, we form three market capitalization-based funds, i.e., large-cap, mid-cap, and small-cap funds, from each of our sample countries and utilize the risk-return characteristics of cap-based funds computed over the 20-year period 1980-1999. Our analysis in this paper comprises two parts. In the first part, we examine the different return generating mechanisms for cap-based funds, the correlation structure of cap-based funds, and their implications for international diversification. In the second part, we conduct the mean-variance analysis of international portfolio investment with cap-based funds.

The key findings of our paper can be summarized as follows. First, our mean-variance spanning tests show that international small-cap funds are not 'spannable' by

[3] In terms of geographical coverage, some funds are global and international while others are regional and national (country). Examples of the existing small-cap oriented international mutual funds include Templeton Global Smaller Companies Fund, Merrill Lynch Global Small Cap Fund, Fidelity International Small Cap Fund, Morgan Stanley International Small Cap Fund, AIM Europe Small Company Fund, FTI European Smaller Companies Fund, Fidelity Japan Smaller Companies Fund, DFA Japanese Small Company Fund, DFA United Kingdom Small Company Fund, etc.

[4] According to the classification by *Strategic Insight*'s fund objective code, there are 57 small-cap oriented international mutual funds as of 2002 in the U.S.

country stock market indices. Small-cap fund returns are driven primarily by local and idiosyncratic factors. As a result, *small-cap funds have relatively low correlations not only with large-cap funds but also with each other*. In contrast, large-cap funds tend to have relatively high correlations with each other, reflecting their common exposure to global factors. During our sample period, for instance, the correlation between the U.S. and Netherlands large-cap funds is 0.61, whereas the correlation between small-cap funds from the two countries is only 0.17. Further, the correlation between the U.S. large- and Netherlands small-cap funds is 0.21. This distinct correlation structure suggests that each small-cap fund may behave as if it were an asset class unto itself. Generally speaking, large-caps are similar, but small-caps are distinct from each other. Our simulations indeed show that a fully diversified international 'large-cap' stock portfolio is about 9.2% as risky (measured by the portfolio variance) as a typical individual stock, but a fully diversified international 'large- and small-cap' stock portfolio is only 3.4% as risky as the average individual stock. Clearly, small-cap stocks can play an effective and unique role in global risk diversification.

Second, in order to assess the potential mean-variance efficiency gains from diversification with small-cap stocks, we solve for the optimal international portfolio using the historical risk-return characteristics of cap-based funds during the period 1980-1999. We use the ten MSCI country indices (proxies for large-cap funds), small-cap funds, and mid-cap funds. Without short sales for foreign stocks, a realistic restriction during much of our sample period, the optimal (tangency) portfolio consists of (i) the U.S. market index and (ii) international small-cap funds. It is noteworthy that neither any foreign market index nor mid-cap fund receives a positive weight in the optimal international portfolio during our sample period; neither does the U.S. small-cap fund. The optimal international portfolio augmented with small-cap funds has a Sharpe performance measure that is statistically significantly larger than that of the U.S. market index as well as that of the optimal portfolio comprising MSCI country indices. Our finding remains robust to a realistic range of 'additional costs' for investing in small-cap funds. In contrast, the optimal international portfolio comprising MSCI country indices has a Sharpe measure during our sample period, which is insignificantly different from that of the U.S. market index. Our key findings generally hold in sub-periods as well.

Similar to the tangency portfolio, the global minimum variance portfolio (MVP), which is computed solely with the covariance matrix, largely consists of the U.S. market index and international small-cap funds, with foreign market indices receiving negligible weights.

The rest of the paper is organized as follows. Section 2 describes the data, fund design, and the risk-return characteristics of cap-based funds. Section 3 (i) tests if small-cap funds can be spanned by country market indices or large-cap funds, (ii) investigates the return-generating mechanism for market cap-based funds, and (iii) assesses via simulations the capacity of small-cap stocks for global risk diversification. Section 4 discusses optimal international allocation strategies with small-cap funds and evaluates the gains from employing such strategies. Section 5 provides robustness checks of our key findings. Lastly, Section 6 offers concluding remarks.

## 2. Data, Fund Design, and Preliminary Analysis

Our data set includes monthly stock prices and returns, number of shares outstanding for exchange-listed companies and MSCI stock market indices from the ten major countries during the period January 1980 – December 1999. There exists a consensus among researchers that investors would not have faced major barriers to international investments during this period. We obtain the firm level data from CRSP for U.S. firms and from Datastream for international firms. Our sample includes all U.S. firms listed on the New York Stock Exchange, American Stock Exchange and Nasdaq and all foreign firms from each of the ten countries for which Datastream provides the necessary data during our sample period. We consider both active and inactive stock files to avoid a survivorship bias. Our findings reported in this paper thus are free from the survivorship bias. We obtain MSCI stock market indices from Datastream. In addition, we obtain the U.S. T-bill rate, which proxies the risk-free interest rate in our analysis, from CRSP.

For the sake of both analytical tractability and consistency with industry practices, we form three market cap-based funds (CBFs), i.e., large-cap, mid-cap, and small-cap funds, from each of our sample countries. To form the CBFs, we rank all our sample firms in each country based on their market capitalization at the end of each year. We then form a ‘large-cap fund’ with the top 20 percent of the largest-cap stocks, a ‘small-

cap fund' with the bottom 20 percent of the smallest-cap stocks, and a 'mid-cap fund' with the rest of stocks in each country. Further, we use the relative market value for each stock to determine its weight in the fund. We thus form three cap-based, value-weighted 'index' funds from each country. We then calculate the monthly (value-weighted) returns for each fund in terms of U.S. dollars. Since there are three funds from each of the ten countries, we generate 30 separate time series of monthly fund returns, in terms of U.S. dollars, over the 20-year period 1980 – 1999. CBFs are updated once a year based on the market capitalization of individual stocks at the end of each year.

Table 1 provides summary statistics for each of our sample CBFs. Specifically, Panel A of Table 1 provides the average number of stocks (No) comprising the fund, annualized mean ($\overline{R}$) and standard deviation ($\sigma$) of returns, the Sharpe ratio (SHP) for each CBF, and the fund's correlation with the (MSCI) U.S. market index ($\rho_{US}$). As can be seen from the panel, the large-(small-) cap fund includes, on average, 211 (200) stocks, whereas the mid-cap fund includes 629 stocks, on average. The number of stocks included in individual funds varies greatly, however, reflecting the different size of stock markets across our sample countries. As can be expected, the U.S. funds include the most stocks and the Italian and Dutch funds the least.

A few things are noteworthy from Panel A of Table 1. First of all, in the majority of countries, the small-cap fund has a higher mean return than large-cap fund, suggesting that the size premium exists in these countries. The "small-cap premium" is most pronounced in Canada and Australia. The U.S. and the Netherlands are the two exceptions to this; the mean return is a bit higher for the large-cap fund than for the small-cap fund in these countries.[5] In the majority of countries, the mid-cap fund has a lower mean return than its large-cap counterpart. This "mid-cap discount" is most pronounced in Germany, Hong Kong, and Italy. In Hong Kong (Italy), for example, the mean return is 22.1% (20.0%) for the large-cap fund and 16.0% (15.7%) for the mid-cap fund. In Germany, the mean return is 14.4% for large-cap, 11.0% for mid-cap, and 14.6% for small-cap funds. Large- and small-cap funds thus have practically the same mean returns in Germany, but the mid-cap fund has a significantly lower mean return. As

[5] Fama and French (1992) also find evidence that the size premium in the U.S. has become weaker in recent years. In fact, they document a negative and insignificant size premium during the period 1981-1990.

shown in the last row of the panel, the cross-country average of mean returns is 16.6% for large-cap funds, 14.8% for mid-cap funds and 21.1% for small-cap funds.

Second, with the exception of two countries, Germany and Italy, the small-cap fund has a greater return volatility than the large-cap fund. Among large-cap funds, the U.S. fund has the lowest volatility. This is a familiar result often attributed to the fact that the U.S. has the largest stock market and the returns are computed in U.S. dollar terms. Among mid-cap funds, however, each of the three foreign funds, i.e., Canada, Germany, and the Netherlands, has a lower volatility than the U.S. fund. Among small-cap funds, both German and Dutch funds have lower volatilities than the U.S. fund. Third, in every country, the large-cap fund has the highest correlation with the U.S. and the small-cap fund the lowest, with the mid-cap fund falling in between. Taking Canada for example, the correlation with the (MSCI) U.S. market index is 0.71 for the large-cap fund, 0.58 for the mid-cap fund, and 0.45 for the small-cap fund. Lastly, the Sharpe performance measures (SHP) indicate that in the majority of countries, the small-cap fund outperforms both the mid- and large-cap funds. The cross-country average Sharpe ratio is 0.46 for large-cap funds, 0.38 for mid-cap funds, and 0.57 for small-cap funds. The U.S. fund is the best performing one among all large-cap funds, but it is the second worst performing among all small-cap funds. In contrast, the Canada fund is the second worst performing among all large-cap funds, but it is the best performing among all small-cap funds. The national fund performance ranking varies greatly across market-cap classes.

Panel B of Table 1 reports the 'average' inter- and intra-category correlations among sample CBFs computed from the full correlation matrix over the 20-year period 1980-1999. If small-cap stocks are thinly traded, the correlations computed with monthly returns may result in an understatement of the true magnitude of correlations for small-cap funds. To alleviate this concern, we compute and present the average correlations using both monthly and quarterly return data in Panel B. Domestic (international) correlations are provided in the upper (lower) triangle. For brevity, we provide the full correlation matrix in Appendix A.

Overall, Panel B suggests that correlations among stocks are strongly influenced by both the country and market-cap classifications. When monthly returns are used, the average 'international' correlation is 0.44 among large-cap funds, 0.39 among mid-cap

funds, and 0.27 among small-cap funds. The intra-category correlation decreases as the market-cap decreases. To further examine this correlation structure, we plot the cumulative distribution function of correlation for each fund category in Panel A of Figure 1. The figure shows that the probability of observing a particular correlation or lower is always higher for small-cap funds than for either mid- or large-cap funds. In fact, there exists a first-order stochastic dominance of the distribution of small-cap fund correlation over those of mid- and large-cap fund correlations. Panel B of Figure 1 provides the results of Davidson-Duclos (2000) test, indicating that the reported stochastic dominance is statistically significant.[6]

Further, the average 'international' correlation of small-cap funds is 0.30 with large-cap funds and 0.31 with mid-cap funds. When quarterly returns are used, the average correlations tend to go up, but only slightly, suggesting that the effect of possible thin-trading of small-cap stocks on the reported correlation structure is not a serious problem. In what follows, we only use monthly return data. The correlation structure presented in Panel B of Table 1 clearly shows that small-cap funds have relatively low correlations not only with large- and mid-cap funds but also with each other. This correlation structure, in fact, suggests that each international small-cap fund may behave as if it were a distinct asset class in and of itself. In terms of reducing the portfolio risk, *international diversification is likely to be more effective with a combination of small- and large-cap stocks than with large-cap stocks alone*.

Panel B of Table 1 also shows that the average 'domestic' (i.e., same country) correlation is 0.87 between the large- and mid-cap funds, 0.66 between the large- and small-cap funds, and 0.83 between the mid- and small-cap funds. In comparison, the average 'international' correlation is 0.39 between the large- and mid-cap funds, 0.30 between the large- and small-cap funds, and 0.31 between the mid- and small-cap funds. The marked difference in domestic vs. international correlations among CBFs implies that domestic cross-cap diversification would be less effective than international cross-cap diversification in reducing the portfolio risk.

## 3. Mean-Variance Spanning Tests: Are Small-Caps Different?

[6] For detailed description of the Davidson-Duclos test, refer to Appendix B.

In this section, we (i) formally check if small-cap funds can be spanned by the MSCI country indices, (ii) examine the return-generating mechanism for market cap-based funds (CBFs) and their risk-return characteristics, and (iii) perform simulations to assess the capacity of small-caps for risk diversification. Although small-cap funds have relatively low pair-wise correlations with large-cap funds or country indices, the former may still be spanned collectively by the latter. If so, small-cap funds are redundant in the portfolio context and thus the 'additional gains' from international diversification with small-caps will be insignificant. If the spanning is rejected, on the other hand, small-cap funds can potentially play an important role in enhancing the gains from international diversification.

### 3.1. Are small-cap funds spanned by MSCI country indices?

Following Huberman and Kandel (1987), we check if small-cap funds can be spanned by MSCI country indices by regressing the 'new asset' (each CBF) on the 'benchmark assets' (ten MSCI country indices) as follows:

$$R_i = \alpha_i + \beta_i^{AU} MSCI^{AU} + ... + \beta_i^{US} MSCI^{US} + \varepsilon_i \quad , \tag{1}$$

where $R_i$ is the return on small-cap fund from the i-th country, $MSCI^{AU}$ ($MSCI^{US}$) denotes the return on the MSCI Australia (U.S.) country index, $\alpha_i$ is the estimated regression intercept for the small-cap fund, $\beta_i^{AU}$ ($\beta_i^{US}$) is the estimated regression coefficient associated with $MSCI^{AU}$ ($MSCI^{US}$) for the fund. The null hypothesis of spanning is equivalent to the joint hypothesis that $\alpha$ is equal to zero and the sum of $\beta$s is equal to one:

$$\alpha_i = 0, \quad \text{and} \quad \Sigma_i \beta_i = 1.$$

When there is only one new asset, as is the case with our analysis, the exact distribution of the likelihood ratio test under the null hypothesis is given by:

$$HK = \left( \frac{1}{V} - 1 \right) \left( \frac{T - K - 1}{2} \right) \quad , \tag{2}$$

where V denotes the ratio of the determinant of the maximum likelihood estimator of the error covariance matrix for the unrestricted model (no spanning) to that of the restricted

model (spanning), T is the number of observations, and K is the number of benchmark assets. The test statistic follows an F distribution with (2, T-K-1) degree of freedom.[7]

Table 2 reports the Huberman-Kandel mean-variance spanning test results for small-cap funds from each of the ten countries. As can be seen from the table, the spanning hypothesis is rejected for Canada, France, Germany, Japan, the Netherlands and the U.K. at the 1 percent level of significance, and for Australia and Italy at the 5 percent level. For the U.S. the spanning hypothesis is rejected at the 10 percent level. For Hong Kong, spanning cannot be rejected at the usual significance levels.[8] The spanning test results confirm that small-cap funds are indeed unique and investors may benefit from adding small-cap funds to their portfolio of country indices[9].

Also noteworthy from Table 2 is the fact that all of the small-cap funds have a significant and positive beta coefficient with respect to their own home country market index but rarely with respect to foreign market indices. It is also interesting to note that with the sole exception of Canada, each foreign small-cap fund has a *negative* beta against the U.S. country index. Table 2 further shows that the estimated alphas are all significantly different from zero, with the exception of Hong Kong and the U.S.

