Assessing Default Probabilities from Structural Credit Risk Models

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Abstract

In this paper, we study the empirical performance of structural credit risk models by examining the default probabilities calculated from these models with different time horizons. The parameters of the models are estimated from firm's bond and equity prices. The models studied include Merton (1974), Merton model with stochastic interest rate, Longstaff and Schwartz (1995), Leland and Toft (1996) and Collin-Dufresne and Goldstein (2001). The sample firms chosen are those that have only one bond outstanding when bond prices are observed. We first find that when the Maximum Likelihood estimation, introduced in Duan (1994), is used to estimate the Merton model from bond prices the estimated volatility is unreasonable high and the estimation process does not converge for most of the firms in our sample. It shows that the Merton (1974) is not able to generate high yields to match the empirical observations. On the other hand, when equity prices are used as input we find find that the default probabilities predicted for investment-grade firms by Merton (1974) are all close to zero. When stochastic interest rates are assumed in Merton model the model performance is improved. Longstaff and Schwartz (1995) with constant interest rate as well as the Leland and Toft (1996) model provide quite reasonable predictions on real default probabilities when compared with those reported by Moody's and S&P. However, Collin-Dufresnce and Goldstein (2001) predicts unreasonably high default probabilities for longer time horizons.

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1 Introduction

Since the seminal work of Merton (1974), many structural credit risk models have been proposed, including Longstaff and Schwartz (1995), Leland and Toft (1996), and Collin-Dufresne and Goldstein (2001), among others. In this type of models, both the equity and the debt of a firm are modeled as contingent claims over the asset value of the issuing firm, and as a result, option pricing theory can be applied. Defaults occur when the firm asset value, which is usually modeled as a diffusion process, reaches a certain barrier either during the life of the debt or at the maturity of the debt. This type of models establish the relationships between the returns of the firm's equity and debt, as well as the yield spreads and the firm's balance sheet information such as leverage ratio.

Structural models can also be used to estimate the default probabilities of the issuing firms. For banks and regulators, timely and accurate predictions of borrowers default probabilities are essential to developing responsive and effective risk management tools. Moreover, the newly adopted Basel II specifically requires financial institutions to use credit risk models that are conceptually sound and empirically validated. Our main aim in this study is to empirically analyze the performance of structural models, including the Merton model, Longstaff and Schwartz (LS) model, Leland and Toft (LT) model, and the Collins-Dufresne and Goldstein (CDG) model, when they are used to estimate the default probabilities of the debt issuing firms.

Many studies have been taken to investigate if structural models can explain yield spreads. They include Jones et al. (1984), Wei and Guo (1997), Anderson et al. (2000), Lyden and Saraniti (2000), Collin-Dufresne et al. (2001), Elton et al. (2001), Cooper and Devydenko (2003), Delianedis and Geske (2003), Huang and Huang (2003), Eom et al (2004), Leland (2004), and Ericsson and Reneby (2005), among others. Huang and Huang (2003) and Eom et al. (2004) provide the most comprehensive comparison among various structural models. By calibrating different models to default probabilities and historical equity premium, Huang and Huang (2003) find that the spread implied by structural models are too low for investment grade bonds. Eom et al (2004) show that the Merton (1974) model and the Geske (1977) model under-predict while the LT model over-predicts the yield spreads. With stochastic interest rate, it is found that the LS model and the CDG model do relatively better than the other models. However, they are sensitive to the choice of interest rate parameters.

The poor empirical performance of structural models, especially in forecasting yield spreads of corporate debts over Treasury bonds for short term debts, are usually explained in the literature by the following: it is believed that yield spreads consist of three distinct components that are attributed to default risk, taxes and liquidity factors. Even though default risk is considered to be the most important factor in determining the yield spreads, empirical studies, such as Elton et al (2001) and Huang and Huang (2003), have argued that while default risk can explain a large proportion of the yield spreads for low grade debts, it only account for a small proportion of the yield spreads for high grade debt. The remaining portion of the spread are attributed to the risk premium compensating the systematic risk of defaults (Elton et al. (2001) and Vassalou and Xing (2004)), as well as to the different tax treatments between Treasury bonds and corporate bonds (Elton et al (2001)).

On the contrary to the approach adopted in Huang and Huang (2003), Cooper and Devydenko (2003), basing on the Merton (1974) model, calibrate on the yield spreads between corporate bonds and otherwise-similar AAA-rated bond rather than using the spread between the corporate bond and the Treasury to predict the expected default loss, given information on leverage, equity risk premium, and equity volatility. Their results are consistent with Elton et al. (2001). Delianedis and Geske (2003) study the the influence of several factors including tax, jump, and liquidity on the level of credit spreads. They show that even with jumps in firm asset value the models are still unable to explain the high yield spreads.

