Quantifying Illiquidity in Emerging Sovereign Market Trades

Abstract

With increasing liquidity of in emerging sovereign debt market, it has become possible to estimate the term structure. However, several frictions that cause individual securities to be priced differently from the average pricing in the market characterize the market. In such a scenario, traditional estimation procedures like ordinary least squares using various functional forms do not perform well. We consider the Indian sovereign debt market as an example for our study. We find that mean absolute deviation is a better estimation procedure in illiquid markets than the ordinary least square. We further find out a novel liquidity weighted objective function for parameter estimation. We model the liquidity function using the exponential and hyperbolic tangent functions and suggest the most robust model for estimating term structures.

Keywords: Finance; Fixed Income Securities; Non-linear Constrained Optimisation

JEL Classifications: G12, G15

1 Introduction

The term structure[1] of interest rates is one of the most fundamental tools used in valuation of fixed income securities. The term structure is, in a trading context, used for identification of differences in the theoretical value of securities relative to their market value. In the monetary policy context, spot interest rates act as an indicator of the market's expectations regarding interest rates. Its slope can provide information about the expected changes in interest rates. Spot interest rates are also inputs in testing theories of the term structure of interest rates, particularly the no-arbitrage class of theories.

With the increase in the liquidity of the emerging sovereign debt market, it has become feasible to estimate the term structure in emerging markets. It has been found that the Nelson-Seigel model (1987) is the most robust measure of the term structure. However, this model does not take account of the illiquidity and other frictions in emerging markets. Hence, the error is high. The standard deviation of the error is of the order of 15 to 20 basis points with respect to market prices. It is as high as 30 to 40 basis points on some days. Both average and maximum standard deviations of errors are unacceptable for trading purposes.

We consider the case of the Government of India securities market which is characterized by several frictions that cause individual securities to be priced differently from the average pricing in the market as indicated by the term structure of interest rates. One of these frictions is related to the accounting and performance measurement norms prevalent in the market. The performance of a trader is judged by the trading profits as measured by the difference between the purchase and sale prices of a security. The coupon plays no part in performance measurement and is accounted as coupon income in the accounts of the organization. There is an incentive for traders to buy discount bonds to show a trading profit. Hence, the demand for the discount bonds is higher which raises their prices above the economic cost as ascertained from the term structure of interest rates. Another friction is that a substantial portion of trading is concentrated in a handful of bonds. The market perceives these bonds to be liquid. The remaining bonds, which are not perceived as liquid, get traded at prices that incorporate their illiquidity.

The primary objective of this paper is to produce a framework that can be used to generate accurate model prices of emerging market sovereign debt instruments. We pose the following questions:

1. Which of the various functional forms available can be best used in emerging markets to generate yield curves on a daily basis?

2. Which method should be used to estimate the functional forms so that the error between model prices and observed prices is minimized?

3. Is liquidity (or lack of it) significant in estimating the term structure? If yes, how do we model liquidity?

4. Should all bonds traded on a day be included in the calibration of the model or should some of the bonds, which are seen to be distorting the price information, be excluded?

The third question, in particular, is quite pertinent to term structure estimation in all emerging markets which typically exist in developing countries. In fact in emerging markets, in the absence of an established term structure of interest rates, the markets model it on their own. However, the drawback in this is that the market determines the model based only on the liquid securities and hence is only partially complete. The classic example is the Indian market where no specific term structure model of interest rate exists. Market operators devise a proxy of the term structure based on the liquid

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1 The term "term structure" is somewhat loose and includes the discount function, the discount rate function (zero-coupon yield curve), and the forward rate curve. Since each is a transformation of the other, the term can be used

securities. However, the large number of illiquid securities in the market causes this market-determined proxy to be subject to large errors.

This paper is organized as follows. The next section is devoted to a literature survey of the modelling structure. Section 3 details the methodology adopted in this paper. Section 4 discusses the data used for estimation. Section 5 presents the results and investigates their implications. Section 6 puts forth recommendations and highlights some of the areas for further research.