### 3.2. Return-generating mechanism for CBFs

To better understand the return behavior of market-cap sorted stocks, we estimate the extent to which returns to cap-based funds (CBFs) are driven by global and country factors. To this end, we employ a simple two-factor model to estimate the global and country betas for each CBF. Specifically, we estimate the global and country betas as follows:

$$R_{ij} = \alpha_{ij} + \beta_{ij}^{W} R^{W} + \beta_{ij}^{C} R_{i}^{C} + \varepsilon_{ij} \quad , \tag{3}$$

where $R_{ij}$ is the return on the j-th fund from the i-th country, $R^W$ is the return on the MSCI world index, a proxy for the return on the global market portfolio, $R_i^C$ is the portion of country i's national stock market index return that is uncorrelated to the return on the global market portfolio; it is the residual from the regressing the MSCI stock market

[7] For the derivation of Equation (2), readers are referred to Kan and Zhou (2001).
[8] When we form the Hong Kong small-cap fund from the smallest 10 percent of companies, the spanning hypothesis is rejected at the 10 percent level.

index return for country i on the MSCI world market index return. The coefficients $\beta_{ij}^{W}$ and $\beta_{ij}^{C}$ in Equation (3) denote the global beta and country beta for the j-th CBF from the i-th country, respectively. They measure the sensitivities of returns on CBFs to the global and country-specific factors.

Once the global and country betas are estimated, we can decompose a CBF's return variance into the following three components: (i) the proportion of the variance attributable to the global factor, (ii) the proportion attributable to the country factor, and (iii) the idiosyncratic risk of the fund, unrelated to either the global or country factor. Stated algebraically, we decompose $\mathrm{Var}(R_{ij})$ as follows:

$$\mathrm{Var}(R_{ij}) = (\beta_{ij}^{W})^2 \mathrm{Var}(R^{W}) + (\beta_{ij}^{C})^2 \mathrm{Var}(R_i^{C}) + \mathrm{Var}(\varepsilon_{ij}) \qquad (4)$$

We then calculate (i), (ii), and (iii) as follows:

(i) global factor proportion $= (\beta_{ij}^{W})^2 \mathrm{Var}(R^{W}) \,/\, \mathrm{Var}(R_{ij})$

(ii) local factor proportion $= (\beta_{ij}^{C})^2 \mathrm{Var}(R_i^{C}) / \mathrm{Var}(R_{ij})$

(iii) idiosyncratic factor proportion $= \mathrm{Var}(\varepsilon_{ij}) / \mathrm{Var}(R_{ij})$

Table 3 presents the estimates of the global and country betas and the idiosyncratic risk measures in Panel A and the variance decompositions in Panel B. Several things are noteworthy. First, regardless of the originating country and market-cap categories, each CBF in our sample has statistically significant global and country beta measures, attesting to the pervasive influences of global and country factors. However, in every country, the large-cap fund has higher global and country betas than the small-cap fund, with the mid-cap fund generally falling in between. In the case of the Netherlands, for example, the global (country) beta is 0.87 (0.85) for the large-cap fund, 0.72 (0.72) for the mid-cap fund, and 0.53 (0.53) for the small-cap fund. For the United States, the global (country) beta is 0.84 (0.99) for the large-cap fund, 0.90 (0.92) for the mid-cap fund, and 0.70 (0.75) for the small-cap fund. As can be seen from the last rows of Panel A, the sample average global (country) beta is 0.95 (0.97) for the large-cap funds, 0.82 (0.82) for the mid-cap funds, and 0.72 (0.71) for the small-cap funds. In contrast, the

[9] Since our empirical test is based on monthly return data, we do not think thin trading is a serious problem for our analysis. As a robustness check, however, we replicate the spanning test using quarterly data. The qualitative results remain unchanged.

sample average idiosyncratic risk measure, $\sigma(\varepsilon)$, is 0.011 for the large-cap funds, 0.032 for the mid-cap funds, and 0.055 for the small-cap funds. Compared with the large- and mid-cap funds, the small-cap funds are clearly driven much less by the world and country factors and much more by their own idiosyncratic factors. Consistent with this pattern, the adjusted R-square declines sharply as the market-cap of the fund declines. On average, the adjusted R-square is 0.970 for the large-cap funds, 0.724 for the mid-cap funds, and 0.415 for the small-cap funds.

The variance decompositions presented in Panel B of Table 3 shows, among other things, that idiosyncratic factors often account for more than 50 percent of the small-cap fund variance but less than 5 percent of the large-cap fund variance. Again, using the Netherlands for example, the global (country) factor accounts for 58.3% (37.9) of the total variance of the large-cap fund, 35.9% (24.2%) for the mid-cap fund and 17.2% (11.4%) for the small-cap fund. This means that the idiosyncratic factor accounts for 71.4% of the variance of the small-cap fund, 39.9% of the mid-cap fund, and only 3.8% of the large-cap fund. Similarly for the United States, the idiosyncratic factor accounts for 69.3% of the variance of the small-cap fund, 33.9% of the mid-cap fund, and only 1.6% of the large-cap fund. As can be seen from the last rows of Panel B, the idiosyncratic risk accounts, on average, for 3.0% of the variance of the large-cap funds, 27.3% of the mid-cap funds, and 58.0% of the small-cap funds. Clearly, large-cap funds are substantially driven by common global factors, whereas small-cap funds are primarily driven by idiosyncratic factors.

### 3.3. Risk diversification with small-cap stocks

The preceding analyses strongly suggest that small-cap stocks can be an effective vehicle for global risk diversification. To assess this potential, we perform an experiment that is similar to Solnik (1974). Specifically, we examine how the portfolio variance is reduced as we add more stocks to the portfolio during the simulation period January 1995 to December 1999. Due to the limited number of eligible sample stocks, we perform the simulation for a sub-period 1995-1999. We consider three diversification strategies: diversification across (i) U.S. large-cap stocks, (ii) international large-cap stocks, and (iii) international large- and small-cap stocks.

For our simulation, we only consider those stocks whose size membership does not change and for which there are no missing data for the entire simulation period. For U.S. large-cap diversification, we randomly pick 300 stocks from a pool of eligible U.S. large-cap stocks satisfying our criteria. We then randomly and repeatedly draw stocks with replacement from these 300 stocks to form equal-weighted portfolios with different numbers of stocks. The average portfolio variance is computed from 500 repetitions. Similar methods are applied to international diversification strategies to compute the average portfolio variance with different numbers of stocks. For international large-cap diversification strategy, we consider the 300 U.S. large-cap stocks (used for U.S. diversification) plus 600 foreign large-cap stocks. The latter comprises 50 Australian, 50 Canadian, 65 French, 65 German, 50 Hong Kong, 37 Italian, 155 Japanese, 28 Dutch, and 100 U.K. stocks. The number of stocks chosen for each country roughly reflects its relative value share in the world market portfolio during the simulation period. Lastly, for international large- and small-cap diversification, we use 900 large-cap stocks, i.e., 300 U.S. and 600 foreign, plus 827 small-cap stocks. The latter comprises 31 Australian, 124 Canadian, 55 French, 86 German, 27 Hong Kong, 32 Italian, 134 Japanese, 17 Dutch, and 53 U.K., and 268 U.S. stocks. These small-cap stocks represent the entire set of small-cap stocks in our sample that satisfy the selection criteria for simulation.

Figure 2 plots the portfolio variance, expressed as a percentage of the variance of a typical (or average) individual stock, as a function of the number of stocks included in the portfolio. Figure 2 shows that the variance of a fully diversified U.S. large-cap stock portfolio is 17.9% of the individual stock variance. On the other hand, the variance of a fully diversified international large-cap portfolio is 9.2% of the individual stock variance. This proportion is roughly comparable to the 11.7% for international diversification reported by Solnik (1974). Lastly, the variance of a fully diversified international 'large- and small-cap' stock portfolio is only about 3.4 % of the individual stock variance. Clearly, our findings here confirm the previous finding that international diversification reduces the portfolio risk beyond what's possible with domestic stocks. Furthermore, our findings show that augmented international diversification with large- and small-cap

stocks will be substantially more effective in reducing the portfolio risk than diversification with large-cap stocks alone.[10]

## 4. International Diversification with Cap-Based Funds

Our findings in the previous section strongly suggest that there will be additional gains in terms of additional risk reduction when investors diversify with small-cap as well as large-cap stocks. In this section, we extend our analysis to examine if small-cap funds can enhance the mean-variance efficiency of international portfolios. We first briefly describe the mean-variance intersection test for the portfolio efficiency gains, and conduct an analysis of the benchmark case, i.e., international diversification with MSCI country indices, against which the augmented diversification strategies will be compared. We then solve for the compositions of optimal international portfolios considering CBFs as well as MSCI country indices, and estimate the 'additional' mean-variance gains from international diversification with CBFs.

### 4.1. The mean-variance intersection test

In order to determine if the additional gains, in terms of mean-variance efficiency, from international diversification with CBFs are indeed significant, we must formally test the significance of the difference between the maximum Sharpe ratio attainable without CBFs and that attainable with CBFs. To do so, we employ the Sharpe ratio test proposed by Glen and Jorion (1993). We consider both the cases where short sales constraints are imposed and where they are not. One of the advantages of the Glen-Jorion test is its ability to allow for a large number of 'new' and 'benchmark' assets.[11]

Following Glen and Jorion (1993), we test the significance of the diversification benefit from adding new assets as follows:

[10] Although the simulation results are from a subperiod 1995-1999, there is no particular reason to believe that the results will be substantially different for the rest of our sample period.

[11] Unlike the mean-variance spanning tests checking if the mean-variance frontier of the benchmark assets plus the new assets coincides with the frontier of the benchmark assets only, the intersection tests performed here check if the two frontiers have one point in common, i.e., an intersection. In the case of spanning, no mean-variance investors can benefit from adding the new assets to the benchmark assets. In the case of intersection, on the other hand, there is one mean-variance utility function for which there is no benefit from adding the new assets. For detailed discussions of this point, refer to De Roon and Nijman (2001).

$$F = \frac{T-(K+N)}{N}\frac{\hat{\theta}_2^2 - \hat{\theta}_1^2}{1+\hat{\theta}_1^2} \quad , \tag{5}$$

where T is the number of observations, K is the number of benchmark assets, N is the number of new assets, and $\hat{\theta}_1$ and $\hat{\theta}_2$ are the maximum sharpe ratios attainable by the benchmark assets and augmented assets (benchmark assets plus new assets), respectively. When short sales are allowed, the test statistic follows a F distribution with (T-K-N, N) degree of freedom.

When short-sale constraints are imposed, however, the test statistic follows an unknown distribution and should be approximated by simulation. As proposed by Glen and Jorion, we derive the expected return, variance, and covariance of both benchmark and new assets from historical data. Then, the expected returns of the new assets are so modified that the new assets are spanned by the benchmark assets. In particular, we modify the expected returns of the new assets to be proportional to their betas to the optimal risky portfolio of benchmark assets. Then, at each simulation experiment, we draw T random samples of joint returns from a multivariate normal distribution with those parameters. From these simulated returns, we solve the optimization problem and calculate the test statistic as before. The process is repeated 2,000 times and the 1, 5, and 10 percent critical values are documented. It is pointed out that the Sharpe ratio test is, in fact, a mean-variance intersection test. Testing whether the maximum Sharpe ratio attainable by benchmark assets is equal to that attainable by augmented assets is equivalent to testing whether the mean-variance frontier spanned by benchmark assets intersects that spanned by the augmented assets at the tangent portfolio.

### 4.2. Diversification with country market indices: The Benchmark Case

Since we are interested in assessing the 'additional gains' from augmented international diversification with small-cap funds, it would be useful to first examine the benchmark case of diversification with country market indices. To that end, we provide basic parameter values for MSCI country stock market indices computed over our 20-year sample period and the composition of optimal international portfolios in Table 4.[12]

[12] Unlike the correlations or covariances, the historical mean returns tend to be highly unstable over time. As a result, the composition of the 'ex post' optimal portfolio solved with historical mean returns tends to be unstable over time and often assigns unrealistic weights to some of the constituent assets. These problems, which are endemic to 'ex post' mean-variance optimization, tend to be mitigated with ex ante

As can be seen from Table 4, pair-wise stock market correlation ranges from 0.23 to 0.72, with an average of 0.44 during our sample period. The U.S. market has relatively low correlations with Japan (0.26) and Italy (0.27), and relatively high correlations with the U.K. (0.56), the Netherlands (0.60), and Canada (0.72). Neighboring markets tend to have relatively high correlations: For instance, the correlation is 0.67 for France-Germany, 0.69 for the Netherlands-U.K, and 0.72 for Canada-U.S. But there are many exceptions to this. Despite a substantial geographical distance, Canada and Australia have a relatively high correlation, 0.60, perhaps reflecting the similar resource-based economies. Despite the geographical proximity, Japan and Hong Kong have a relatively low correlation, 0.24. Similarly, the correlation is relatively low, 0.37, for Italy-U.K. Overall, Japan has the lowest average correlation, 0.29, with other markets and the Netherlands the highest, 0.54. This reflects the relatively insular nature of the Japanese economy and highly multinational nature of the Dutch economy, respectively.

During our 20-year sample period, the mean monthly return ranges from 1.01% for Canada to 1.81% for Hong Kong. The standard deviation of returns ranges from 4.29% for the U.S. to 9.74% for Hong Kong. Clearly, Hong Kong is a high risk and high return market. The world systematic risk (beta), on the other hand, ranges from 0.83 for the U.S. to 1.22 for Japan. The Sharpe performance measures indicate that the U.S. is the best performing market, closely followed by the Netherlands. Other markets lag substantially behind the two best performing markets in terms of the risk-adjusted performance measure. Canada and Australia register the worst performances in terms of Sharpe measure.

The last two columns of Table 4 provide the compositions of optimal (tangent) international portfolios. When short-sales are not allowed, the optimal portfolio is dominated by the U.S. and Dutch markets. Specifically, the optimal portfolio consists of investing 53.2% in the U.S., 34.5% in the Netherlands, 7.7% in Italy, 3.1% in Hong Kong, and 1.5% in Japan. The Sharpe performance measure for the optimal international portfolio is 0.243, which is compared with 0.217 for the U.S. market index. This

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optimization allowing for the parameter uncertainty. In spite of these problems, we conduct ex post mean-variance optimization in this section as it can be useful for documenting the potential gains from a particular investment strategy. The purpose of this section is to document such 'potential gains' in terms of mean-variance efficiency from international investment with small-cap stocks.

difference in the Sharpe ratios is found to be statistically insignificant. This means that during our sample period, U.S. investors could not have gained significantly from international diversification with country indices. When short-sales are allowed, the Canadian, French and German markets receive negative weights in the optimal portfolio. The Sharpe ratio is a bit higher with short-sales, 0.290. But again, it is statistically insignificantly different from that of the U.S. market index. This result is in contrast to the previous findings that tend to support international diversification.[13]

### 4.3. The optimal global asset allocation

Given that significant gains might be achieved by diversification across international CBFs, we examine the optimal global asset allocation with MSCI country indices and CBFs in a Markowitz framework. In particular, we solve for the optimal international portfolio by maximizing the mean excess return per standard deviation of returns. As a proxy for the risk-free interest rate, we use the average one-month U.S. T-bill rate, 0.554%, over our sample period. The optimal portfolio is thus the one with the highest Sharpe ratio among all feasible portfolios.