In this study, we use equity and bond prices to estimate the model parameters by the maximum likelihood estimation developed in Duan (1994), when the likelihood function is available. When the likelihood function can not be derived for some models the parameters are chosen to fit the observed prices in order to predict default probabilities. We compare the predicted default probabilities from each structural model, grouped by rating classes, with the historical default probabilities over different time horizon reported by both Moody's and S&P to assess the model performance.

Our results show that the one-year default probability from the Merton (1974) model are close to zero for most of the investment-grade firms. However, it tends to over estimate default probabilities for non-investment-grade firms. Its performance is improved when a stochastic term structure is assumed, where the default probabilities from the model with stochastic interest rate. We also find that the default probabilities calculated the LS model with constant interest rate and the LT model are very close to the real world observations. However, with a mean-reverting capital structure assumed, the CDG model over predicts default probabilities to a quite large extent.

2 Structural Models and Default Probabilities

The core concept of the structure models, which originated in the seminal work of Merton (1974), is to treat firm's equity and debt as contingent claims written on its asset value. Default is modeled as either when the underlying asset process reaches the default threshold or when the asset level is below the debt face value at the maturity date. More specifically, the asset value is assumed to follow a diffusion process in the following form:

$$\frac{dV_t}{V_t} = (\mu_v - \delta)dt + \sigma_v dW_t^v \tag{1}$$

where μ_v is the expected asset return, δ is the asset payout ratio, σ_v is the volatility of firm asset value, and W_t^v is a Brownian motion. Structural models can be distinguished as either have exogenous default barrier or endogenous default barrier.

2.1 Merton (1974) Model

In the Merton model a firm's equity is treated as an European call option written on the firm's asset value. It is assumed that the issuing firm has only one outstanding bond, and thus that firm does not default prior to the debt maturity date. In addition, the term structure of risk-free interest rate r, firm's asset volatility σ_v and asset risk premium π_v are assumed to be constants. At maturity time T, the payoff of the equity is

$$E(V, T) = \max(0, V_T - F),$$

and the payoff of the risky bond is

$$B(V, T) = \min(V_T, F) = F - \max(0, F - V_T)$$

where F denotes the face value of the promised payments of debt. The equity value can then be written as,

$$E_t(V, T) = Ve^{-\delta(T-t)}N(d_1) - Fe^{-r(T-t)}N(d_2) \tag{2}$$

where

$$d_1 = \frac{[\ln(V/F) + (r - \delta + \sigma_v^2/2)(T - t)]}{\sigma_v\sqrt{T - t}},\ d_2 = d_1 - \sigma_v\sqrt{T - t},$$

δ is the asset payout ratio. The value of the risky bond is equal to the difference between the asset value and the equity value,

$$B_t(V, T) = V_t e^{-\delta(T-t)}N(-d_1) + Fe^{-r(T-t)}N(d_2). \tag{3}$$

The yield spread over risk-free bond can be expressed as,

$$s = -\frac{1}{T-t}\ln(B/F) - r \tag{4}$$

The asset volatility σ_v and the equity volatility, σ_e satisfy the following equation,

$$\sigma_e = \frac{V}{E}\frac{\partial E}{\partial V} = \frac{\sigma_v e^{-\delta(T-t)}N(d_1)V}{E} \tag{5}$$

The asset risk premium π^v and the equity risk premium π^e can be linked by

$$\pi^v = \frac{\pi^e \sigma_v}{\sigma_e} \tag{6}$$

Under the empirical probability measure, the probability of default over time interval $[t, T]$ is derived as,

$$DP_{\text{Merton}} = P(V_T < F_T) = P\left(z_T \leq -\frac{\ln(V_t/F) + (\mu_v - \delta - \frac{\sigma_v^2}{2})(T-t)}{\sigma_v\sqrt{T-t}}\right) \tag{7}$$

where z follows a standard normal distribution. The quantity

$$-\frac{\ln(V_t/F) + (\mu_v - \frac{\sigma_v^2}{2})(T-t)}{\sigma_v\sqrt{T-t}}$$

is referred to as the *distance-to-default* by Moody's KMV. It is usually calculated by the the relevant three-year asset value, asset volatility and the face value of debt, proxied by the sum of the total short-term debt plus half of the long-term debt.

2.2 Merton(1974) with Stochastic Interest Rate

Merton model can easily be extended to the case where the risk-free interest rate is stochastic. Consider the case the interest rate follows the Vasicek (1977) process,

$$dr = \kappa_r(\bar{r} - r)dt + \sigma_r dW_t^r \tag{8}$$

where κ_r is the rate of mean reversion, $\bar{r}$ is the long term mean and σ_r is the short rate volatility, W_t^r is the standard Brownian motion and the instantaneous correlation between dW_t^v and dW_t^r is $\rho_{vr}dt$. All the parameters in this model are assumed to be constant.