2 Literature Survey

Most of the numerical methods to estimate the term structure explicitly constrain cash flows from different bonds due at the same time to be discounted at the same rate. This leads to two distinct methods of estimation: one that focuses on flexibility of the curve (accuracy) and the other on the smoothness based on the notion of a particular shape of the term structure. The effort is to reach a compromise between flexibility and smoothness. The trade-off is inherent in the problem itself and any method evolved will have to grapple with the problem of deciding on the extent of trade-off.

The numerical theory underlying the approximation of yield curves using various functional forms is based on the Weierstrass theorem, which holds that a continuously differentiable function can be approximated in some interval (to within an arbitrary error) to some polynomial defined over the same interval.

McCulloch's (1971) method tries a spline approximation to yield curve with the Weierstrass theorem as the basis. This method requires the specification of a basis function that is crucial. The discount function is expressed as a linear combination of basis functions, which is estimated by regressing on the price data of the bond. Certain properties of the discount function are mandated:

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interchangeably. In this paper, we use "term structure" to mean the discount rate function unless otherwise specified.

1. The discount function must be positive
2. The discount function must be monotonically non-decreasing
3. The discount function must be equal to unity at $t = 0$

McCulloch himself suggested a simple polynomial for the basis functions. However, these functions have a uniform resolution. Hence they fit better where the point set is dense as compared to the range where point sets are scarce. McCulloch uses quadratic splines[2], but this leads to oscillations in forward rate curves, a phenomenon which McCulloch calls “knuckles”.

The method to avoid this effect is to increase the order of the estimating function and use (for example) a *cubic spline*. McCulloch (1975) presents the simplest implementation of a cubic spline. It can be quite flexible, as it does not constrain the discount function to be non-increasing; however, the forward rates may turn out to be negative.

Mastronikola (1991) suggests a more complex cubic spline wherein the first and second derivatives of the adjoining functions are constrained to be equal at the knot points. By constraining the end points, each cubic equation is unique and hence the entire curve is unique. The short end of the curve is constrained to have a constant slope while the long end is made flat. But cubic splines too have the disadvantage of producing estimates of forward rates that are rather unstable.

In order to avoid the problem of improbable looking forward curves with cubic splines, a method that uses exponential splines to produce an asymptotically flat forward rate curve is used. This model is as capable of modelling the term structure, as are ordinary polynomial splines. Also there is an added computational burden of estimating the non-linear rather than linear model. Hence use of ordinary splines, rather than exponential splines, is recommended.

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2 Splines can be thought of as a number of polynomials joined smoothly at the point of join. These join points are called knot points and smooth means that at these points the first and second derivatives of the curve exist.

But there are important concerns regarding the choice of basis functions as suggested by McCulloch. The contention is that these basis functions can generate a regression matrix with columns that are perfectly collinear, resulting in possible inaccuracies owing to subtraction of large numbers. Use of B-splines as a solution is advocated. These are functions that are identically zero over a large portion of the approximation space and prevent the loss of accuracy because of cancellation. Steeley (1991) suggests the use of B-splines, which he shows to be more convenient and an alternative to the much-involved Bernstein (1926) polynomials. Eom, Subrahmanyam, and Uno (1998) use B-splines successfully to model the tax and coupon effects in the Japanese bond market.

Considering the problems of unbounded forward rates with the above methods, Nelson and Siegel (1987) tried smoothing the forward rate, which they modelled as an exponential polynomial.

Svensson (1995) proposes a modification to Nelson and Siegel’s (1987) forward rate model. Bliss (1997) has proposed estimating the Nelson and Siegel model using a non-linear, constrained optimization procedure that accounts for the bid and ask prices of bonds as well.

Adams and Van Deventer (1994), while continuing to focus on the forward rate function, take a fundamentally different approach to estimating the term structure of interest rates. Their criterion for the best fitted curve is in terms of the maximum smoothness for forward rates.