We first solve for the optimal international portfolio with MSCI country indices and small-cap funds. The results are presented in Panel A of Table 5. When short-sales are not allowed, a realistic constraint in international investment, the optimal portfolio consists of investing 26.0% in the U.S. MSCI country index and 74.0% in eight foreign small-cap funds from Australia (1.2%), Canada (22.3%), Germany (10.8%), Hong Kong (4.5%), Italy (9.0%), Japan (12.5%), the Netherlands (2.0%), and the U.K (11.7%). Only France and U.S. small-cap funds are excluded from the optimal portfolio.[14] Remarkably, no foreign MSCI 'country index' receives any positive weight in the optimal portfolio. This particular composition of optimal international portfolio implies that it is more desirable to combine foreign small-cap funds, rather than foreign market indices, with the U.S. market index to enhance the portfolio efficiency. The fact that eight out of ten small-

[13] The same result is found to hold for each sub-period, 1980-89 and 1990-99, as well. In other words, the Sharpe ratio of optimal benchmark portfolio (comprising MSCI country indices) is not statistically significantly different from that of the U.S. market index in both sub-periods.

[14] During much of our sample period, it was difficult to take a short position in foreign stocks. In Hong Kong, for instance, investors were allowed to short a small number of large-cap stocks, but not small-cap

cap funds are included in the optimal portfolio reflects the relatively low correlations among these funds. The U.S. small-cap fund is excluded from the optimal portfolio due to a relatively high correlation with the U.S. market index as well as a modest return. France's small-cap fund is excluded from the optimal portfolio due to its relatively high correlations with other European small-cap funds, especially German and Netherlands funds. When short-sales are allowed, on the other hand, all 'foreign' small-cap stocks as well as three MSCI country indices, i.e., Hong Kong, the Netherlands, and the U.S., receive positive weights in the optimal portfolio, whereas the U.S. small-cap fund and the remaining MSCI country indices receive negative weights.

At the bottom of Panel A, we report the mean, standard deviation, and the Sharpe ratio for the optimal international portfolio. The optimal international portfolio without (with) short-sales has a Sharpe ratio of 0.322 (0.464), which is statistically significantly greater than the Sharpe ratio for the optimal international portfolio comprising only MSCI country indices, 0.243 (0.290). Both an increased return and reduced risk contribute to the higher Sharpe ratio for the augmented international optimal portfolio. This means that the gains from the augmented international diversification with small-cap funds are significant. Figure 3 illustrates the preceding analysis. Note from the lower panel of Figure 3 that several small-cap funds are located above the efficient frontier spanned by MSCI country indices.

Panel B of Table 5 reports the composition of optimal international portfolio for the case where investors diversify across MSCI country indices and mid-cap funds. When short-sales are not allowed, the US country index receives a dominant weight (52.2%). In addition, country indices of the Netherlands, Hong Kong, and Italy receive positive weights. In contrast to the case of diversification with small-cap funds, only four mid-cap funds are included in the optimal portfolio, with a combined weight of 17.8%. The associated Sharpe ratio is 0.245, which is less than that attainable with small-cap funds and statistically insignificantly different from that attainable with MSCI country indices only. A similar situation prevails when short sales are allowed. Overall, the extra gains from the augmented international diversification with mid-cap funds are insignificant.

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stocks. In Japan, investors were required, until recently, to receive permission from the Ministry of Finance to short stocks.

Next, we evaluate the additional gains from international diversification considering both small- and mid-cap funds simultaneously, in addition to MSCI country indices. Table 6 provides detailed statistical results. Several interesting findings emerge from this exercise. First, when short sales are not allowed, mid-cap funds receive zero weights in the optimal portfolio, suggesting that mid-cap funds are 'redundant' once investors hold country indices and small-cap funds. Second, the US index is again the only country index receiving a positive weight in the optimal portfolio. Eight of the ten small-cap funds receive positive weights; the U.S. and France are the exceptions. Third, since mid-cap funds are redundant, the maximum attainable Sharpe ratio by MSCI country indices, mid- and small-cap funds is the same as that attainable by just MSCI country indices and small-cap funds. Fourth, when short sales are allowed, investors can significantly benefit from short-selling mid-cap funds and thereby increasing investments in small-cap funds and country indices. In the optimal portfolio, mid-cap funds receive a combined weight of - 447%, small-cap funds 408% and MSCI country indices 139%. These rather extreme investment weights stem from the assumption of unrestricted short sales. The optimal portfolio has a Sharpe ratio of 0.624, which is significantly higher than that for any other portfolio that we have considered so far. During our sample period, mid-cap funds only play a significant role in international investment as a shorting opportunity.

Table 7 provides a summary of the Sharpe ratio test results for eight sets of benchmark assets and new assets. Column 1 reports the benchmark assets and new assets considered in the test, with the former stated in the first row and the latter in the second row. Columns 2 and 3 report the maximum attainable Sharpe ratios for the benchmark and augmented assets, respectively, with short-sales allowed. The test statistic (F-stat) is reported in column 4, with the p-value in the parenthesis. Columns 5, 6, and 7, report the same set of statistics, but with no short-sales restrictions imposed. The simulated 1, 5, and 10 percent critical values are reported in columns 8, 9, and 10.

The test results reported in Table 7 can be summarized as follows. First, investors who hold the US market index do not benefit significantly by adding foreign country indices, regardless of whether short-sales constraints are imposed. Second, investors who hold the US country index benefit significantly if they add foreign small-cap funds to

their portfolio. This result is robust to short-sales constraints. Third, investors who hold a well-diversified portfolio of international country indices benefit significantly by adding small-cap funds to their portfolio. This result is again robust to short-sale constraints. Fourth, investors who hold a well-diversified portfolio of country indices and small-cap funds benefit significantly from adding mid-cap funds to their portfolio only when short sales are allowed. Otherwise, there are no significant gains from adding mid-cap funds.

## 5. Robustness Check and Discussion

In the preceding sections, we show that investors can significantly benefit from augmented international diversification with small-cap stocks. In this section, we check the robustness of this finding and also discuss a few related issues. In particular, we (i) assess the gains from augmented diversification over sub-sample periods, (ii) examine the composition of the global minimum variance portfolio, and (iii) examine how our results are sensitive to a range of 'additional costs' for small-cap stock investment.

### 5.1. Sub-period results

In this sub-section, we repeat our mean-variance portfolio analysis over two sub-periods, i.e., 1980-1989 and 1990-1999. For each sub-period, we solve for the optimal international portfolio first with MSCI country indices only and then with small-cap funds as well. For brevity, we focus on the case of no short sales, a more realistic case.

During the first sub-period 1980-1989, the optimal benchmark portfolio consists of investing in the market indices of Italy (10.9%), Japan (51.8%), the Netherlands (14.0%), and the U.S. (23.3%). On the other hand, the optimal augmented portfolio consists of investing 5% in the U.S. market index and 95% in the small-cap funds of France (0.4%), Hong Kong (1.9%), Italy (13.4%), Japan (56.1%), the Netherlands (10.0%), and the U.K. (13.2%). In both the optimal benchmark and augmented portfolios, Japan receives a dominant weight, reflecting the prolonged appreciation of Japanese stock markets during the 1980s. During the second sub-period 1990-1999, the optimal benchmark portfolio is comprised of Hong Kong (3.3%), the Netherlands (29.5%), and the U.S. (67.2%) market indices. The optimal augmented portfolio, on the other hand, consists of investing 40.8% in the U.S. market index and 59.2% in small-cap funds of Australia (7.4%), Canada (31.9%), Germany (18.6%), and Hong Kong (1.4%). A

significant weight accorded to the U.S. market index is attributable to the robust performance of U.S. stock markets during the 1990s.

As is the case with the overall sample period, neither foreign stock market indices nor the U.S. small-cap fund receive any positive weights in the optimal augmented portfolios in either sub-period, attesting to the potentially important role that foreign small-cap funds play in international diversification. In the first (second) sub-period, the optimal augmented portfolio has a Sharpe ratio of 0.449 (0.445), which is significantly higher than that of the optimal benchmark portfolio, 0.274 (0.308). It is noted that the Sharpe ratio of the optimal benchmark portfolio is not significantly different from that of the U.S. market index in both sub-periods.[15] Since the Sharpe ratio of U.S. market index is only 0.151 (0.293) in the first (second) sub-period, the optimal augmented international portfolio significantly outperforms the U.S. market index in both sub-periods. The gains from augmented diversification with small-cap funds thus remain robust across sub-sample periods.

**5.2. Small-cap funds and the global minimum variance portfolio**

Apart from the tangency portfolio, the global minimum variance portfolio (MVP) is another notable portfolio located on the minimum variance frontier. It is thus useful to briefly examine the composition of the MVP. Table 8 provides the MVP comprising MSCI country indices in Panel A and the augmented MVP comprising MSCI country indices and small-cap funds in Panel B. Again, we consider no short-sales case. A few things are noteworthy. As can be seen from Panel A, the U.S. country index tends to receive a large weight in the MVP, i.e., 60.5% during the overall sample period and 59.4% (48.7%) during the first (second) sub-sample period. In addition, the German, Italian, Japanese, and Dutch country indices tend to receive substantial weights. Once small-cap funds are considered, however, foreign country indices are largely excluded from the MVP. Panel B shows that during the overall sample period 1980-1999, the MVP consists of investing roughly 40% in the U.S. market index and 60% in small-cap funds. During the first sub-period 1980-1989, the MVP comprises the U.S. market index (41.6%), foreign country indices (6.7%), and foreign small-cap funds (51.7%). During

[15] For the significance test of the difference in the Sharpe ratios of the optimal 'benchmark' portfolio and the U.S. market index, F-statistic (P-value) is 0.627 (0.14) for the first sub-period 1980-89 and 0.100 (0.63) for the second sub-period 1990-99.

the second sub-period 1990-1999, on the other hand, the MVP comprises the U.S. market index (34.5%), Australian market index (6.0%), and small-cap funds (59.5%). Panel B also shows that the standard deviation of the augmented MVP is significantly less than that of the MVP comprising only country indices. This holds not only for the overall sample period but also for each of the sub-sample periods.

The composition of the MVP is interesting for another reason. Due to the difficulty in differentiating funds based on the future expected returns, the historical MVP, which is solved solely based on the covariance matrix, is often regarded as an ex ante mean-variance efficient portfolio. Substantial weights allocated to small-cap funds in the MVP thus suggest that these funds can play an important role in *ex ante* efficient portfolios, as well as in *ex post* efficient portfolios as we have documented in the previous section. A full-fledged *ex ante* analysis, however, is deferred to future research.[16]

**5.2. Effect of additional costs for small-cap funds**

In the preceding analyses, we have not allowed for the possibility that investors may incur higher transaction costs for investing in small-cap stocks than for investing in large-cap stocks. If investors incur excessive transaction costs, the extra gains from international diversification with small-cap stocks can be illusory. To examine this issue, we compare the actual trading costs of small- vs. large-cap stocks. In an extensive study of trading costs in 37 countries, Chiyachantana et al. (2003) document that the difference in one-way trading costs between small-cap and large-cap stocks averages 0.55% to 0.64% based on the 2001 institutional transaction data, considering both explicit commission costs and implicit price impact costs. In addition, they show that trading costs have been steadily declining over time. Their finding suggests that for 100% annual turnover, for instance, the 'additional' trading costs for small-caps can be as high as 1.10% - 1.28%. For passively managed index-style funds with lower turnover ratios, the

[16] It is also pointed out here that in this study, investors are assumed to bear exchange rate uncertainty, rather than trying to hedge against it. For a recent discussion of these issues, readers are referred to Larsen and Resnick (2000).

additional trading costs will be less than the range.[17] The additional costs will be even lower for investors who pursue 'buy and hold' strategies.

For individual investors who use mutual funds for diversification, expense ratios are an important component of investment costs. It would thus be instructive to examine the expense ratios of existing mutual funds. To that end, we examine the CRSP mutual fund database that covers all U.S.-based mutual funds and is free from survivorship bias. We find that during the period 1992-99, the average expense ratio is 1.94% for small-cap oriented international funds (as classified by *Strategic Insight*) and 1.84% for the rest of international funds. Thus, expense ratios can be higher for small-cap funds than for large-cap funds but the difference do not appear to be significant.

To assess the effect of 'additional costs' for investing in small-cap stock investment, we impose what amounts to a proportional tax on small-cap funds at the start of each year. As with Stulz (1981), this tax is meant to capture broadly whatever additional cost investors may face when they invest in small-cap stocks. We then solve for the optimal international portfolio comprising country market indices (with no costs) and small-cap funds (with proportional costs) with short sales restrictions, and examine the portfolio efficiency. We repeat this analysis using different levels of additional costs for small-cap funds during our sample period. The results are provided in Table 9.

Table 9 shows different levels of proportional (additional) tax or cost per annum for small-cap funds in the first column. For each level of additional cost for small-cap funds, the table provides the optimal portfolio weights for the small-cap funds vs. country market indices, the Sharpe ratio of the optimal augmented portfolio, and the extra percentage return per annum on the augmented portfolio over the U.S. market index at the U.S.-equivalent risk level. This 'extra return' is computed as the difference in the Sharpe ratio between the optimal augmented international portfolio and the U.S. market index, multiplied by the standard deviation of the U.S. market index returns.

Table 9 shows that as the additional transaction costs for small-cap funds increase, the optimal portfolio weights allocated to small-cap funds continue to fall, as

[17] Examination of the turnover ratios for existing mutual funds shows that the turnover is likely to be less than 100%. For instance, according to *Fidelity Mutual Fund Guide 2003*, the turnover for Fidelity fund family is 85% for International Small Cap Fund and 50% for Japan Smaller Companies Fund in 2002. It is noted that these funds do not profess to be index funds.

does the Sharpe ratio of the optimal portfolio. At an additional cost of 1.5% (3.0%) per annum, for instance, the small-cap funds receive a 63.0% (53.3%) weight in the optimal portfolio as opposed to a 74.0% weight at no additional cost. At the same time, the Sharpe ratio declines from 0.322 with zero transaction costs to 0.303 (0.285). When the extra transaction costs are 3.5% per annum, the small-cap funds and MSCI country indices receive approximately equal weights in the optimal portfolio. This implies that unless the extra costs for small-caps are excessive, small-cap funds will receive significant weights in the optimal portfolio. Small-cap funds receive zero weight in the optimal portfolio once the transaction cost reaches 12% per annum. The relationship between portfolio weights and the extra costs for small-cap funds is illustrated in Figure 4.

Table 9 also shows that the gains from augmented international diversification with small-cap funds remain statistically significant so long as the additional transaction costs do not exceed 2% per annum. In view of the finding reported by Chiyachantana et al. (2003), the difference in trading costs between small and large international stocks is not likely to exceed 2% per annum unless the turnover exceeds 150%. Overall, the additional gains from augmented diversification with small-cap funds may remain significant unless the additional transaction costs are excessive. The last column of Table 9 provides the extra return accruing to the augmented optimal international portfolio above the return to the U.S. market index at the U.S. equivalent risk level. The extra return is 5.41% per annum when there are no additional costs for small-cap funds. The extra return declines to 4.74% (4.11%) at an additional transaction cost of 1.0% (2%). Even if the additional transaction cost exceeds 2% level, investors continue to optimally allocate substantial weights to the small-cap funds. At an additional cost of 3.5%, for example, investors still allocate a 50% weight to the small-cap funds and reap an extra return of 3.22% per annum at the U.S. equivalent risk level, as opposed to an 1.36% extra return when the optimal portfolio is exclusively comprised of MSCI country indices. Thus, there can be continuous economic gains although they may not be statistically significant. However, it is important to control transaction costs to maximize the extra gains from investing in small-cap funds.