The value of a risk-free discount bond $\bar{B}(r, t, T)$ is given by

$$\bar{B}(r, t, T) = e^{A(t,T) - C(t,T)r(t)} \tag{9}$$

where

$$\begin{aligned} C(t,T) &= \frac{1}{\kappa_r}(1 - e^{-\kappa_r(T-t)}), \\ A(t,T) &= \frac{1}{\kappa_r^2}\left(C(t,T) - (T-t)\right)\left(\kappa_r^2\bar{r} - \frac{\sigma_r^2}{2}\right) - \frac{1}{4\kappa_r}\sigma_r^2 C(t,T)^2. \end{aligned}$$

If we assume that the firm asset value V is tradable, the expected rate of return on firm's value and risk-free rate are connected through $\mu_v - \lambda^v \sigma_v = r$, where λ^v denotes the market price of risk of firm asset. Here we further assume that the market price of risk of asset is not constant and described by,

$$d\lambda_t^v = \kappa_\lambda(\bar{\lambda}^v - \lambda_t^v)dt + \sigma_\lambda dW_t^\lambda \tag{10}$$

where the instantaneous correlation coefficient between dW_t^v and dW_t^λ is $\rho_{v\lambda}dt$ and the correlation coefficient between dW_t^r and dW_t^λ is $\rho_{r\lambda}dt$.

If we let

$$y = -\int_t^T r_s ds, \quad x = \frac{V_T}{V_t},$$

and $\tau = T - t$ then the value of equity can be written as,[1]

$$S_t = \exp[\mu_{\ln(x)} + \mu_y + \frac{1}{2}(\sigma^2_{\ln(x)} + \sigma_y^2 + 2Cov_{\ln(x),y})]V_t N(d_1) - \bar{B}(t,T)FN(d_2) \tag{11}$$

where

$$\begin{aligned}
\mu_{\ln(x)} &= \mu_v\tau - \frac{\sigma_v^2}{2}\tau - \delta\tau - \sigma_v[\lambda^v - (\bar{\lambda}^v - \lambda_t^v)C_\lambda^v(\tau)] \\
\mu_y &= -\bar{r}\tau + (\bar{r} - r_t)C_r(\tau) \\
\sigma^2_{\ln(x)} &= \frac{\sigma_v^2\sigma_\lambda^2}{\kappa_\lambda^2}[\tau - C_{\lambda^v}(\tau) - \frac{1}{2}\kappa_\lambda C_r(\tau)^2] + \sigma_v^2\tau - \rho_{v\lambda}\frac{\sigma_v^2\sigma_\lambda^2}{\kappa_\lambda}[\tau - C_{\lambda^v}(\tau)] \\
\sigma_y^2 &= \frac{\sigma_r^2}{\kappa_r^2}[\tau - C_r(\tau) - \frac{1}{2}\kappa_r C_r(\tau)^2] \\
Cov(\ln(x), y) &= \rho_{r\lambda}\frac{\sigma_r\sigma_v\sigma_\lambda}{\kappa_r\kappa_\lambda}[\tau - C_{\lambda^v}(\tau) - C_r(\tau) + C_{\lambda^v,r}(\tau)] - \rho_{vr}\frac{\sigma_v\sigma_r}{\kappa_r}[\tau - C_r(\tau)] \\
C_{\lambda^v,r}(\tau) &= \frac{1}{\kappa_r + \kappa_\lambda}(1 - \exp(-(\kappa_r + \kappa_\lambda)\tau) \\
d1 &= \frac{\ln(\frac{V_t}{F}) + \mu_{\ln(x)} + \sigma^2_{\ln(x)} + Cov_{\ln(x),y}}{\sigma_{\ln(x)}} \\
d2 &= d1 - \sigma_{\ln(x)}
\end{aligned}$$

The above equation can be re-written as,

$$S_t = \bar{B}(t,T)\exp\left[\mu_{\ln(x)} + \frac{1}{2}(\sigma^2_{\ln(x)} + 2Cov(\ln(x), y))\right] V_t N(d_1) - \bar{B}(t,T)FN(d_2) \tag{12}$$

Correspondingly, from $V_t = S_t + B_t$ and equation (11), the bond price can be written as,

$$B_t = V_t[1 - \exp(\mu_{\ln(x)} + \mu_y + \frac{1}{2}\sigma^2_{\ln(x)} + \frac{1}{2}\sigma_y^2 + Cov_{\ln(x),y})N(d_1)] + \bar{B}(r,t,T)FN(d_2) \tag{13}$$

[1]Derivation available upon request

2.3 Exogenous Default Barrier Models

2.3.1 Constant Interest Rates

Black and Cox (1976) treat the firm's equity as a down-and-out call option on firm's value. In their model, firm defaults when its asset value hits a pre-specified default barrier, V^*, which can be either a constant or a time varying variable. The default barrier is assumed to be exogenously determined. When the risk-free interest rate, asset payout ratio, asset volatility and risk premium are all assumed to be constant, the cumulative default probability over a time interval $[t, t+\tau]$ can be calculated as

$$\begin{aligned} DP_{\text{Black-Cox}}(t, t+\tau) &= N\left(-\frac{\ln(\frac{V_t}{V^*}) + (\mu_v - \delta - \sigma_v^2/2)\tau}{\sigma_v\sqrt{\tau}}\right) \\ &+ \exp\left(-\frac{2\ln(\frac{V_t}{V^*})(\mu_v - \delta - \sigma_v^2/2)}{\sigma_v^2}\right) N\left(-\frac{\ln(\frac{V_t}{V^*}) - (\mu_v - \delta - \sigma_v^2/2)\tau}{\sigma_v\sqrt{\tau}}\right). \end{aligned} \tag{14}$$