Splines and the Nelson and Seigel(1987), and Svennson(1995) forms constitute the two polar extremes in the continuum of methods that emphasize flexibility (accuracy) and smoothness respectively. There has been the middle of the road approaches too. Fisher, Nychka, and Zervos (1995) propose using a cubic spline with roughness penalty to extract the forward rate curve. This method tries to model smoothness and flexibility in the same objective function and allows weights to be attached to each. Varying this weight decides the extent of the trade-off required. The roughness penalty is chosen by a generalized cross-validation method to regulate the trade-off. This method performs better than McCulloch’s in the medium and long bonds but ends up with excessive smoothing on the short side.

According to Bliss (1997), the method of Fisher *et al.* (1995) tends to mis-price short maturity securities. This is because it attaches the same penalty across maturities. Waggoner (1997) proposes using a variable roughness penalty for different maturities called the VRP (Variable Roughness Penalty) method. This method provides better results than that of Fisher on the short side and performs as well on the medium and long bonds.

All these works concentrate on estimating the yield curves in developed markets where information for securities at different maturities is readily available. However, in less developed markets the government debt market is not liquid and derivatives markets for government debt do not exist. In such a scenario, functional forms discussed earlier can be used but with some modifications. We propose that the effect of liquidity be incorporated into the estimation procedure. Traditional parameter estimation has been done either by minimizing the mean of the squares of the error between the observed and calculated prices or by using maximum likelihood estimation. Bolder and Streliski (1999) use objective functions which penalize to a greater extent the errors on such bonds which fall out of the bid-ask spread. They use maximum likelihood estimation incorporated in the errors by assuming the standard deviation to be equal to half the bid-ask spread. They also use weighted least square estimation. They calculate the weights by dividing the individual security price error with the bid–ask spread and raising the quotient to the power of λ which they call the 'penalty parameter'. Other modified objective functions include incorporating penalties for roughness in the yield curve.

However, in illiquid markets like India where only about a handful of liquid securities get traded in a day, illiquid bonds must also be included in the estimation procedure. Hence the estimation methods must incorporate the effect of liquidity premiums on illiquid bonds. We attempt to estimate the parameters by minimizing the mean absolute deviation between the observed and calculated prices. We also use the weighted least squares and weighted mean absolute deviation to estimate the parameters. The weights have been assigned based on the liquidity of individual securities.

3 Analysis

This section discusses the methodology adopted in our work. Subsection 3.1 explains the various objective functions used for the estimation and details the rationale for the same. Subsection 3.2 narrates the functional forms used for estimation.

3.1 Objective Functions

Since the objective of the project is to generate spot interest rates so that the GOISEC can be priced as accurately as possible, the error between the observed price and the price calculated from the model is the basis for optimization. Our hypothesis is that the mean absolute deviation is a more appropriate objective function than the mean squared error for the following reasons:

- The term structure of interest rates enables traders identify over-priced and under-priced securities. The payoff to traders to identify underpriced or overpriced securities in the market and take advantage of them is a linear function of the pricing error in the security. Hence, the function that needs to be optimized must be a linear function in the error. optimization of the mean squared error, on the other hand, has no economic rationale apart from modelling convenience.

- In the GOISEC market, as in most other markets, the class of traded securities comprises both liquid ones, which get traded frequently, and illiquid ones that report one-off trades. Liquid securities tend to typically have finer bid-ask spreads compared to illiquid ones. But in an illiquid government debt market the number of liquid securities is low and hence illiquid securities must also be included in estimating the term structure. The market, however, tends to charge a liquidity premium on illiquid securities and hence we expect illiquid securities to be priced more inaccurately by a model that ignores liquidity premiums. Hence we expect the pricing errors to be larger on illiquid securities than on the more liquid ones. In such a scenario, using a squared error criterion tends to accentuate pricing anomalies since large error terms resulting from the presence of liquidity premiums contribute more to the objective function than to the errors on liquid securities.

Having argued that the mean absolute deviation is a better optimization parameter, we go one step further. Errors are caused for two reasons: (a) curve fitting; (b) presence of liquidity premium. Now, the errors due to curve fitting arise from the calculations and should be avoided. But the error due to the presence of liquidity premium is reflective of market conditions and one does not want to ignore them. Assigning equal weights to both types of errors will give undue weight to the kind of error that creeps in due to curve fitting. Hence, we hypothesize that a weighted error function, with weights based on liquidity, would lead to better estimation than that using equal weights. However, note that this is valid only when equal weights bias the results and hence using weights based on liquidity premium is better. On the other hand, if equal weights do not bias the results, assigning weights based on liquidity premium also does not bias the results. Thus, as an overall picture, it is better to use weights based on the liquidity premium.