Another practical difficulty in implementing small-cap diversification strategies arises from the limited size of small-cap markets. The limited market size suggests that it would not be practical for large institutional investors to allocate a significant portion of their funds to small-cap stocks. Thus, it may be prudent for large investors to impose restrictions on the portfolio weights for small-cap stocks. By contrast, small investors especially individual investors would not face such restrictions. Small investors thus can fully benefit from small-cap diversification strategies as documented in this paper. For large investors, however, the gains from international diversification with small-cap stocks can be limited.

## 6. Concluding Remarks

In this paper, we evaluated the potential of small-cap stocks as a vehicle for international portfolio diversification. To that end, we first formed three market cap-based index funds (CBFs) from ten major countries with well-developed, open capital markets and examined risk-return characteristics of these funds. We found that small-cap funds have low correlations not only with large-cap funds but also with each other. In contrast, large-cap funds tend to have relatively high correlations with each other, reflecting common exposures to global factors. Consistent with this correlation structure, we found that small-cap funds cannot be 'spanned' by country stock market indices that are dominated by large-cap stocks.

When we formed the optimal international portfolio using MSCI country indices, small- and mid-cap funds, only the U.S. country index and foreign small-cap funds receive positive weights; neither any foreign country indices nor any mid-cap funds receive positive weights in the optimal portfolio. When short-sales are allowed, mid-cap funds tend to receive negative weights allowing extra positive investments in small-cap funds and selected country indices. Overall, our findings indicate that investors can reap significant additional gains from international diversification if they consider foreign small-cap stocks. In contrast, the gains from international diversification with country market indices were found to be statistically insignificant during our sample period 1980-1999. The same result held during each of the two sub-sample periods. It is important,

however, to control investment costs to actually reap the additional benefits from diversifying with international small-cap stocks.

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**Table 1. Market Cap-Based Funds: Risk-Return Characteristics**

Panel A reports the average number of stocks (No), annualized mean return ($\overline{R}$), annualized standard deviation ($\sigma$), Sharpe ratio (SHP), and correlation with U.S. MSCI country index $\rho_{us}$, for each large- mid-, and small-cap fund. The sample period is from January 1980 to December 1999. The sample countries include ten developed countries, i.e., Australia, Canada, France, Germany, Hong Kong, Italy, Japan, the Netherlands, the U.K., and the U.S. The risk free interest rate is proxied by the average one-month Treasury-Bill rate during the sample period, 0.554%. At the beginning of each year, from 1980 to 1999, we rank all firms in each country based on their market capitalization values measured at the end of the previous year. Then, we form three cap-based funds for each country: large-, mid-, and small-cap funds. The large-cap fund consists of the 20 percent of stocks with the largest market capitalization values; the small-cap fund consists of the 20 percent of stocks with the smallest capitalization values; the mid-cap fund contains the rest of stocks. During the year, we calculate each portfolio's monthly value-weighted return. A firm's weight in the portfolio is proportional to its market capitalization value at the end of the previous month. Panel B reports the average correlations among cap-based funds, using monthly (B.1) and quarterly (B.2) return data. The numbers within the upper triangle are intra-country correlations, while the others are inter-country correlations.

***Panel A. Risk, Return, and Performance***

| *Countries* | *Large-Cap Funds* | | | | | *Mid-Cap Funds* | | | | | *Small-Cap Funds* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *No* | $\overline{R}$ | $\sigma$ | *SHP* | $\rho_{us}$ | *No* | $\overline{R}$ | $\sigma$ | *SHP* | $\rho_{us}$ | *No* | $\overline{R}$ | $\sigma$ | *SHP* | $\rho_{us}$ |
| Australia | 92 | 14.9% | 25.7% | 0.32 | 0.45 | 274 | 15.5% | 23.0% | 0.38 | 0.36 | 92 | 24.9% | 33.1% | 0.55 | 0.22 |
| Canada | 188 | 10.9% | 17.9% | 0.24 | 0.71 | 565 | 10.3% | 16.8% | 0.22 | 0.58 | 185 | 24.6% | 22.5% | 0.80 | 0.45 |
| France | 99 | 15.3% | 21.9% | 0.40 | 0.46 | 295 | 15.4% | 19.6% | 0.45 | 0.36 | 95 | 17.2% | 21.9% | 0.48 | 0.27 |
| Germany | 102 | 14.4% | 20.1% | 0.39 | 0.41 | 308 | 11.0% | 16.5% | 0.26 | 0.29 | 102 | 14.6% | 16.5% | 0.48 | 0.19 |
| Hong Kong | 52 | 22.1% | 34.3% | 0.45 | 0.38 | 159 | 16.0% | 35.9% | 0.26 | 0.31 | 53 | 27.6% | 39.7% | 0.53 | 0.26 |
| Italy | 41 | 20.0% | 27.7% | 0.48 | 0.26 | 118 | 15.7% | 26.0% | 0.35 | 0.24 | 41 | 23.2% | 27.2% | 0.61 | 0.21 |
| Japan | 179 | 15.6% | 24.2% | 0.37 | 0.22 | 541 | 15.0% | 25.9% | 0.32 | 0.20 | 179 | 23.1% | 27.8% | 0.59 | 0.13 |
| Netherlands | 42 | 18.4% | 16.2% | 0.73 | 0.61 | 127 | 17.0% | 17.3% | 0.60 | 0.39 | 41 | 16.3% | 18.4% | 0.52 | 0.20 |
| U.K. | 160 | 17.3% | 19.1% | 0.56 | 0.54 | 474 | 16.6% | 19.2% | 0.52 | 0.40 | 152 | 24.0% | 23.7% | 0.73 | 0.31 |
| U.S. | 1157 | 17.4% | 15.1% | 0.71 | 0.99 | 3429 | 15.6% | 18.8% | 0.48 | 0.81 | 1063 | 15.9% | 21.7% | 0.43 | 0.55 |
| Average | 211 | 16.6% | 22.2% | 0.46 | 0.50 | 629 | 14.8% | 21.9% | 0.38 | 0.39 | 200 | 21.1% | 25.3% | 0.57 | 0.28 |

***Panel B. Average Correlations among Cap-Based Funds***

***B.1. Monthly Result***

| *Cap-Based Funds* | *Large-Cap* | *Mid-Cap* | *Small-Cap* |
|---|---|---|---|
| | Domestic correlations | | |
| Large-Cap | 0.44 | 0.87 | 0.66 |
| Mid-Cap | 0.39 | 0.39 | 0.83 |
| Small-Cap | 0.30 | 0.31 | 0.27 |
| | International correlations | | |

***B.2.Quarterly Result***

| *Cap-Based Funds* | *Large-Cap* | *Mid-Cap* | *Small-Cap* |
|---|---|---|---|
| | Domestic correlations | | |
| Large-Cap | 0.47 | 0.87 | 0.66 |
| Mid-Cap | 0.41 | 0.40 | 0.85 |
| Small-Cap | 0.32 | 0.33 | 0.28 |
| | International correlations | | |

**Table 2. Mean-Variance Spanning Tests for Small-Cap Funds**

The table reports the results of the Huberman-Kandel mean-variance spanning test on the returns of the small-cap fund from each country. The sample consists of portfolios from ten developed countries, i.e., Australia (AU), Canada (CN), France (FR), Germany (GE), Hong Kong (HK), Italy (IT), Japan (JP), the Netherlands (NE), the U.K. (UK), and the U.S. (US). At the beginning of each year, from 1980 to 1999, we rank all firms in each country based on their market capitalization values measured at the end of the previous year. Then, we form three cap-based funds for each country: large-, mid-, and small-cap funds. The large-cap fund consists of the 20 percent of stocks with the largest market capitalization values; the small-cap fund consists of the 20 percent of stocks with the smallest capitalization values; the mid-cap fund includes the rest of stocks. During the year, we calculate each portfolio's value-weighted monthly return. A firm's weight in the portfolio is proportional to its market capitalization value at the end of the previous month. To investigate whether investors could expand their mean-variance efficient frontier by investing in small-cap funds, we use MSCI country indices as the benchmark assets. Then, we conduct the spanning test on each small-cap fund. Specifically, we run the OLS regression: $R_i = \alpha_i + \beta_i^{AU} MSCI^{AU} + \ldots + \beta_i^{US} MSCI^{US} + \varepsilon_i$ , where, R denotes the return of the small-cap fund, $MSCI^J$ denotes the return of the MSCI country index for country J, $\alpha$ is the estimated intercept of the regression, $\beta^J$ is the estimated regression coefficient associated with $MSCI^J$, $\varepsilon$ is the error term, and J denotes country AU, CN, …, or US. ***, **, * denote the 1, 5, 10 percent significance levels, respectively. The last two columns of the table report the F-statistic and p-value for the spanning test with the null hypothesis that the small-cap fund is spanned by the ten MSCI country indices, which is equivalent to the joint hypothesis that $\alpha$ is equal to zero and the sum of $\beta$s is equal to one.

| ***Small-Cap Fund*** | $\alpha$ | $\beta^{AU}$ | $\beta^{CN}$ | $\beta^{FR}$ | $\beta^{GE}$ | $\beta^{HK}$ | $\beta^{IT}$ | $\beta^{JP}$ | $\beta^{NE}$ | $\beta^{UK}$ | $\beta^{US}$ | ***F-stat*** | ***P-value*** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Australia | 0.014*** | 0.797*** | 0.317** | 0.062 | -0.227* | 0.013 | 0.113 | -0.005 | 0.184 | -0.078 | -0.459*** | 4.496 | 0.012 |
| Canada | 0.013*** | 0.152*** | 0.660*** | -0.025 | -0.185** | -0.024 | 0.108** | -0.026 | 0.131 | -0.053 | 0.007 | 8.604 | 0.000 |
| France | 0.006** | 0.045 | 0.072 | 0.782*** | 0.011 | -0.027 | 0.035 | 0.024 | -0.036 | -0.015 | -0.232** | 11.984 | 0.000 |
| Germany | 0.005** | 0.016 | -0.095 | 0.099* | 0.374*** | -0.060** | 0.006 | 0.033 | 0.299*** | -0.002 | -0.180** | 42.473 | 0.000 |
| Hong Kong | 0.009 | 0.080 | 0.013 | 0.067 | -0.046 | 0.754*** | -0.051 | 0.033 | 0.082 | 0.020 | -0.138 | 1.364 | 0.258 |
| Italy | 0.008** | -0.040 | 0.098 | 0.053 | 0.121 | -0.030 | 0.692*** | 0.049 | -0.183 | 0.138 | -0.087 | 3.537 | 0.031 |
| Japan | 0.010** | 0.126* | -0.151 | 0.030 | 0.005 | 0.000 | -0.001 | 0.744*** | -0.052 | 0.051 | -0.058 | 5.592 | 0.004 |
| Netherlands | 0.005* | -0.005 | -0.058 | 0.189*** | 0.072 | -0.008 | 0.093** | 0.064 | 0.441*** | -0.030 | -0.224** | 22.093 | 0.000 |
| U.K. | 0.010*** | 0.061 | 0.174* | -0.017 | 0.038 | -0.002 | 0.045 | 0.098* | 0.036 | 0.594*** | -0.245* | 4.665 | 0.010 |
| U.S. | 0.002 | 0.147** | 0.217** | -0.062 | -0.113 | 0.019 | 0.120** | -0.052 | 0.090 | -0.124 | 0.577*** | 2.341 | 0.099 |

**Table 3. Two-Factor Regression Analysis and the Variance Decomposition for Cap-Based Funds**

The table provides the results from estimating the two-factor regression equation and variance decomposition for international cap-based funds. In particular, at the beginning of each year, from 1980 to 1999, we rank all firms in each country based on their market capitalization values measured at the end of the previous year. Then, we form three cap-based funds for each country: large-, mid-, and small-cap funds. The large-cap fund consists of the 20 percent of stocks with the largest market capitalization values; the small-cap fund comprises the 20 percent of stocks with the smallest capitalization values; and the mid-cap fund includes the rest of stocks. During the year, we calculate each portfolio's value-weighted monthly return. A firm's weight in the portfolio is proportional to its market capitalization value at the end of the previous month. Panel A provides the estimation results of the two-factor regression equation:

$$R_{ij} = \alpha_{ij} + \beta_{ij}^{W} R^{W} + \beta_{ij}^{C} R_{i}^{C} + \varepsilon_{ij} \text{ ,}$$

where i = AU, CN,…,US, j = large-, mid-, or small-cap fund, $R^W$ is the return on the MSCI World Market Index, and $R_i^C$ is the residual obtained from regressing country i's stock market index return on $R^w$. $\sigma(\varepsilon)$ and adj-$R^2$ denote the standard deviation of the regression error and the adjusted R-square of the regression, respectively. Panel B provides the decomposition of the variance [Var(R)] of each cap-based fund into three components: (i) the proportion of the variance attributable to the volatility of the world portfolio, (ii) the proportion attributable to the volatility of the country portfolio, and (iii) the idiosyncratic variance or the variance attributable to the fund itself.