2.3.2 Stochastic Interest Rates

Longstaff and Schwartz (1995) extends the Black-Cox model to the case when the risk-free interest rate is stochastic and follows the Vasicek (1977) process. The default boundary, V^*, is pre-determined. When default occurs bondholders receive a fraction of $(1-\omega)$ of the face value of the debt at maturity. In the original LS model the payout ratio of the asset value process is assumed to equal zero. Here we assume the asset value follows the process in (1). In their model, the asset risk premium is assumed to be constant and the interest rate risk premium is of an affine form in r_t. The value of a risky discount bond with maturity T in the LS model is given as,

$$B(X, r, t, T) = \bar{B}(r, t, T)(1 - \omega Q_t(X, r, T)) \tag{15}$$

where $Q(\cdot)$ is the risk-neutral default probability and $X = V/V_*$ is the ratio of the asset value to the default boundary. One can derive the valuation formula for a risky bond that pays semi-annual coupons at an annual rate of c. Let T_i, $i = 1, ..., 2(T-t)$, denote the i-th coupon payment date, and the value of the bond is derived as,

$$\begin{aligned} B(X, r, t, T)_{\text{coupon}} = & \frac{c}{2} \sum_{i=1}^{2(T-t)-1} \bar{B}(r, t, T_i)(1 - \omega_{\text{coupon}} Q_t^{T_i}(r, T_i) \\ & + \left(1 + \frac{c}{2}\right) \bar{B}(r, t, T)(1 - \omega Q_t^T(r, T)) \end{aligned} \tag{16}$$

where ω_{coupon} is the loss rate on coupon,[2] and $Q_t^{T_i}(T_i)$ is the time-t default probability over $[t, T_i]$ under the T_i-forward measure. The default probability $Q_t^{T_i}$ can be calculated analytically as in

[2] In practice, coupon payments due after the default event are typically written down completely and thus ω_{coupon} is often set to equal to 1.

Section 3.[3] The yield to maturity for this risky coupon bond y_c can be calculated through

$$B(X,r,t,T)_{\text{coupon}} = e^{-y_c(T-t)} + \frac{cF}{2}\sum_{i=1}^{2(T-t)} e^{-y_c T_i} \tag{17}$$

The risk-free T-sport rate r_c can be also implied in the same way,

$$\bar{B}(r,t,T)_{\text{coupon}} = e^{-r_c(T-t)} + \frac{cF}{2}\sum_{i=1}^{2(T-t)} e^{-r_c T_i} \tag{18}$$

The credit spread is defined as

$$s_c = y_c - r_c \tag{19}$$

2.3.3 Mean-Reverting Leverage Ratio

In the LS model, the default boundary is presumed to be a monotonic function of the amount of outstanding debt. Since asset value follows geometric Brownian motion and increases exponentially over time while the debt level remains constant it leads to a exponential decline of the expected leverage ratios. However, this is not consistent with the empirical observations that most of firms do keep stable leverage ratios (e.g. see Wang (2005)). Collin-Dufresne and Goldstein (2001) extends the model by considering a general model that generates mean-reverting leverage ratios. In their model, the risk-free interest rate is assumed to follow the same process as in (8), and the log-default threshold is assumed to follow the process,

$$d\ln V_t^* = \kappa_l[\ln V_t - \nu - \phi(r_t - \bar{r}) - \ln V_t^*)]dt \tag{20}$$

After applying Ito's lemma we obtain a mean-reverting log-leverage process under the physical measure as,

$$dl_t = \kappa_l(\bar{l}^P - l_t)dt - \sigma_v dW_t^{vQ} \tag{21}$$

where

$$\bar{l}^P = \frac{-\mu_v + \delta + \sigma_v^2/2}{\kappa_l} - \nu + \phi(\bar{r} - r) \tag{22}$$

where we let, $\mu_v = \pi^v + r$. The asset payout ratio and the asset risk premium are assumed to be constant in their model.

[3] Since the LS model can be nested in the CDG we will present the close-form solution for the default probability in the following section.

In order to calculate the default probability DP_{CDG}, we need to calculate the conditional and unconditional moments of the bivariate normal distribution of (l_t, r_t). The derivations of the conditional moments of $(\ln(X_t), r_t)$ are shown in the Appendix.[4]

The value of a coupon bond can be calculated similarly as in the LS model. Notice that we can treat the LS model as a special case by setting $\kappa_l = 0$.