A reciprocal of the bid-ask spread is ideally the best liquidity function to use. However, available data do not report the bid-ask spreads in individual securities. Hence we try to model the liquidity using a function with two factors: the volume of trade in a security and the number of trades in that security. We highlight the necessity to use both factors through the following example. Consider two securities each of which report a trade volume of INR 500 million. One of them reports a single trade of INR 500 million and the other reports 10 trades of INR 50 million each. The second security is more liquid and hence would have a smaller liquidity premium associated with it than the first one. Hence, its price would be more reflective of the market prices than the first one. To quantify the liquidity weights, we use two variations:

The weight of the i^{th} security W_i is given by

$$W_i = \frac{\left(\left(1 - e^{(-v_i/v_{max})}\right) + \left(1 - e^{(-n_i/n_{max})}\right)\right)}{\sum_i Wi}$$

$$W_i = \frac{\left(\tanh\left(-v_i/v_{max}\right) + \tanh\left(-n_i/n_{max}\right)\right)}{.\sum_i Wi}$$

(1)

where v_i and n_i are the volume of trade and the number of trades in the i^{th} security while v_{max} and n_{max} are the maximum volume of trades and the maximum number of trades among all the securities traded for the day respectively.

We use the exponential and the hyperbolic tangent function to incorporate asymptotic behaviour in the liquidity function. The relatively liquid securities would have v_i/v_{max} and n_i/n_{max} close to 1 and hence the weights of liquid securities would not be significantly different. However, the weights would fall at a fast rate as liquidity decreases. This is the behaviour that we wish liquidity function to accomplish.

Thus we use the following six objective functions:

- Mean squared error (RMSE)
- Mean absolute deviation (MAD)
- Liquidity weighted mean squared error using the hyperbolic tangent function (LRMSE-T)
- Liquidity weighted mean squared error using the exponential function (LRMSE-E)
- Liquidity weighted mean absolute deviation using the hyperbolic tangent function (LMAD-T)
- Liquidity weighted mean squared error using the exponential function (LMAD-T)

3.2 Model Selection

The basis for judging model performance is linked to the expectations from the model. Our expectations are:

1. Accurate pricing of GOISEC
2. Robustness of the model in producing stable spot interest rate curves

Given the lack of interest rate derivative markets in India from which forward rates can be obtained, the general equilibrium models of Vasicek (1977), Cox, Ingersoll and Ross (1985), Brennan, and Schwartz (1979) are difficult to implement. Hence we do not consider this class of models.

Among the models discussed in Section 2, we use the Nelson, Seigel, and Svennson model (the generalized version of the Nelson Siegel model) with a premium on smoothness, cubic B-splines, and cubic splines with VRP. B-spline models are superior to cubic splines models. Hence, in the class of models emphasizing flexibility, we use B-splines. We also test smoothing splines with the VRP method to include the class of models that use a compromise between accuracy and smoothness.

3.2.1 Nelson Siegel Svensson Model

The Nelson, Siegel, and Svensson model derives the forward rate in a functional form and determines the discount function from it to avoid oscillations in the forward rate. This method has the advantage of estimating lesser number of parameters and ensures a smooth forward curve.

The forward rate function, *F(m),* is modelled as follows:

$$\begin{aligned} F(m) = {} & \beta_0 + \beta_1 \exp^{\left(-m/\tau_1\right)} + \beta_2 \left[\left(-m/\tau_1\right)\exp^{\left(-m/\tau_1\right)}\right] \\ & + \beta_3 \left[\left(-m/\tau_2\right)\exp^{\left(-m/\tau_2\right)}\right] \end{aligned} \tag{2}$$

The parameters:

β_0 is positive and is the asymptotic value of f(m).

β_1 determines the starting value of the curve in terms of deviation from the asymptote. It also defines the basic speed with which the curve tends towards its long-term trend.