| *Country* | *Cap-based Fund* | *Panel A: Two-Factor Regression* | | | | *Panel B: Variance Decomposition (%)* | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\beta^W$ *(t-stat)* | $\beta^C$ *(t-stat)* | $\sigma(\varepsilon)$ | *adj-$R^2$* | *Var (R)* | *Global* | *Country* | *Fund* |
| Australia | L-Cap | 0.98 (55.18) | 1.03 (83.07) | 0.011 | 0.977 | 0.005 | 29.9 | 67.8 | 2.3 |
| | M-Cap | 0.77 (17.72) | 0.86 (28.27) | 0.028 | 0.823 | 0.004 | 23.3 | 59.2 | 17.5 |
| | S-Cap | 0.75 (6.38) | 0.85 (10.41) | 0.075 | 0.381 | 0.009 | 10.6 | 28.1 | 61.4 |
| Canada | L-Cap | 0.88 (72.85) | 0.91 (71.28) | 0.008 | 0.978 | 0.003 | 49.9 | 47.8 | 2.2 |
| | M-Cap | 0.71 (18.56) | 0.76 (18.74) | 0.024 | 0.744 | 0.002 | 36.9 | 37.7 | 25.4 |
| | S-Cap | 0.72 (9.21) | 0.75 (9.10) | 0.050 | 0.409 | 0.004 | 21.0 | 20.5 | 58.6 |
| France | L-Cap | 0.99 (53.55) | 0.99 (61.34) | 0.012 | 0.965 | 0.004 | 41.8 | 54.8 | 3.5 |
| | M-Cap | 0.78 (19.09) | 0.80 (22.40) | 0.026 | 0.783 | 0.003 | 33.0 | 45.5 | 21.5 |
| | S-Cap | 0.68 (10.45) | 0.81 (14.27) | 0.041 | 0.565 | 0.004 | 19.9 | 37.0 | 43.1 |
| Germany | L-Cap | 0.81 (56.03) | 0.93 (77.56) | 0.009 | 0.975 | 0.003 | 33.4 | 64.1 | 2.5 |
| | M-Cap | 0.58 (14.18) | 0.64 (18.84) | 0.026 | 0.699 | 0.002 | 25.4 | 44.8 | 29.9 |
| | S-Cap | 0.45 (8.14) | 0.52 (11.26) | 0.035 | 0.444 | 0.002 | 15.4 | 29.5 | 55.1 |
| Hong Kong | L-Cap | 1.10 (50.85) | 1.01 (97.26) | 0.014 | 0.981 | 0.010 | 21.1 | 77.0 | 1.9 |
| | M-Cap | 1.00 (11.99) | 0.91 (22.92) | 0.053 | 0.736 | 0.011 | 15.9 | 58.0 | 26.1 |
| | S-Cap | 0.92 (6.87) | 0.76 (11.83) | 0.086 | 0.437 | 0.013 | 11.1 | 33.0 | 55.9 |
| Italy | L-Cap | 0.91 (36.71) | 1.01 (66.56) | 0.016 | 0.960 | 0.006 | 22.4 | 73.7 | 3.9 |
| | M-Cap | 0.80 (15.75) | 0.87 (28.13) | 0.032 | 0.813 | 0.006 | 19.4 | 62.0 | 18.6 |
| | S-Cap | 0.77 (9.21) | 0.72 (14.09) | 0.053 | 0.541 | 0.006 | 16.3 | 38.2 | 45.5 |
| Japan | L-Cap | 1.17 (48.35) | 0.99 (48.14) | 0.015 | 0.951 | 0.005 | 47.8 | 47.4 | 4.8 |
| | M-Cap | 1.05 (16.21) | 0.91 (16.45) | 0.041 | 0.690 | 0.006 | 34.1 | 35.1 | 30.8 |
| | S-Cap | 0.85 (9.03) | 0.82 (10.16) | 0.060 | 0.433 | 0.006 | 19.3 | 24.5 | 56.2 |
| Netherlands | L-Cap | 0.87 (60.50) | 0.85 (48.74) | 0.009 | 0.962 | 0.002 | 58.3 | 37.9 | 3.8 |
| | M-Cap | 0.72 (14.61) | 0.72 (11.98) | 0.031 | 0.597 | 0.002 | 35.9 | 24.2 | 39.9 |
| | S-Cap | 0.53 (7.56) | 0.53 (6.14) | 0.045 | 0.280 | 0.003 | 17.2 | 11.4 | 71.4 |
| U.K. | L-Cap | 0.98 (59.19) | 0.95 (52.55) | 0.011 | 0.963 | 0.003 | 53.9 | 42.5 | 3.6 |
| | M-Cap | 0.85 (18.01) | 0.80 (15.38) | 0.030 | 0.700 | 0.003 | 40.7 | 29.6 | 29.7 |
| | S-Cap | 0.82 (9.56) | 0.64 (6.80) | 0.054 | 0.362 | 0.005 | 24.4 | 12.4 | 63.3 |
| U.S. | L-Cap | 0.84 (98.66) | 0.99 (72.65) | 0.005 | 0.984 | 0.002 | 63.8 | 34.6 | 1.6 |
| | M-Cap | 0.90 (18.12) | 0.92 (11.55) | 0.032 | 0.658 | 0.003 | 47.0 | 19.1 | 33.9 |
| | S-Cap | 0.70 (8.50) | 0.75 (5.70) | 0.052 | 0.301 | 0.004 | 21.2 | 9.5 | 69.3 |
| ***Average*** | ***L-Cap*** | ***0.95*** | ***0.97*** | ***0.011*** | ***0.970*** | ***0.004*** | ***42.2*** | ***54.7*** | ***3.0*** |
| | ***M-Cap*** | ***0.82*** | ***0.82*** | ***0.032*** | ***0.724*** | ***0.004*** | ***31.2*** | ***41.5*** | ***27.3*** |
| | ***S-Cap*** | ***0.72*** | ***0.71*** | ***0.055*** | ***0.415*** | ***0.006*** | ***17.6*** | ***24.4*** | ***58.0*** |

**Table 4. Optimal International Portfolios: The Benchmark Case of MSCI Country Stock Market Indices**
**(Sample Period: 1980.1 – 1999.12)**

The table reports optimal international portfolios comprising ten Morgan Stanley Capital International (MSCI) country indices: MSCI Australia (AU), Canada (CN), France (FR), Germany (GE), Hong Kong (HK), Italy (IT), Japan (JP), Netherlands (NE), U.K. (UK), and U.S. (US) indices. The sample period is from January 1980 to December 1999. The risk free interest rate is assumed to equal the average one-month Treasury-Bill rate over the sample period, 0.554%. The table also reports the correlation matrix for the ten country indices, and the mean return ( $\overline{R}$ ), standard deviation ($\sigma$), Sharpe ratio and estimated world beta ($\beta^W$) for each of the ten country indices, where index returns are measured at monthly frequency and are in dollar terms. The last two columns of the table provide the portfolio weights, mean return, standard deviation, and Sharpe ratio for the optimal portfolios, with and without short sales. Also provided are the F-statistics and p-values for the hull hypothesis that the Sharpe ratio of the optimal international portfolio comprising MSCI country indices is the same as that of the U.S. country index.

| | ***Correlation Matrix*** | | | | | | | | | | | | | | ***Optimal Portfolio Weights*** | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***MSCI Country Index*** | ***AU*** | ***CN*** | ***FR*** | ***GE*** | ***HK*** | ***IT*** | ***JP*** | ***NE*** | ***UK*** | ***US*** | $\overline{R}$ ***(%)*** | $\sigma$ ***(%)*** | ***Sharpe Ratio*** | $\beta^W$ | ***With Short Sales*** | ***Without Short Sales*** |
| Australia (AU) | 1.00 | | | | | | | | | | 1.16 | 7.12 | 0.085 | 0.95 | 0.042 | 0.000 |
| Canada (CN) | 0.60 | 1.00 | | | | | | | | | 1.01 | 5.59 | 0.082 | 0.96 | -0.726 | 0.000 |
| France (FR) | 0.35 | 0.43 | 1.00 | | | | | | | | 1.41 | 6.30 | 0.136 | 1.00 | -0.058 | 0.000 |
| Germany (GE) | 0.31 | 0.38 | 0.67 | 1.00 | | | | | | | 1.34 | 6.17 | 0.128 | 0.88 | -0.138 | 0.000 |
| Hong Kong (HK) | 0.45 | 0.45 | 0.28 | 0.32 | 1.00 | | | | | | 1.81 | 9.74 | 0.129 | 1.10 | 0.079 | 0.031 |
| Italy (IT) | 0.23 | 0.34 | 0.47 | 0.40 | 0.27 | 1.00 | | | | | 1.49 | 7.74 | 0.121 | 0.90 | 0.146 | 0.077 |
| Japan (JP) | 0.30 | 0.30 | 0.41 | 0.32 | 0.24 | 0.37 | 1.00 | | | | 1.27 | 7.05 | 0.101 | 1.22 | 0.037 | 0.015 |
| Netherlands (NE) | 0.41 | 0.57 | 0.63 | 0.69 | 0.45 | 0.40 | 0.40 | 1.00 | | | 1.62 | 5.07 | 0.211 | 0.91 | 0.563 | 0.345 |
| U.K. (UK) | 0.54 | 0.57 | 0.55 | 0.49 | 0.46 | 0.37 | 0.41 | 0.69 | 1.00 | | 1.45 | 5.61 | 0.160 | 1.00 | 0.013 | 0.000 |
| U.S. (US) | 0.45 | 0.72 | 0.48 | 0.42 | 0.39 | 0.27 | 0.26 | 0.60 | 0.56 | 1.00 | 1.49 | 4.29 | 0.217 | 0.83 | 1.044 | 0.532 |
| *Average Correlation* | *0.40* | *0.48* | *0.47* | *0.44* | *0.37* | *0.41* | *0.29* | *0.54* | *0.52* | *0.46* | | | | | | |

| ***Portfolio Performance*** | | |
|---|---|---|
| $\overline{R}$ (%) | 1.94 | 1.54 |
| $\sigma$ (%) | 4.77 | 4.06 |
| Sharpe Ratio | 0.290 | 0.243 |
| F-stat | 0.904 | 0.290 |
| (p-value) | (0.641) | (0.377) |

**Table 5. Optimal Augmented International Portfolios: Small-Cap vs. Mid-Cap Funds**

Panel A (Panel B) of the table reports the optimal international portfolios comprising MSCI country indices, and small-cap (mid-cap) funds, with and without short sales. Specifically, we calculate the monthly returns of the MSCI country indices and small- and mid-cap funds for Australia, Canada, France, Germany, Hong Kong, Italy, Japan, the Netherlands, the U.K., and the U.S. from January 1980 to December 1999. To construct small- and mid-cap funds, at the beginning of each year, from 1980 to 1999, we rank all firms in each country based on their market capitalization values at the end of the previous year. The large-cap fund consists of the 20 percent of stocks with the largest market capitalization values, the small-cap fund consists of the 20 percent of stocks with the smallest capitalization values, and the mid-cap fund consists of the rest of stocks. During the year, we calculate each portfolio's value-weighted monthly return. A firm's weight in the portfolio is proportional to its market capitalization value at the end of the previous month. The risk free interest rate is proxied by the average one-month Treasury-Bill rate over the sample period, 0.554%. The portfolio weights, mean return, standard deviation, and Sharpe ratio for each optimal portfolio are reported in the table. The F-statistic and p-value for the null hypothesis that the maximum Sharpe ratio attainable with the corresponding portfolio is the same as that attainable with the portfolio comprising only MSCI country indices are reported at the bottom of the table.

| ***Panel A: The optimal international portfolio of small-cap funds and MSCI country indices*** | | | ***Panel B: The optimal international portfolio of mid-cap funds and MSCI country indices*** | | |
|---|---|---|---|---|---|
| ***Funds*** | ***Portfolio weights*** | | ***Funds*** | ***Portfolio weights*** | |
| | ***with short sales*** | ***without short sales*** | | ***with short sales*** | ***Without short sales*** |
| Australia Small-Cap | 0.131 | 0.012 | Australia Mid-Cap | 0.761 | 0.013 |
| Canada Small-Cap | 0.574 | 0.223 | Canada Mid-Cap | -0.431 | 0.000 |
| France Small-Cap | 0.070 | 0.000 | France Mid-Cap | 0.464 | 0.000 |
| Germany Small-Cap | 0.094 | 0.108 | Germany Mid-Cap | -0.621 | 0.000 |
| Hong Kong Small-Cap | 0.047 | 0.045 | Hong Kong Mid-Cap | -0.242 | 0.000 |
| Italy Small-Cap | 0.214 | 0.090 | Italy Mid-Cap | -0.126 | 0.000 |
| Japan Small-Cap | 0.176 | 0.125 | Japan Mid-Cap | 0.090 | 0.025 |
| Netherlands Small-Cap | 0.031 | 0.020 | Netherlands Mid-Cap | 0.251 | 0.120 |
| U.K. Small-Cap | 0.248 | 0.117 | U.K. Mid-Cap | 0.341 | 0.020 |
| U.S. Small-Cap | -0.356 | 0.000 | U.S. Mid-Cap | -0.272 | 0.000 |
| Australia MSCI | -0.150 | 0.000 | Australia MSCI | -0.544 | 0.000 |
| Canada MSCI | -0.842 | 0.000 | Canada MSCI | -0.413 | 0.000 |
| France MSCI | -0.139 | 0.000 | France MSCI | -0.415 | 0.000 |
| Germany MSCI | -0.066 | 0.000 | Germany MSCI | 0.187 | 0.000 |
| Hong Kong MSCI | 0.049 | 0.000 | Hong Kong MSCI | 0.261 | 0.023 |
| Italy MSCI | -0.103 | 0.000 | Italy MSCI | 0.242 | 0.057 |
| Japan MSCI | -0.153 | 0.000 | Japan MSCI | -0.131 | 0.000 |
| Netherlands MSCI | 0.291 | 0.000 | Netherlands MSCI | 0.503 | 0.220 |
| U.K. MSCI | -0.180 | 0.000 | U.K. MSCI | -0.224 | 0.000 |
| U.S. MSCI | 1.066 | 0.260 | U.S. MSCI | 1.317 | 0.522 |
| ***Portfolio performance*** | | | ***Portfolio performance*** | | |
| $\overline{R}$ (%) | 2.810 | 1.780 | $\overline{R}$ (%) | 2.480 | 1.510 |
| $\sigma$ (%) | 4.860 | 3.810 | $\sigma$ (%) | 5.500 | 3.900 |
| Sharpe Ratio | 0.464 | 0.322 | Sharpe Ratio | 0.350 | 0.245 |
| F-stat | 2.655 | 0.924 | F-stat | 0.769 | 0.020 |
| (p-value) | (0.044) | (0.012) | (p-value) | (0.768) | (0.867) |

**Table 6. Optimal Augmented International Portfolios: The Case of Three Cap-Based Funds**

Panel A (Panel B) of the table reports the optimal international portfolios comprising small- and mid-cap funds and MSCI country indices (large-cap funds), with and without short sales. Specifically, we calculate the monthly returns of the MSCI country indices and small- and mid-cap funds for Australia, Canada, France, Germany, Hong Kong, Italy, Japan, the Netherlands, the U.K., and the U.S. from January 1980 to December 1999. To construct small-, mid-, and large-cap funds, at the beginning of each year, from 1980 to 1999, we rank all firms in each country based on their market capitalization values at the end of the previous year. The large-cap fund consists of the 20 percent of stocks with the largest market capitalization values, the small-cap fund consists of the 20 percent of stocks with the smallest capitalization values, and the mid-cap fund includes the rest of stocks. During the year, we calculate each portfolio's value-weighted monthly return. A firm's weight in the portfolio is proportional to its market capitalization value at the end of the previous month. The risk free interest rate is proxied by the average one-month Treasury-Bill rate over the sample period, 0.554%. The portfolio weights, mean return, standard deviation, and Sharpe ratio for each optimal portfolio are reported in the table. The F-statistic and p-value at the bottom of Panel A (Panel B) test the null hypothesis that the maximum Sharpe ratio attainable with the corresponding portfolio is the same as that attainable with the portfolio comprising only MSCI country indices (only large-cap funds).