2.4 Endogenous Default Barrier Models

Leland (1994) and Leland and Toft (1996) assume that firm defaults when its asset value reaches an endogenous default boundary. In order to avoid default a firm would issue equity to service its debt and at default the value of equity goes to zero. The optimal default boundary is chosen by the shareholders to maximize the value of equity at default-triggering asset level. Leland (1994) postulates that the term structure, dividend payout rate and asset risk premium are constants. In the event of default equity holders get nothing and debt holders receive a fraction $(1-\omega)$ of the firm's asset value. Under these assumptions, the value of a perpetual bond that pays semi-annual coupons at an annual rate of c and the optimal default boundary can be calculated analytically.

Leland and Toft (1996) relax the assumption of the infinite maturity of debt while keeping the same assumptions for the term structure of interest rate and the fraction of loss upon default. Under risk neutral valuation, the value of debt is the sum of the expected discounted value of the coupon flow and the repayment of principal, and the expected value of the fraction of assets which will go to debt upon default:

$$B(V,T)_{LT} = \frac{cF}{r} + \left(F - \frac{cF}{r}\right)\left(\frac{1-e^{-rT}}{rT} - I(T)\right) + \left((1-\omega)V^* - \frac{cF}{r}\right)J(T) \tag{23}$$

where

$$\begin{aligned}
I(T) &= \frac{1}{rT}(G(T) - e^{-rT}\tilde{F}(T)), \\
G(T) &= (X)^{-a+z}N(q_1(T)) + (X)^{-a-z}N(q_2(T)) \\
\tilde{F}(T) &= G(T)|z = a \\
J(T) &= \frac{1}{z\sigma_v\sqrt{T}}[-(X)^{-a+z}N(q_1(T))q_1(T) + (X)^{-a-z}N(q_2(T))q_2(T)].
\end{aligned}$$

with

$$a = \frac{r-\delta}{\sigma_v^2} - \frac{1}{2}, \quad b = \ln(X), \quad X = \frac{V}{V^*}, \quad z = (a^2 + \frac{2r}{\sigma_v^2})^{1/2},$$

[4]Their derivations are shown because there are some typos in the formulae presented in Eom et al. (2004) and Huang and Huang (2003).

$$q_1(T) = \frac{-b - z\sigma_v^2 T}{\sigma_v \sqrt{T}}, \quad q_2(T) = \frac{-b + z\sigma_v^2 T}{\sigma_v \sqrt{T}},$$

The default boundary takes the following form,

$$V_{LT}^* = \frac{(cF/r)(A/(rT) - B) - AF/(rT) - \tau cF(a+z)/r}{1 + \omega(a+z) - (1-\omega)B} \tag{24}$$

where

$$A = 2ae^{-rT}N(a\sigma_v\sqrt{T}) - 2zN(z\sigma_v\sqrt{T}) - \frac{2}{\sigma_v\sqrt{T}}n(z\sigma_v\sqrt{T}) + \frac{2e^{-rT}}{\sigma_v\sqrt{T}}n(a\sigma_v\sqrt{T}) + (z-a)$$

$$B = -(2z + \frac{2}{z\sigma_v^2 T})N(z\sigma_v\sqrt{T}) - \frac{2}{\sigma_v\sqrt{T}}n(z\sigma_v\sqrt{T}) + (z-a) + \frac{1}{z\sigma_v^2 T}$$

with $n(\cdot)$ as the standard normal density function and τ as the marginal tax rate. The default probability takes the similar form as (14) with the default boundary changed to V_{LT}^*. The credit spread is defined as $cF/B(V,T) - r$ or it can be derived in the same way as in (17), (18), and (19).

3 Data Sample

Treasury Yield

Monthly observations on the yield of 3- and 6-month constant maturity U.S. Treasury bills, 1-, 2-, 3-, 5-, 7- and 10-year constant maturity Treasury Notes, and 20-year as well as 30-year constant maturity Treasury Bonds from January 1983 to December 2004 are downloaded from the Federal Reserve Board. We choose 1983 as our starting year to estimate the Vasicek (1977) model based on the fact that several empirical studies have shown there is a regime change in U.S. interest rates in the early 1980's.[5] We have missing observations for yields on the 20-year constant maturity Treasury Bond from 1987 to 1993. In addition, monthly observations for yields on 30-year constant maturity Treasury Bond ended in February 2002. For these reasons, we strict our sample for the estimation of the riskfree rate to the time period between January 1983 and February 2002.

Corporate Bonds

Datastream provides weekly bond prices for which Merrill Lynch is the main data provider. It contains daily evaluated bid pricse, which Datastream recorded as market prices, for bonds issued with the amount outstanding above $100 million from 1989. It started providing ask price and mean price only from February 2003. We restrict our sample period for issuance firms from January 1989

[5] See Butler et al. (2004)) Duffy and Engle-Warnick (2004) .

to December 2004 and focus on bonds that were issued by nonfinancial firms.[6] Bonds issued by regulated utility firms (gas and electric) with SIC code between 4900 and 4999 are also excluded from our sample as the risk of these bonds is directly related to the decisions of the utility commissions (see Eom et al (2004)).