τ_1 must be positive and specifies the position of the first hump or the U-shape on the curve.

β_2 decides the magnitude and direction of the hump. If this is positive, a hump occurs at τ_I whereas if it is negative, the U-shape occurs at τ_I.

τ_2 must also be positive and defines the position of the second hump or the U-shape on the curve.

β_3 like β_2, determines the magnitude and direction of the hump.

3.2.2 B-splines

We define the following with respect to B- splines:

n = number of control points

m = number of elements in knot vector

p = degree

T is a knot vector; **T** = { t_0, t_1,….t_m }; where **T** is a non-decreasing sequence with $t_i \varepsilon [0,1]$, and defines control points $\mathbf{P_0,\ldots,P_n}$.

We also define the degree as $p \equiv m - n - 1$.

Based on above terms, the basis function, $N_{i,o}(t)$ is defined as follows:

$$N_{i,0}(t) = \begin{matrix} 1 & t_i \le t < t_{i+1} \\ 0 & otherwise \end{matrix}$$

(3)

$$N_{i,p}(t)=\left(\left(t-t_i\right)\Big/\left(t_{i+p}-t_I\right)\right)N_{i,p-1}(t)+$$
$$\left(\left(t_{i+p+1}-t\right)\Big/\left(t_{i+p+1}-t_i+1\right)\right)N_{i+1,p-1}(t)$$

(4)

Then the curve defined by

C(t)= $\sum \mathbf{P}_i N_{i,p}(t)$ is a B-spline.

The discount function between any two-knot points s_j and s_{j+1} is defined as:

$$P(m)=\sum_j a_j B_j^g(m)$$

(5)

where B^g is a g-order B-spline.

To obtain a smooth forward curve a spline function of at least order three must be used.

3.2.3 Variable Roughness Penalty (Smoothing Splines)

The smoothing splines approach is an extension of the splines approach suggested by McCulloch (1971).

Cubic splines generate oscillations in the forward rate term structure. Since oscillation in the forward rate is an unexpected behaviour, this method is not acceptable.

Since the forward curve is well behaved in the short maturity segment, the penalty is less. In the long maturity segment, the penalty is the highest.

The objective is to minimize the function:

$$\sum_{j}(P_i - P_i\hat{})^2 + \lambda\int_0^k [f'(t)dt$$

(6)

The first term represents the goodness of fit (using ordinary least squares) while the second term is the roughness penalty. The values of □ suggested by McCulloch (1971) are:

$$\lambda(t) = \begin{matrix} 0.1 & 0 \le t < 1 \\ 100 & 1 \le t < 10 \\ 100000 & 10 \le t \end{matrix}$$

(7)

where t is measured in years. These values of λ are somewhat close to the square of the number of years.

4 Data

The two primary sources of data on the traded prices of GOISEC are the subsidiary general ledger (SGL) data published by the Reserve Bank of India (RBI) and the daily trade data released by the National Stock Exchange (NSE). The SGL data has the advantage of being comprehensive. However, till recently, the SGL data had the problem that the prices corresponded to the settlement dates and not contract dates. Thus SGL data would include trades contracted a couple of days earlier thus distorting the price information. However, more recently RBI has started providing the contract dates along with the settlement dates and hence the data are most reliable.

Since the trades in GOISECS are not settled at NSE, data provided by NSE are based on the trades reported to them by the brokers. Hence the NSE data are less than comprehensive. Moreover, the prices are reported based on the contract date and information about the settlement date is not always provided. Hence there may be distortions in the data because of the differences in the settlement date. But the NSE data are the best since the errors owing to differences in settlement date are considerably less as compared to the error owing to differences in contract date. The reason being this data (even though it is not entirely comprehensive) has at least got both; the settlement and contract dates while the SGL data just had the settlement dates. Thus the distortion in price information by the use of the NSE data (though slightly incomplete in terms of the number of trades) is not too much. The daily weighted average prices (weights being the volume of trade) are used for estimation.