| ***Panel A: The optimal risky portfolio of small- And mid-cap funds and MSCI country indices*** | | | ***Panel B: The optimal risky portfolio of small-, mid-, and large-cap funds*** | | |
|---|---|---|---|---|---|
| | ***Portfolio weights*** | | | ***Portfolio weights*** | |
| ***Funds*** | ***with short sales*** | ***without short sales*** | ***Funds*** | ***with short sales*** | ***without short sales*** |
| Australia Small-Cap | 0.073 | 0.012 | Australia Small-Cap | 0.063 | 0.010 |
| Canada Small-Cap | 0.918 | 0.223 | Canada Small-Cap | 0.888 | 0.246 |
| France Small-Cap | 0.178 | 0.000 | France Small-Cap | 0.205 | 0.000 |
| Germany Small-Cap | 0.940 | 0.108 | Germany Small-Cap | 0.884 | 0.117 |
| Hong Kong Small-Cap | 0.390 | 0.045 | Hong Kong Small-Cap | 0.400 | 0.049 |
| Italy Small-Cap | 0.383 | 0.090 | Italy Small-Cap | 0.309 | 0.095 |
| Japan Small-Cap | 0.953 | 0.125 | Japan Small-Cap | 0.923 | 0.130 |
| Netherlands Small-Cap | -0.074 | 0.020 | Netherlands Small-Cap | -0.095 | 0.022 |
| U.K. Small-Cap | 0.356 | 0.116 | U.K. Small-Cap | 0.357 | 0.127 |
| U.S. Small-Cap | -0.037 | 0.000 | U.S. Small-Cap | 0.052 | 0.000 |
| Australia Mid-Cap | 0.262 | 0.000 | Australia Mid-Cap | 0.157 | 0.000 |
| Canada Mid-Cap | -1.358 | 0.000 | Canada Mid-Cap | -1.205 | 0.000 |
| France Mid-Cap | -0.013 | 0.000 | France Mid-Cap | 0.226 | 0.000 |
| Germany Mid-Cap | -1.213 | 0.000 | Germany Mid-Cap | -1.153 | 0.000 |
| Hong Kong Mid-Cap | -0.694 | 0.000 | Hong Kong Mid-Cap | -0.764 | 0.000 |
| Italy Mid-Cap | -0.264 | 0.000 | Italy Mid-Cap | -0.355 | 0.000 |
| Japan Mid-Cap | -1.119 | 0.000 | Japan Mid-Cap | -1.049 | 0.000 |
| Netherlands Mid-Cap | 0.490 | 0.000 | Netherlands Mid-Cap | 0.456 | 0.000 |
| U.K. Mid-Cap | -0.463 | 0.000 | U.K. Mid-Cap | -0.631 | 0.000 |
| U.S. Mid-Cap | -0.094 | 0.000 | U.S. Mid-Cap | -0.308 | 0.000 |
| Australia MSCI | -0.231 | 0.000 | Australia Large-Cap | -0.125 | 0.000 |
| Canada MSCI | -0.056 | 0.000 | Canada Large-Cap | -0.190 | 0.000 |
| France MSCI | -0.362 | 0.000 | France Large-Cap | -0.648 | 0.000 |
| Germany MSCI | 0.294 | 0.000 | Germany Large-Cap | 0.265 | 0.000 |
| Hong Kong MSCI | 0.353 | 0.000 | Hong Kong Large-Cap | 0.425 | 0.000 |
| Italy MSCI | 0.026 | 0.000 | Italy Large-Cap | 0.194 | 0.000 |
| Japan MSCI | 0.181 | 0.000 | Japan Large-Cap | 0.162 | 0.000 |
| Netherlands MSCI | 0.304 | 0.000 | Netherlands Large-Cap | 0.318 | 0.000 |
| U.K. MSCI | 0.180 | 0.000 | U.K. Large-Cap | 0.380 | 0.000 |
| U.S. MSCI | 0.698 | 0.260 | U.S. Large-Cap | 0.860 | 0.202 |
| ***Portfolio performance*** | | | ***Portfolio performance*** | | |
| Mean (%) | 5.030 | 1.780 | Mean (%) | 5.010 | 1.790 |
| SD (%) | 7.170 | 3.810 | SD (%) | 6.940 | 3.920 |
| Sharpe Ratio | 0.624 | 0.322 | Sharpe Ratio | 0.643 | 0.315 |
| F-stat | 2.959 | 0.440 | F-stat | 3.046 | 0.433 |
| (p-value) | (0.003) | (0.022) | (p-value) | (0.003) | (0.036) |

**Table 7. Mean-Variance Intersection Tests for Internationally Diversified Portfolios: A Summary**

The table reports the results of Sharpe ratio tests on internationally diversified portfolios comprising assets from MSCI country indices and international cap-based funds. The sample period is from January 1980 to December 1999. The sample countries include Australia, Canada, France, Germany, Hong Kong, Italy, Japan, the Netherlands, the U.K., and the U.S. We conduct the Sharpe ratio test on each of the eight pairs of benchmark assets and augmented assets (benchmark assets plus new assets) to examine whether the maximum Sharpe ratio attainable with the latter is significantly greater than that attainable with the former. Specifically, we calculate the following test statistic:

$$F = \frac{T-(K+N)}{N}\frac{\hat{\theta}_2^2-\hat{\theta}_1^2}{1+\hat{\theta}_1^2} \quad ,$$

where $\hat{\theta}_1$ and $\hat{\theta}_2$ are the maximum Sharpe ratio attainable with benchmark assets and augmented assets, respectively, T is the number of observations, K is the number of benchmark assets, and N is the number of new assets. When short sales constraints are not imposed, the test statistic follows a F distribution with (T-K-N, N) degrees of freedom. When short sale constraints are imposed, the test statistic follows an unknown distribution and must be approximated by simulation. Column 1 reports the benchmark assets and new assets considered in the test, with the former stated in the first row and the latter in the second row. Columns 2 and 3 report the maximum attainable Sharpe ratios for the benchmark and augmented assets, respectively, with no restriction imposed. The test statistic (F-stat) is reported in Column 4, with the p-value in the parenthesis. Columns 5, 6, and 7, report the same set of statistics as those reported in Columns 2, 3, and 4, but with short sale constraints imposed. The simulated 1, 5, and 10 percent critical values, based on 2000 simulations, are reported in Columns 8, 9, and 10, respectively.

| | | *With Short Sales* | | | *Without Short Sales* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | *Benchmark Assets and New Assets* | *Sharpe Ratio* | | | *Sharpe Ratio* | | | | | |
| | | *Bench-mark* | *Augm-ented* | *F-stat (p-value)* | *Bench-mark* | *Augm-ented* | *F-stat (p-value)* | *Critical Values 1%* | *5%* | *10%* |
| (1) | MSCI US | 0.217 | 0.290 | 0.904 | 0.217 | 0.243 | 0.290 | 1.322 | 0.909 | 0.724 |
| | MSCI country indices | | | (0.641) | | | (0.377) | | | |
| (2) | MSCI US | 0.217 | 0.464 | 1.856 | 0.217 | 0.322 | 0.623 | 0.772 | 0.544 | 0.446 |
| | MSCI country indices and small-cap funds | | | (0.057) | | | (0.031) | | | |
| (3) | MSCI US | 0.217 | 0.350 | 0.829 | 0.217 | 0.245 | 0.142 | 0.716 | 0.469 | 0.382 |
| | MSCI country indices and mid-cap funds | | | (0.746) | | | (0.485) | | | |
| (4) | MSCI country indices | 0.290 | 0.464 | 2.655 | 0.243 | 0.322 | 0.924 | 0.958 | 0.678 | 0.544 |
| | Small-cap funds | | | (0.044) | | | (0.012) | | | |
| (5) | MSCI country indices | 0.290 | 0.350 | 0.769 | 0.243 | 0.245 | 0.020 | 0.824 | 0.550 | 0.432 |
| | Mid-cap funds | | | (0.768) | | | (0.867) | | | |
| (6) | MSCI country indices and small-cap funds | 0.464 | 0.624 | 3.019 | 0.322 | 0.322 | 0.000 | 0.654 | 0.430 | 0.338 |
| | Mid-cap funds | | | (0.028) | | | (1.000) | | | |
| (7) | MSCI country indices and small-cap funds | 0.464 | 0.678 | 2.010 | 0.322 | 0.322 | 0.000 | 0.369 | 0.250 | 0.208 |
| | Large- and mid-cap funds | | | (0.034) | | | (1.000) | | | |
| (8) | MSCI country indices and mid-cap funds | 0.350 | 0.624 | 5.008 | 0.245 | 0.322 | 0.861 | 0.760 | 0.488 | 0.401 |
| | Small-cap funds | | | (0.004) | | | (0.006) | | | |

**Table 8. Minimum Variance Portfolios**

Panel A of the table reports the minimum variance portfolios comprising MSCI country indices, and Panel B reports those comprising MSCI country indices as well as small-cap funds. We calculate the monthly returns of the MSCI country indices and small-cap funds for Australia, Canada, France, Germany, Hong Kong, Italy, Japan, the Netherlands, the U.K., and the U.S. from January 1980 to December 1999. To construct small-cap funds, at the beginning of each year, from 1980 to 1999, we rank all firms in each country based on their market capitalization values at the end of the previous year. The small-cap fund consists of the 20 percent of stocks with the smallest capitalization values. During the year, we calculate each portfolio's value-weighted monthly return. A firm's weight in the portfolio is proportional to its market capitalization value at the end of the previous month. The portfolio weights, mean return, standard deviation, and Sharpe ratio of each minimum variance portfolio are reported in the table for the entire sample period as well as for the two sub-periods. The F-statistic and p-value for testing the null hypothesis that the variance of the minimum variance portfolio comprising MSCI country indices and small-cap funds is greater or equal to that of the minimum variance portfolio comprising only MSCI country indices are reported at the bottom of Panel B.

| ***Funds*** | ***Panel A: The MVP of MSCI Countries Indices*** | | | ***Panel B: The MVP of Small-cap Funds and MSCI Countries Indices*** | | |
|---|---|---|---|---|---|---|
| | ***1980-1999*** | ***1980-1989*** | ***1990-1999*** | ***1980-1999*** | ***1980-1989*** | ***1990-1999*** |
| Australia Small-Cap | n.a. | n.a. | n.a. | 0.013 | 0.039 | 0.000 |
| Canada Small-Cap | n.a. | n.a. | n.a. | 0.088 | 0.016 | 0.105 |
| France Small-Cap | n.a. | n.a. | n.a. | 0.000 | 0.000 | 0.146 |
| Germany Small-Cap | n.a. | n.a. | n.a. | 0.302 | 0.195 | 0.314 |
| Hong Kong Small-Cap | n.a. | n.a. | n.a. | 0.000 | 0.000 | 0.000 |
| Italy Small-Cap | n.a. | n.a. | n.a. | 0.000 | 0.000 | 0.000 |
| Japan Small-Cap | n.a. | n.a. | n.a. | 0.053 | 0.178 | 0.000 |
| Netherlands Small-Cap | n.a. | n.a. | n.a. | 0.111 | 0.090 | 0.018 |
| U.K. Small-Cap | n.a. | n.a. | n.a. | 0.014 | 0.000 | 0.010 |
| U.S. Small-Cap | n.a. | n.a. | n.a. | 0.013 | 0.000 | 0.001 |
| Australia MSCI | 0.031 | 0.000 | 0.127 | 0.000 | 0.000 | 0.060 |
| Canada MSCI | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| France MSCI | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Germany MSCI | 0.086 | 0.114 | 0.000 | 0.000 | 0.000 | 0.000 |
| Hong Kong MSCI | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Italy MSCI | 0.064 | 0.067 | 0.050 | 0.000 | 0.011 | 0.000 |
| Japan MSCI | 0.125 | 0.225 | 0.007 | 0.004 | 0.056 | 0.000 |
| Netherlands MSCI | 0.074 | 0.000 | 0.205 | 0.000 | 0.000 | 0.000 |
| U.K. MSCI | 0.014 | 0.000 | 0.125 | 0.000 | 0.000 | 0.000 |
| U.S. MSCI | 0.605 | 0.594 | 0.487 | 0.403 | 0.416 | 0.345 |
| Portfolio performance | | | | | | |
| $\overline{R}$ (%) | 1.447 | 1.686 | 1.350 | 1.474 | 1.906 | 1.226 |
| $\sigma$ (%) | 3.898 | 4.055 | 3.494 | 3.299 | 3.606 | 2.649 |
| Sharpe Ratio | 0.229 | 0.236 | 0.271 | 0.279 | 0.326 | 0.310 |
| F-stat | n.a. | n.a. | n.a. | 1.397 | 1.265 | 1.740 |
| (p-value) | n.a. | n.a. | n.a. | (0.005) | (0.101) | (0.001) |

**Table 9. The Effects of Additional Transaction Costs for Small-Cap Funds**

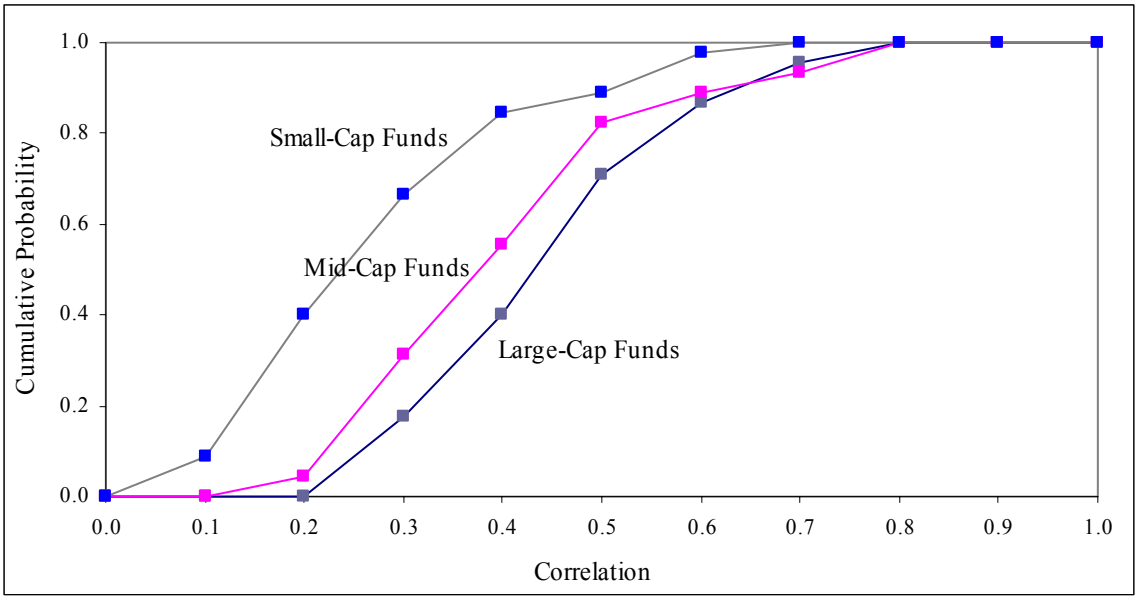
The table examines the effect of transaction costs on the diversification benefits of international small-cap funds. In particular, we impose transaction costs on small-cap funds, but not on MSCI country indices. The transaction costs we impose represent the 'additional' transaction costs associated with the investment in small-cap funds as opposed to county indices. Column 1 reports the additional annualized transaction cost that we impose on each small-cap fund. Columns 2 and 3 report the optimal portfolio weights for MSCI country indices and small-cap funds, respectively. When solving for the optimal portfolio, we assume the risk free interest rate equals the average one-month Treasury-Bill rate over the sample period, 0.554%. We also assume that short sales are prohibited. The Sharpe ratio for the optimal portfolio is reported in Column 4. Superscript a, b, and c, denote the 1%, 5%, and 10% significance level, respectively, testing for the null hypothesis that the maximum Sharpe ratio attainable with the augmented portfolio, comprising MSCI country indices and small-cap funds, is the same as that attainable with the benchmark portfolio, comprising only MSCI country indices. The reported significance level is based on 2000 simulations. The last column reports the extra return a U.S. investor could receive if she, given the domestic risk level, invests in the augmented portfolio as opposed to the U.S. domestic country index.