We have obtained information on bond issuing date, redemption date, dollar amount issued, coupon payment schedules, derivative features, whether the bond is sinkable, whether the bond is convertible, whether the coupon is floating rate and the most recent long-term credit ratings assigned by both S&P and Moody's. These static information on bonds is obtained on May 20, 2005. In order to retrieve a clean measure of corporate bond yields we follow the approaches adopted by previous studies (Elton et al (2001) and Eom et al (2004)) to eliminate bonds with special features such as callability/putability, a sinking fund schedule, floating rate coupons, and odd frequency of coupon payments such as quarterly coupons or monthly coupons. Thus we keep only straight bonds with no options features. We also exclude bonds that do not have credit ratings from either S&P or Moody's or have ratings lower than CCC- in S&P measure or Caa3 in Moody's measure.

Next we exclude bonds with maturities of under one year.[7] In order to keep capital structure simple, we include a firm in our analysis only if the firm has only one bond outstanding at the time when market price is observed,[8] and Datastream has kept observations of their prices for at least 100 weeks. The bond issuance information is also manually checked with the SDC U.S. Market New Issue database to ensure the bonds included in our sample are indeed the single outstanding bonds for each firm. Since the bond price must be close to its par value when bonds are close to maturity we do not keep the observations of the last 6-month to maturity date. All bonds in our sample are senior unsecured.

Due to the availability of bond prices provided by Datastream, we are able to obtain weekly evaluated bid price for most of the bonds after year 1995. The focus time period of this study is from 1996 to 2004. Information on corporate bonds obtained from Datastream is matched to the COMPUSTAT and CRSP by CUSIPs and they are manually checked by company names. A firm

[6] In contrast, Lyden and Saraniti (2000) include both nonfinancial and financial firms in their sample. As studies have shown, financial firms usually have unique financial characteristics (e.g. they keep leverage ratios as high as 90% while industrial firms usually have leverage ratios about 35%). In order to reduce the heterogeneity of our sample firms it is better to keep our focus on industrial firms only.

[7] Warga (1991) suggests that bonds with such short maturities are highly unlikely to be traded. This practice was also adopted in studies such as Eom et al (2004) and Driesson (2005).

[8] Jones et al. (1984) show that in the contingent claim analysis for corporate liability the presence of multiple debt issues increases the complexity of the problem dramatically.

is dropped from our sample if its accounting information is not recorded in Compustat or if it does not have outstanding common stocks. We are able to obtain a sample of 55 single bonds issued by 55 firms with a total of 6,787 weekly observations.

Finally, historical average cumulative default probabilities for different ratings classes are obtained from the latest report produced by both Moody's and S&P (see Hamilton et al. (2005) and Vassa et al. (2005)).

4 Estimation Method

There are usually two approaches to estimate the structural models. One is from stock market as well as balance sheet information (Jones et al. (1984), Ronn and Verma (1986), Duan and Simonato (2002), Delianedis and Geske (2003), and Ericsson and Reneby (2005)). The other approach uses information from bond market or credit derivative market (Wei and Guo (1997), Cooper and Davydenko (2004), and Longstaff et al. (2004)). In this section, we use information from both the equity market and the bond market for our empirical implementation.

4.1 The Merton (1974) Model

From the perspective of estimation procedures and methodology we can distinguish among four approaches that have been employed in the past to deal with the Merton type of models. First, a proxy for asset value may be computed as the sum of the market value of the firm's equity and the book value of liabilities. Asset volatility can be derived by computing the annualized volatility of the asset returns from the quarterly balance sheet from COMPUSTAT. This approach is adopted by studies such as Brockman and Turtle (2003) and Eom et al. (2004).

The second approach to estimate the initial value of the asset or the initial leverage ratio and the asset volatility is to solve the system equations of (2) and (5) simultaneously. This method has been employed by earlier studies such as Jones et al. (1984) and Ronn and Verma (1986) and later by Cooper and Davydenko (2003) and Delianedis and Geske (2003), among others. However, as outlined in Crosbie and Bohn (2002), equation (5) holds only instantaneously since in reality both the leverage ratio and hedge ratio $N(d_1)$ are not stable. Thus this approach forces a stochastic variable to be constant. Instead they illustrate an iterative procedure of backing out the current leverage ratio and the equity volatility though equation (2) (see also Ronn and Verma (1986)). This approach has been experimented by studies such as Du and Suo (2004) and Vassalou and Xing (2004).