We ignore all the bonds that are traded below par. The reason for this is the accounting and trader performance measurement norm prevailing in the Indian debt market. The performance of a trader is judged by the trading profits as measured by the difference between the purchase and sale price of a security. The coupon interest received plays no part in performance measurement and is accounted as coupon income in the accounts of the organization. There is, thus, an incentive for traders to buy discount bonds to show a trading profit. Hence the demand for discount bonds is higher which raises their prices above the economic cost as ascertained from the market yield. Therefore, including the discount securities would distort price information and introduce unnecessary noise.

We ignore securities with residual maturity less than one year since these get traded as money market instruments. Further, GOISEC with less than a year to maturity trade at yields, which are significantly different from T-bills of comparable maturities owing to the preference for discount instruments in the market.

In the GOISEC market, the market lot for trade is Rs. 50 million. Hence all trades which are less than Rs. 50 million or are not multiples are ignored. This is because these trades, being odd lot trades, would distort the price information.

5 Results

We employ in-sample tests using the mean absolute error between the observed prices and calculated prices as the criterion. We calculate the mean absolute error and the standard deviation in the absolute errors for each of the days for which the models are fitted. The B-splines and the VRP method were found to be quite unstable for estimation as they produced very large errors while they fitted the price data quite well on other days. Hence these two methods were not found to be robust for the purposes of estimation.

Hence, we pursue our estimation tests using the NSS method. We use the paired *t*-test to test for significance in the difference in errors and standard deviations using the various optimization methods. All tests are carried out at 95% level of confidence. The results are shown in Tables 1-9.

We use the following notations in Tables 1-9

MAD	Mean absolute Deviation
RMSE	Root Mean Squared Error
WMAD-E	Weighted Mean Absolute Deviation – Exponential
WMAD-HT	Weighted Mean Absolute Deviation – Hyperbolic Tangent
WRMSE-E	Weighted Root Mean Squared Error-Exponential
WRMSE-HT	Weighted Root Mean Squared Error - Hyperbolic Tangent

In Table 1, we find that that MAD estimation gives smaller absolute errors and smaller standard deviations than RMSE estimation. Similarly Table 2 and 3 shows that MAD gives smaller error and smaller standard deviation than WMAD-E and WMAD-HT. In Table 4 we find a comparison of WMAD-E and WMAD-HT. The values of mean and standard deviation are very close to each other and the differences in the mean and standard deviation are in the fourth decimal places. In Tables 5 and 6, we find that RMSE gives smaller mean and smaller variance than WRMSE-E and WRMSE-

HT. Table 7 makes a paired two-sample comparison of WRMSE-E and WRMSE-HT. In this case we find that WRMSE-HT has smaller mean and standard deviation than WRMSE-HT. Table 8 denotes a paired comparison between WRMSE-HT and WMAD-HT. In Table 9, we show paired comparison between WRMSE-E and WMAD-E. In both the tables we find that WMAD-E is better than RMSE-E and WMAD-HT is better than WRMSE-HT. From these observations, we conclude than MAD gives better estimation than RMSE. In case we plan to give weights, WMAD-HT is slightly better than WMAD-E.

6. Conclusion

In a government bond market where the number of liquid bonds is quite small (as in the GOISEC market), term structure estimation needs to model the liquidity (or the lack of it) in individual bonds. We find that a liquidity function based on the number of trades in the security and the total volume of trade in the security models the liquidity fairly well. Estimation using this weighted objective function ensures that liquid bonds in the market are priced efficiently. We also find that in illiquid markets, minimizing the mean absolute deviation is a better estimation procedure than minimizing the root mean squared error.

For modelling the liquidity of individual securities, we find the hyperbolic tangent function to be a better approximation than an exponential function. Further work in this area can be done to include modelling of liquidity premiums in individual securities.

The term structure-modelling framework developed in this paper can be extended to estimating the term structure for corporate bond markets, which are typically illiquid in most emerging market debt.