| *Additional transaction cost* | *Optimal portfolio weights* | | *Portfolio Performance* | |
|---|---|---|---|---|
| | *MSCI indices* | *Small-cap funds* | *Sharpe ratio* | $\Delta R_{US}$ |
| 0.0% | 0.260 | 0.740 | 0.322[a] | 5.41% |
| 0.5% | 0.284 | 0.716 | 0.315[b] | 5.06% |
| 1.0% | 0.325 | 0.675 | 0.309[b] | 4.74% |
| 1.5% | 0.370 | 0.630 | 0.303[c] | 4.42% |
| 2.0% | 0.411 | 0.589 | 0.297[c] | 4.11% |
| 2.5% | 0.439 | 0.561 | 0.291 | 3.81% |
| 3.0% | 0.467 | 0.533 | 0.285 | 3.51% |
| 3.5% | 0.497 | 0.503 | 0.280 | 3.22% |
| 4.0% | 0.529 | 0.471 | 0.274 | 2.93% |
| 4.5% | 0.562 | 0.438 | 0.269 | 2.66% |
| 5.0% | 0.597 | 0.403 | 0.264 | 2.43% |
| 5.5% | 0.632 | 0.368 | 0.261 | 2.26% |
| 6.0% | 0.677 | 0.323 | 0.257 | 2.07% |
| 6.5% | 0.727 | 0.273 | 0.254 | 1.90% |
| 7.0% | 0.777 | 0.223 | 0.251 | 1.76% |
| 7.5% | 0.824 | 0.176 | 0.249 | 1.64% |
| 8.0% | 0.858 | 0.142 | 0.247 | 1.55% |
| 8.5% | 0.891 | 0.109 | 0.246 | 1.49% |
| 9.0% | 0.922 | 0.078 | 0.245 | 1.44% |
| 9.5% | 0.940 | 0.060 | 0.244 | 1.41% |
| 10.0% | 0.956 | 0.044 | 0.244 | 1.40% |
| 10.5% | 0.972 | 0.028 | 0.244 | 1.38% |
| 11.0% | 0.985 | 0.015 | 0.244 | 1.37% |
| 11.5% | 0.999 | 0.001 | 0.244 | 1.37% |
| 12.0% | 1.000 | 0.000 | 0.243 | 1.36% |

**Figure 1. Pair-wise Correlations among Large-, Mid-, and Small-Cap Funds**

Panel A of the figure plots the cumulative distribution function (CDF) of intra-category correlations among large- (bottom curve), among mid- (middle curve), and among small-cap funds (upper curve). To form large-, mid-, and small-cap funds, at the beginning of each year, from 1980 to 1999, we rank all firms in each country based on their market capitalization values measured at the end of the previous year. The large-cap fund consists of the 20 percent of stocks with the largest market capitalization values; the small-cap fund consists of the 20 percent of stocks with the smallest market capitalization values; the mid-cap fund comprises the rest of stocks. During the year, we calculate each portfolio's monthly value-weighted return. A firm's weight in the portfolio is proportional to its market capitalization value at the end of the previous month. Panel B tests for the first order stochastic dominance, where r refers to the pairwise correlation, $D_S(r)$, $D_M(r)$, and $D_L(r)$ are the cumulative distribution functions for the intra-category correlations among small-, among mid-, and among large-cap funds, respectively. $T_{SM}(r)$, $T_{SL}(r)$, and $T_{ML}(r)$ are the test statistics testing for the null (alternative) hypothesis that the CDF of the intra-category correlations of small-cap funds equal (dominate) that of mid-cap funds, that the CDF of the intra-category correlations of small-cap funds equal (dominate) that of large-cap funds, and that the CDF of the intra-category correlations of mid-cap funds equal (dominate) that of large-cap funds, respectively.

***Panel A Plot of the Cumulative Distribution Function***




***Panel B Significance Tests of the First Order Stochastic Dominance***

| | *CDF* | | | *Test Statistics* | | |
|---|---|---|---|---|---|---|
| ***r*** | ***$D_S(r)$*** | ***$D_M(r)$*** | ***$D_L(r)$*** | ***$T_{SM}(r)$*** | ***$T_{SL}(r)$*** | ***$T_{ML}(r)$*** |
| 0.1 | 0.089 | 0.000 | 0.000 | 2.095 | 2.095 | - |
| 0.2 | 0.400 | 0.044 | 0.000 | 5.477 | 4.488 | 1.447 |
| 0.3 | 0.667 | 0.311 | 0.178 | 5.403 | 3.610 | 1.490 |
| 0.4 | 0.844 | 0.556 | 0.400 | 4.892 | 3.151 | 1.495 |
| 0.5 | 0.889 | 0.822 | 0.711 | 2.162 | 0.904 | 1.257 |
| 0.6 | 0.978 | 0.889 | 0.867 | 2.012 | 1.718 | 0.322 |
| 0.7 | 1.000 | 0.933 | 0.956 | 1.447 | 1.793 | -0.461 |
| 0.8 | 1.000 | 1.000 | 1.000 | - | - | - |
| 0.9 | 1.000 | 1.000 | 1.000 | - | - | - |
| 1.0 | 1.000 | 1.000 | 1.000 | - | - | - |

**Figure 2. The Portfolio Risk and Cap-Based Diversification**

The figure examines the relationship between portfolio variance and international cap-based diversification. Each curve in the figure plots the portfolio variance, expressed as a percentage of the average stock variance, as a function of the number of securities included in the portfolio. The upper, middle, and lower curves plot the portfolio variance when investors diversify among U.S. large-cap stocks, across international large-cap stocks, and across international large- and small-cap stocks, respectively. The sample countries include Australia (AU), Canada (CN), France (FR), Germany (GE), Hong Kong (HK), Italy (IT), Japan (JP), the Netherlands (NE), the U.K. (UK), and the U.S. (US). Since we form cap-based portfolios on an annual basis, the size membership of a stock in our sample may change from time to time. To conduct this analysis, we include only stocks which have no missing observations during the period 1995-1999 and whose size memberships do not change over that period. With this criterion, we obtain a sample of 99 (31), 175 (124), 101 (55), 108 (86), 66 (27), 37 (32), 227 (134), 28 (17), 106 (53), and 771 (268) securities for AU-, CN-, FR-, GE-, HK-, IT-, JP-, NE-, UK-, and US-large (small) cap funds, respectively. To form U.S. large-cap portfolios, we first randomly draw 300 stocks from the 771 U.S. large-cap stocks. Then, we randomly and repeatedly draw stocks with replacement from these 300 stocks to form equal-weighted portfolios with different numbers of stocks. The average portfolio variance is calculated from 500 repetitions. Using a similar methodology, we conduct the experiment on international large-cap funds by first randomly drawing 50, 50, 65, 65, 50, 37, 155, 28, and 100 securities from AU-, CN-, FR-, GE-, HK-, IT-, JP-, NE-, and UK-large cap stocks, respectively. Then, portfolios with different number of securities are constructed from the selected large-cap stocks in ten countries. The average portfolio variance is again calculated based on 500 repetitions. International large- and small-cap portfolios are formed in a similar manner with stocks drawn from the previously selected international large-cap stocks and from the entire pool of small-cap stocks.

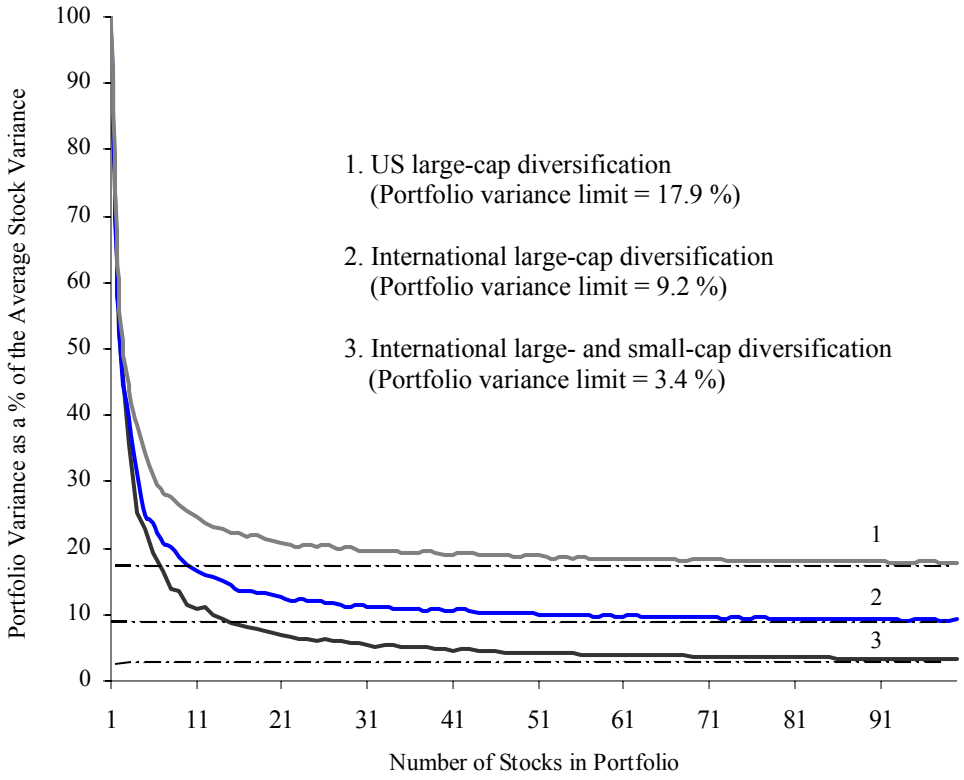
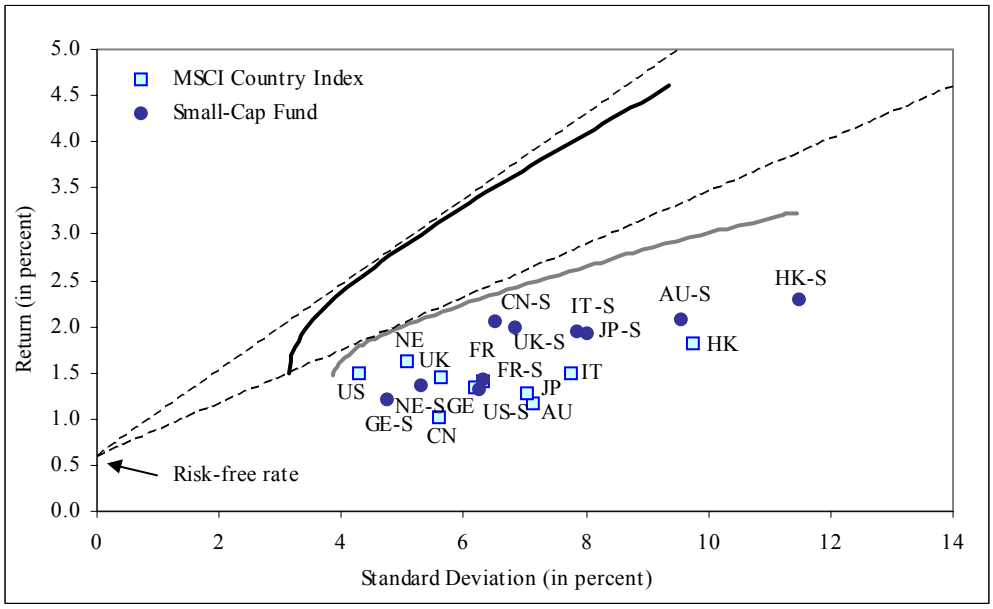


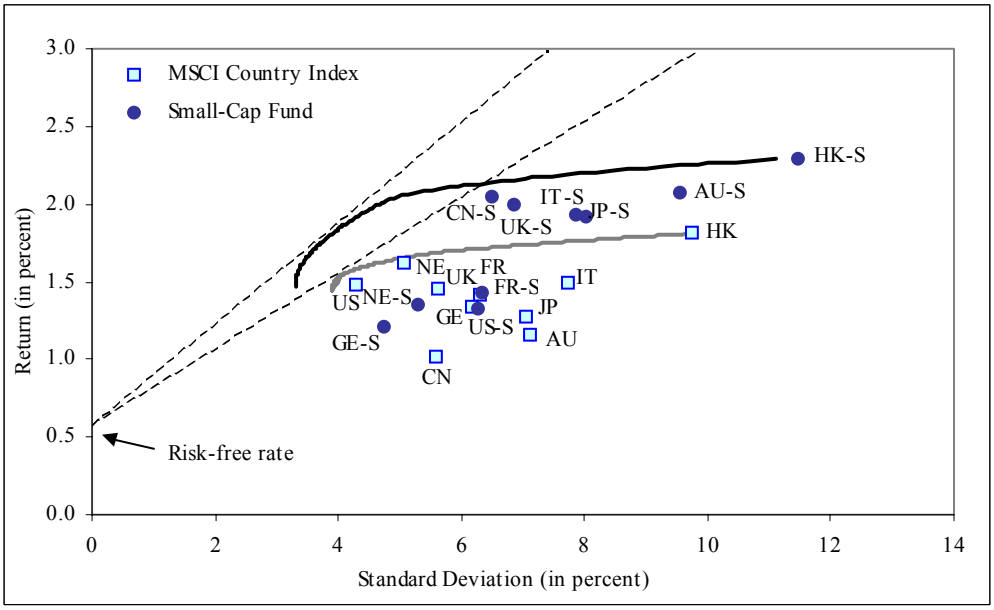
**Figure 3. Efficient Frontiers of International Portfolios: The Effect of Small-Cap Funds**

The figure plots the efficient frontiers spanned by MSCI country indices and small-cap funds during the period 1980-1999. The sample countries include Australia, Canada, France, Germany, Hong Kong, Italy, Japan, the Netherlands, the U.K., and the U.S. The upper (lower) graph plots the frontiers when short sales are allowed (not allowed). In the graph, the lower curve is the efficient frontier spanned by MSCI country indices, whereas the upper curve is the one spanned by MSCI country indices and small-cap funds. The dotted line in the graph connects the risk-free rate to the tangent portfolio. The square (round) dots in the graph denote the mean-standard deviation locations of the MSCI country indices (small-cap funds), denoted as AU, …, US (AU-S, …, US-S).

***With Short Sales***



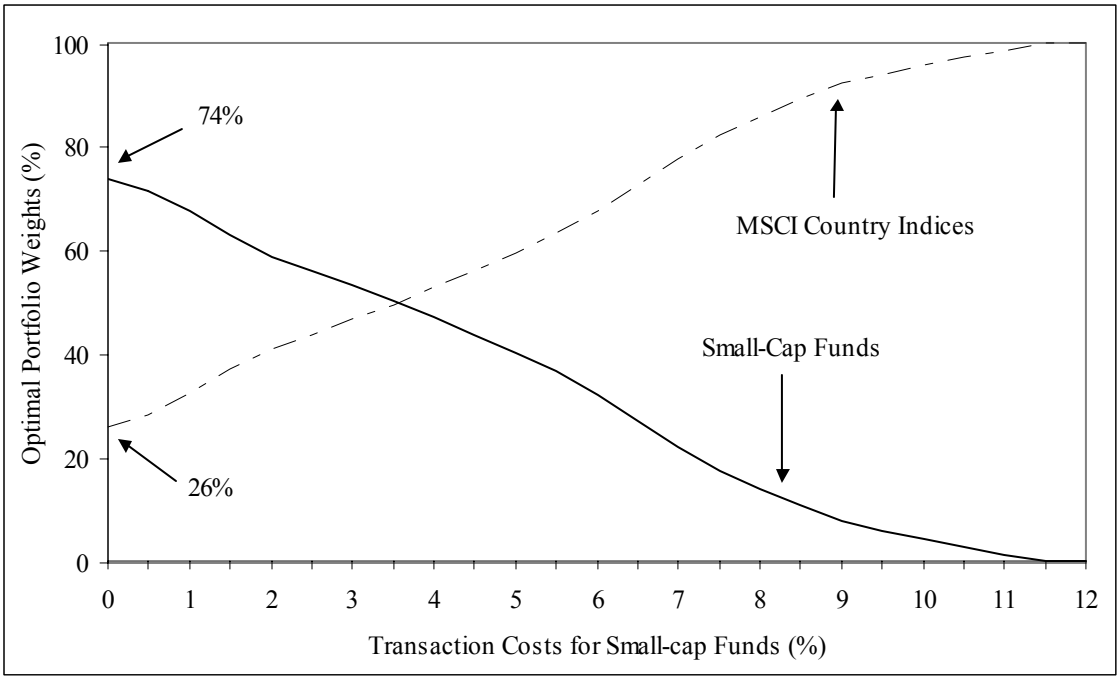

***Without Short Sales***




Lower curve: The efficient frontier spanned by MSCI indices
Upper curve: The efficient frontier spanned by MSCI indices and small-cap funds

**Figure 4. Optimal Portfolio Weights with Transaction Costs for Small-Cap Funds**

In the figure, we plot the weights assigned to international small-cap funds (solid curve) and MSCI country indices (dotted curve) in the optimal portfolio, with different levels of annualized transaction costs imposed on the former but not the latter. The transaction costs imposed represent the differential transaction costs between small-cap funds and county indices. The sample countries include Australia, Canada, France, Germany, Hong Kong, Italy, Japan, the Netherlands, the U.K., and the U.S. The sample period is from January 1980 to December 1999. When the differential transaction cost between small-cap funds and MSCI country indices is zero, the former accounts for 74% of the optimal portfolio and the latter 26%. When their differential transaction cost is 3.5%, they account for equal amount of weight in the optimal portfolio. Small-cap funds maintain a positive weight in the optimal portfolio until their transaction cost exceeds that of MSCI country indices by 12%.