Another estimation approach is the maximu likelihood estimation proposed by Duan (1994). A likelihood function based on the observed equity price is derived by employing the transformed data principle to obtain the parameters related to unobserved firm's asset. Maximum likelihood estimates and statistical inference can be directly obtained from maximizing the log-likelihood function. This approach has been applied to several corporate bond pricing models by Ericsson and Reneby (2005). One of the distinctive advantages of the maximum likelihood estimation is that it directly provides an estimate for the drift of the unobserved asset value process under the physical probability measure, which is critical to obtaining the default probability of the firm.[9] In this section we follow Duan (1994) to obtain parameters associated with the asset value process.

In structural models, $\ln(V_{t_i})$ is assumed to be normally distributed and its conditional moments are given by

$$\begin{aligned} E_{t_{i-1}}\left[\ln\left(\frac{V_{t_i}}{V_{t_{i-1}}}\right)\right] &= \left(\mu_v - \delta - \frac{1}{2}\sigma_v^2\right)\Delta t = \alpha_v \Delta t, \\ Var_{t_{i-1}}\left[\ln\left(\frac{V_{t_i}}{V_{t_{i-1}}}\right)\right] &= \sigma_v^2 \Delta t, \end{aligned} \tag{25}$$

the log-likelihood function for $\ln(V_{t_i})$ can be, therefore, written as,

$$\begin{aligned} & L_{\ln(V_{t_i})}(V_{t_i}, i = 1, 2, \cdots, n; \mu_v, \sigma_v) = \\ & \quad -\frac{n-1}{2}\ln(2\pi) - \frac{n-1}{2}\ln(\sigma_v^2 \Delta t) - \frac{1}{2\sigma_v^2 \Delta t}\sum_{i=2}^{n}\left[\ln\left(\frac{V_{t_i}}{V_{t_{i-1}}}\right) - \alpha_v \Delta t\right]^2. \end{aligned} \tag{26}$$

Since both bonds and equity are derivatives written on firm's asset, we are able to use the observed bond prices or the equity prices and the transformed log-likelihood function to estimate the parameters associated with the asset value process. From equation (3),

$$\frac{\partial B_t(V,T)}{\partial \ln(V_t)} = V_t e^{-\delta(T-t)} N(-d_1), \quad \frac{\partial E_t(V,T)}{\partial \ln(V_t)} = V_t e^{-\delta(T-t)} N(d_1).$$

Applying the results in Duan et al (2004), we can write the log-likelihood function for the bond price as

$$\begin{aligned} & L(B_{t_i}, i = 1, 2, \cdots, n; \mu_v, \sigma_v) = -\frac{n-1}{2}\ln(2\pi) - \frac{n-1}{2}\ln(\sigma_v^2 \Delta t) \\ & -\sum_{i=2}^{n}\ln(\hat{V_{t_i}}) - \sum_{i=2}^{n}\ln(N(-\hat{d}_1)) + \sum_{i=2}^{n}\delta(T - t_i) - \frac{1}{2\sigma_v^2 \Delta t}\sum_{i=2}^{n}\left[\ln\left(\frac{\hat{V_{t_i}}(\sigma_v)}{\hat{V_{t_{i-1}}}(\sigma_v)}\right) - \alpha_v \Delta t\right]^2 \end{aligned} \tag{27}$$

[9]Duan et al (2004) show that the KMV approach turns out to produce the same point estimate as the maximum likelihood estimate. However, the advantage of the maximum likelihood estimation over the KMV approach is that it not only produces asymptotically convergent estimates but also provide sampling error of the estimate to allow for statistical inference to assess the quality of parameter estimates.

where $\hat{V}_t(\sigma_v)$ is the unique solution to equation (3) at each time t. When the value of equity is used, the log-likelihood function for equity can be obtained as,

$$L(E_{t_i}, i = 1, 2, \cdots, n; \mu_v, \sigma_v) = -\frac{n-1}{2}\ln(2\pi) - \frac{n-1}{2}\ln(\sigma_v^2 \Delta t)$$
$$-\sum_{i=2}^{n}\ln(\hat{V}_{t_i}) - \sum_{i=2}^{n}\ln(N(\hat{d}_1)) + \sum_{i=2}^{n}\delta(T - t_i) - \frac{1}{2\sigma_v^2\Delta t}\sum_{i=2}^{n}\left[\ln\left(\frac{\hat{V}_{t_i}(\sigma_v)}{\hat{V_{t_{i-1}}}(\sigma_v)}\right) - \alpha_v \Delta t\right]^2 \quad (28)$$

We should notice that in the Merton model, the bonds are assumed to be zero coupon bonds. However, most of the corporate bonds observed in reality are coupon bearing bonds. Therefore we must first stripe out the coupons from the bond prices observed in order to get a clean measure of the zero coupon bond price. This is accomplished by the following formula

$$B_t^{\mathrm{zc}} = B_t^{\mathrm{coupon}} - \left(\exp[-\frac{r}{2}\times \mathrm{rem}(\frac{T-t}{2})]\frac{cF}{2} + \sum_{i=1/2}^{\mathrm{minint}(\frac{\mathrm{T-t}}{2})} \exp\left[-\frac{r}{2}\times(T-i)\right]\frac{cF}{2}\right)$$

where $\mathrm{rem}(\frac{T-t}{2})$ denote the remainder term when $T-t$ is divided by 2, and $\mathrm{minint}(\frac{T-t}{2})$ denotes the minimum integer obtained after $T-t$ is divided by 2.