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Table 1 t-Test: Paired Two Sample for Means (MAD and RMSE)		
	MAD	**RMSE**
Mean	0.137320233	0.167163307
Variance	0.003281801	0.006848428
Observations	270	270
Pearson Correlation	0.549153875	
Hypothesised Mean Difference	0	
Df	269	
t Stat	-4.460757003	
P(T<=t) one-tail	9.97407E-06	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	1.99481E-05	
t Critical two-tail	2.254018909	

Table 2 t-Test: Paired Two Sample for Means (MAD and WMAD-E)		
	MAD	***WMAD-E***
Mean	0.137320233	0.141179955
Variance	0.003281801	0.003538834
Observations	270	270
Pearson Correlation	0.951584003	
Hypothesised Mean Difference	0	
Df	269	
t Stat	-2.212253972	
P(T<=t) one-tail	0.014517398	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	0.029034797	
t Critical two-tail	2.254018909	

Table 3 t-Test: Paired Two Sample for Means (MAD and WMAD-HT)		
	MAD	***WMAD-HT***
Mean	0.137320233	0.141177129
Variance	0.003281801	0.003526343
Observations	270	270
Pearson Correlation	0.955459833	
Hypothesised Mean Difference	0	
Df	269	
t Stat	-2.307059123	
P(T<=t) one-tail	0.011470519	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	0.022941038	
t Critical two-tail	2.254018909	

Table 4 t-Test: Paired Two Sample for Means (WMAD-E and WMAD-HT)		
	WMAD-E	***WMAD-HT***
Mean	0.141179955	0.141177129
Variance	0.003538834	0.003526343
Observations	270	270
Pearson Correlation	0.988771369	

Hypothesised Mean Difference	0	
Df	269	
t Stat	0.003328035	
P(T<=t) one-tail	0.49867535	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	0.9973507	
t Critical two-tail	2.254018909	

Table 5
t-Test: Paired Two Sample for Means (RMSE and WRMSE-E)

	RMSE	***WRMSE-E***
Mean	0.167163307	0.290252015
Variance	0.006848428	0.022421391
Observations	270	270
Pearson Correlation	0.294077267	
Hypothesised Mean Difference	0	
Df	269	
t Stat	-8.707317274	
P(T<=t) one-tail	1.8685E-14	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	3.737E-14	
t Critical two-tail	2.254018909	

Table 6
t-Test: Paired Two Sample for Means (RMSE and WRMSE-E)

	RMSE	***WRMSE-HT***
Mean	0.167163307	0.213281377
Variance	0.006848428	0.019014788
Observations	270	270
Pearson Correlation	0.293167075	
Hypothesised Mean Difference	0	
Df	269	
t Stat	-3.493256714	
P(T<=t) one-tail	0.000345304	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	0.000690608	
t Critical two-tail	2.254018909	

Table 7
t-Test: Paired Two Sample for Means (WRMSE-E and *WRMSE-HT*)

	WRMSE-E	***WRMSE-HT***
Mean	0.290252015	0.213281377
Variance	0.022421391	0.019014788
Observations	270	270
Pearson Correlation	0.476931138	
Hypothesised Mean Difference	0	
Df	269	
t Stat	5.474984786	
P(T<=t) one-tail	1.41472E-07	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	2.82944E-07	
t Critical two-tail	2.254018909	

Table 8
t-Test: Paired Two Sample for Means (WRMSE-HT and WMAD-HT)

	WRMSE-HT	***WMAD-HT***
Mean	0.213281377	0.141177129
Variance	0.019014788	0.003526343
Observations	270	270
Pearson Correlation	0.604336353	
Hypothesised Mean Difference	0	
Df	269	
t Stat	6.725395826	
P(T<=t) one-tail	4.19751E-10	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	8.39501E-10	
t Critical two-tail	2.254018909	

Table 9
t-Test: Paired Two Sample for Means (WRMSE-E and WMAD-E)

	WRMSE-E	***WMAD-E***
Mean	0.290252015	0.141179955
Variance	0.022421391	0.003538834
Observations	270	270
Pearson Correlation	0.320922935	
Hypothesised Mean Difference	0	
Df	269	
t Stat	10.98894806	
P(T<=t) one-tail	1.16954E-19	
t Critical one-tail	1.968819561	
P(T<=t) two-tail	2.33908E-19	
t Critical two-tail	2.254018909	