## Appendix A. The Correlation Matrix for Cap-Based Funds

The appendix reports the correlation matrix for international large- (L), mid- (M), and small-cap (S) funds. The sample countries include Australia (AU), Canada (CN), France (FR), Germany (GE), Hong Kong (HK), Italy (IT), Japan (JP), the Netherlands (NE), the U.K. (UK), and the U.S. (US). At the beginning of each year, from 1980 to 1999, we rank all firms in each country based on their market capitalization values measured at the end of the previous year. Then, we form three cap-based funds for each country: Large-, mid-, and small-cap funds. The large-cap fund consists of the 20 percent of stocks with the largest market capitalization values; the small-cap fund consists of the 20 percent of stocks with the smallest capitalization values; and the mid-cap fund include the rest of stocks. During the year, we calculate each portfolio's monthly value-weighted return. A firm's weight in the portfolio is proportional to its market capitalization value at the end of the previous month.

| *Size Portfolio* | *AU-L* | *AU-M* | *AU-S* | *CN-L* | *CN-M* | *CN-S* | *FR-L* | *FR-M* | *FR-S* | *GE-L* | *GE-M* | *GE-S* | *HK-L* | *HK-M* | *HK-S* | *IT-L* | *IT-M* | *IT-S* | *JP-L* | *JP-M* | *JP-S* | *NE-L* | *NE-M* | *NE-S* | *UK-L* | *UK-M* | *UK-S* | *US-L* | *US-M* | *US-S* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***AU-L*** | 1.00 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| ***AU-M*** | 0.91 | 1.00 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| ***AU-S*** | 0.63 | 0.76 | 1.00 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| ***CN-L*** | 0.60 | 0.56 | 0.42 | 1.00 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| ***CN-M*** | 0.60 | 0.62 | 0.48 | 0.89 | 1.00 | | | | | | | | | | | | | | | | | | | | | | | | | |
| ***CN-S*** | 0.44 | 0.52 | 0.52 | 0.66 | 0.79 | 1.00 | | | | | | | | | | | | | | | | | | | | | | | | |
| ***FR-L*** | 0.37 | 0.30 | 0.24 | 0.42 | 0.35 | 0.25 | 1.00 | | | | | | | | | | | | | | | | | | | | | | | |
| ***FR-M*** | 0.34 | 0.31 | 0.25 | 0.37 | 0.35 | 0.25 | 0.89 | 1.00 | | | | | | | | | | | | | | | | | | | | | | |
| ***FR-S*** | 0.28 | 0.26 | 0.21 | 0.30 | 0.27 | 0.19 | 0.76 | 0.87 | 1.00 | | | | | | | | | | | | | | | | | | | | | |
| ***GE-L*** | 0.31 | 0.24 | 0.14 | 0.37 | 0.31 | 0.14 | 0.65 | 0.64 | 0.51 | 1.00 | | | | | | | | | | | | | | | | | | | | |
| ***GE-M*** | 0.29 | 0.23 | 0.14 | 0.30 | 0.30 | 0.11 | 0.64 | 0.70 | 0.62 | 0.87 | 1.00 | | | | | | | | | | | | | | | | | | | |
| ***GE-S*** | 0.19 | 0.15 | 0.09 | 0.18 | 0.20 | 0.03 | 0.55 | 0.62 | 0.55 | 0.71 | 0.88 | 1.00 | | | | | | | | | | | | | | | | | | |
| ***HK-L*** | 0.45 | 0.43 | 0.32 | 0.43 | 0.40 | 0.28 | 0.29 | 0.22 | 0.17 | 0.33 | 0.21 | 0.14 | 1.00 | | | | | | | | | | | | | | | | | |
| ***HK-M*** | 0.41 | 0.42 | 0.30 | 0.40 | 0.41 | 0.32 | 0.25 | 0.24 | 0.20 | 0.28 | 0.22 | 0.17 | 0.88 | 1.00 | | | | | | | | | | | | | | | | |
| ***HK-S*** | 0.34 | 0.35 | 0.24 | 0.32 | 0.34 | 0.29 | 0.23 | 0.23 | 0.21 | 0.22 | 0.17 | 0.14 | 0.68 | 0.87 | 1.00 | | | | | | | | | | | | | | | |
| ***IT-L*** | 0.23 | 0.22 | 0.22 | 0.33 | 0.33 | 0.27 | 0.45 | 0.44 | 0.39 | 0.41 | 0.39 | 0.32 | 0.26 | 0.24 | 0.18 | 1.00 | | | | | | | | | | | | | | |
| ***IT-M*** | 0.23 | 0.22 | 0.22 | 0.33 | 0.33 | 0.22 | 0.42 | 0.45 | 0.41 | 0.40 | 0.40 | 0.32 | 0.25 | 0.24 | 0.17 | 0.90 | 1.00 | | | | | | | | | | | | | |
| ***IT-S*** | 0.19 | 0.17 | 0.17 | 0.28 | 0.28 | 0.20 | 0.39 | 0.45 | 0.41 | 0.37 | 0.41 | 0.35 | 0.19 | 0.18 | 0.12 | 0.73 | 0.81 | 1.00 | | | | | | | | | | | | |
| ***JP-L*** | 0.29 | 0.30 | 0.20 | 0.27 | 0.25 | 0.16 | 0.39 | 0.40 | 0.33 | 0.30 | 0.34 | 0.31 | 0.23 | 0.20 | 0.19 | 0.35 | 0.32 | 0.33 | 1.00 | | | | | | | | | | | |
| ***JP-M*** | 0.30 | 0.29 | 0.20 | 0.22 | 0.23 | 0.16 | 0.35 | 0.38 | 0.30 | 0.28 | 0.35 | 0.32 | 0.19 | 0.20 | 0.17 | 0.28 | 0.29 | 0.35 | 0.83 | 1.00 | | | | | | | | | | |
| ***JP-S*** | 0.26 | 0.24 | 0.18 | 0.15 | 0.17 | 0.11 | 0.27 | 0.30 | 0.26 | 0.20 | 0.29 | 0.27 | 0.15 | 0.15 | 0.13 | 0.23 | 0.26 | 0.32 | 0.66 | 0.91 | 1.00 | | | | | | | | | |
| ***NE-L*** | 0.42 | 0.36 | 0.25 | 0.58 | 0.53 | 0.34 | 0.68 | 0.63 | 0.48 | 0.74 | 0.68 | 0.60 | 0.44 | 0.39 | 0.32 | 0.42 | 0.40 | 0.35 | 0.42 | 0.37 | 0.26 | 1.00 | | | | | | | | |
| ***NE-M*** | 0.32 | 0.29 | 0.19 | 0.40 | 0.42 | 0.24 | 0.63 | 0.69 | 0.56 | 0.68 | 0.71 | 0.64 | 0.37 | 0.37 | 0.30 | 0.45 | 0.44 | 0.42 | 0.43 | 0.39 | 0.29 | 0.81 | 1.00 | | | | | | | |
| ***NE-S*** | 0.19 | 0.15 | 0.11 | 0.23 | 0.25 | 0.13 | 0.50 | 0.61 | 0.55 | 0.50 | 0.61 | 0.59 | 0.20 | 0.23 | 0.18 | 0.38 | 0.39 | 0.43 | 0.35 | 0.35 | 0.32 | 0.56 | 0.79 | 1.00 | | | | | | |
| ***UK-L*** | 0.54 | 0.48 | 0.34 | 0.58 | 0.52 | 0.37 | 0.56 | 0.53 | 0.42 | 0.50 | 0.49 | 0.38 | 0.45 | 0.40 | 0.33 | 0.39 | 0.41 | 0.36 | 0.42 | 0.38 | 0.29 | 0.69 | 0.56 | 0.38 | 1.00 | | | | | |
| ***UK-M*** | 0.48 | 0.48 | 0.38 | 0.53 | 0.53 | 0.43 | 0.49 | 0.51 | 0.44 | 0.43 | 0.45 | 0.37 | 0.39 | 0.38 | 0.33 | 0.40 | 0.42 | 0.38 | 0.45 | 0.44 | 0.36 | 0.60 | 0.55 | 0.41 | 0.86 | 1.00 | | | | |
| ***UK-S*** | 0.41 | 0.43 | 0.37 | 0.43 | 0.42 | 0.35 | 0.38 | 0.40 | 0.34 | 0.34 | 0.33 | 0.26 | 0.33 | 0.32 | 0.29 | 0.33 | 0.32 | 0.28 | 0.40 | 0.33 | 0.28 | 0.46 | 0.42 | 0.30 | 0.62 | 0.82 | 1.00 | | | |
| ***US-L*** | 0.46 | 0.38 | 0.23 | 0.74 | 0.62 | 0.48 | 0.46 | 0.36 | 0.27 | 0.40 | 0.29 | 0.20 | 0.39 | 0.32 | 0.27 | 0.26 | 0.25 | 0.22 | 0.23 | 0.20 | 0.13 | 0.61 | 0.39 | 0.21 | 0.54 | 0.41 | 0.32 | 1.00 | | |
| ***US-M*** | 0.48 | 0.45 | 0.30 | 0.72 | 0.73 | 0.59 | 0.37 | 0.34 | 0.25 | 0.32 | 0.26 | 0.19 | 0.39 | 0.38 | 0.32 | 0.29 | 0.28 | 0.22 | 0.24 | 0.20 | 0.13 | 0.53 | 0.40 | 0.22 | 0.48 | 0.43 | 0.36 | 0.86 | 1.00 | |
| ***US-S*** | 0.39 | 0.42 | 0.40 | 0.55 | 0.66 | 0.66 | 0.24 | 0.22 | 0.16 | 0.15 | 0.12 | 0.08 | 0.30 | 0.31 | 0.26 | 0.24 | 0.24 | 0.18 | 0.14 | 0.14 | 0.09 | 0.33 | 0.27 | 0.17 | 0.33 | 0.38 | 0.34 | 0.59 | 0.82 | 1.00 |

## Appendix B. Tests of the First-Order Stochastic Dominance

This appendix describes the methodology we use to test the significance of the first-order stochastic dominance among the three sets of intra-category correlations, i.e., the correlations among large, among mid- and among small-cap funds. We plot the cumulative distribution function (CDF) for each of the three sets of correlations in Panel A of Figure 1. The figure shows that the CDF of small-cap correlations lies above that of the mid-cap and large-cap correlations, except where the correlation approaches 1. If risk-averse agents prefer low security correlations to high correlations, the CDF plot shows that the small-cap correlations first-order stochastically dominate the mid-cap and large-cap correlations. To formally test the significance of the stochastic dominance, we employ the methodology proposed by Davidson and Duclos (2000). Specifically, to test whether distribution A dominates distribution B, we calculate the following statistics:

$$\hat{D}_A^S(r) = \frac{1}{N(s-1)!}\sum_{i=1}^{N}(r - w_{i,A})_+^{S-1} \quad , \tag{B.1}$$

$$\hat{D}_B^S(r) = \frac{1}{N(s-1)!}\sum_{i=1}^{N}(r - w_{i,B})_+^{S-1} \tag{B.2}$$

$$\hat{V}_A^S(r) = \frac{1}{N}\left[\frac{1}{((s-1)!)^2}\frac{1}{N}\sum_{i=1}^{N}(r - w_{i,A})_+^{2(S-1)} - D_A^S(r)^2\right] \quad , \tag{B.3}$$

$$\hat{V}_B^S(r) = \frac{1}{N}\left[\frac{1}{((s-1)!)^2}\frac{1}{N}\sum_{i=1}^{N}(r - w_{i,B})_+^{2(S-1)} - D_B^S(r)^2\right] \quad , \tag{B.4}$$

$$\hat{V}_{AB}^S(r) = \frac{1}{N}\left[\frac{1}{((s-1)!)^2}\frac{1}{N}\sum_{i=1}^{N}(r - w_{i,A})_+^{S-1}(r - w_{i,B})^{S-1} - D_A^S(r)D_B^S(r)\right] \quad , \tag{B.5}$$

$$\hat{V}^S(r) = \hat{V}_A^S(r) + \hat{V}_B^S(r) - 2\hat{V}_{AB}^S(r) \quad , \tag{B.6}$$

$$T^S(r) = \frac{\hat{D}_A^S(r) - \hat{D}_B^S(r)}{\sqrt{\hat{V}^S(r)}}, \tag{B.7}$$

where superscript s denotes the order of stochastic dominance, r is a random variable representing the correlation of cap-based funds in the current paper, N is the number of observations. $W_{i,A}$ and $W_{i,B}$ are the paired correlations for cap-based funds. For instance, for comparing the distributions of small-cap (A) and large-cap (B) correlations, $W_{i,A}$ and $W_{i,B}$ denote the small-cap and large-cap correlations, respectively, for a pair of countries x and y. $(r-W_{i,A})_+$ is $\max(0, r-W_{i,A})$ and $(r-W_{i,B})_+$ is $\max(0, r-W_{i,B})$. For s = 1, $\hat{D}_A^S(r)$ and $\hat{D}_B^S(r)$ are the consistent estimators of the CDFs for distributions A and B, respectively, which are

equivalent to their CDFs. $\hat{V}_A^S(r)$, $\hat{V}_B^S(r)$, and $\hat{V}^S(r)$ are the consistent estimators of the variance for $\hat{D}_A^S(r)$, $\hat{D}_B^S(r)$, and $(\hat{D}_A^S(r) - \hat{D}_B^S(r))$, respectively. Davidson et al. show that under the null of $D_A^S(r) = D_B^S(r)$, $T^S(r)$ is asymptotically distributed as a standard normal variable. To test the alternative hypothesis that the small-cap correlations dominate large-cap correlations, we employ the Bishop-Formby-Thistle (BFT, 1992) union-intersection procedure. Specifically, we consider ten fixed values for r, i.e., 0.1, 0.2, …, 1.0. According to the BFT procedure, if $T(r_i)$ is significantly positive for some r's but not significantly negative for any r's, the alternative hypothesis is accepted. We report the test results in Panel B.