There have been debates on how to measure the face value of debt in Merton (1974) model. The simplest approach is to set the face value of debt equal to the total amount of bond outstanding. However, it has been shown that this approach tends to underestimate the credit risk of the bond. Another approach is to set the debt face value equal to the total amount of short-term and long-term liabilities. However, as argued by KMV, the probability of the asset value falling below the total face value of debt may not reflect an accurate measure of the actual default probability. Instead they set the face value of debt equal to the total amount of short-term debt plus half of the long-term debt. In this study, we will use three different measures independently and compare their performance.

The payout ratio of asset δ is calculated as a weighted average of bond's coupon rate and dividend payout ratio on equity where the weights are taken according to the leverage ratio measured as the book value of total debt to the sum of book value of debt and market value of equity. The risk free interest rate is set equal to the annual average of weekly observation of one-year constant maturity Treasury note for the year when bond prices are observed.

4.2 Merton (1974) with Stochastic Interest Rate

We apply a two-stage MLE estimation as that adopted in Duan and Simonato (2002). In the first stage, the MLE is applied to obtain the parameter estimates for the Vasicek process. The parameters μ_v, σ_v and the market price of risk λ, which are assumed to be constants, are estimated in the second stage by the MLE.

First Stage: Parameter Estimation of the Vasicek (1977) Process

The parameters to be estimated in equation (8) are $\theta = (\kappa_r, \bar{r}, \sigma_r)$. By following Duan (1994) we are able to obtain the first and second conditional moment for the short rate as,

$$E(r_{t+1}|r_t) = \bar{r} + (r_t - \bar{r})e^{-\kappa_r}, \quad Var(r_{t+1}|r_t) = \frac{\sigma_r^2}{2\kappa_r}(1 - e^{-2\kappa_r}).$$

The log-likelihood function for the short rate $r_t, t = 1, , , , .n$ is written as,

$$\begin{aligned} L(r_t, t = 1, ..., n; \theta) &= -\frac{n-1}{2}\ln(2\pi) - \frac{n-1}{2}\ln(Var(r_t|r_{t-1})) \\ &\quad -\frac{1}{2Var(r_t|r_{t-1})}\sum_{t=2}^{n}[r_t - E(r_t|r_{t-1})]^2 \end{aligned} \tag{29}$$

From the risk-free bond price formula in (9), we are able to obtain the yield to maturity $y(r)$ as

$$y_t = -\frac{1}{T-t}\ln(\bar{B}(r, t, T)) = -\frac{1}{T-t}A(t, T) + \frac{1}{T-t}C(t, T)r_t. \tag{30}$$

The above equation defines a data transformation from the unobserved short rate process to the observed yield process. As shown in Duan et al (2004), the resulting likelihood function for the observed yield process becomes the likelihood function of the unobserved short rate process multiplied by the Jacobian of the transformation evaluated at the implied value for the short rate. Since the transformation from the yield to the short rate is of element-by-element nature the resulting log-likelihood function of y_t is written as,

$$\begin{aligned} L(y_t, t = 1, ..., n; \theta) &= (n-1)\ln(T-t) - (n-1)\ln(C(t, T; \theta)) - \frac{n-1}{2}\ln(2\pi) \\ &\quad -\frac{n-1}{2}\ln(Var(\hat{r}_t|r_{t-1}; \theta)) \\ &\quad -\frac{1}{2Var(\hat{r}_t|r_{t-1}; \theta)}\sum_{t=2}^{n}[\hat{r}_t - E(\hat{r}_t|\hat{r}_{t-1}; \theta)]^2 \end{aligned} \tag{31}$$

where

$$\hat{r}_t \equiv \frac{1}{C(t, T)}[(T-t)y_t + A(t, T)]$$

The parameters can be estimated by maximizing the likelihood function.

Second Stage: Estimation of the Parameters Related to the Asset Value Process

In this stage we apply the maximum likelihood estimation method to obtain the parameters that are related to the asset value process and the market price of risk of asset. The parameters to be estimated are $\theta = (\mu_v, \sigma_v, \kappa_\lambda, \sigma_\lambda, \bar{\lambda}_v, \rho_{r\lambda}, \rho_{v\lambda})$. In order to keep the problem simple we assume constant market price of risk, λ_v and thus $\rho_{v\lambda}$ equal to zero. The correlation ρ_{rv} is proxied by the correlation between daily returns of firm's asset, which is defined as the sum of the market value of equity and the book value of total debt, and the changes of 1-year constant maturity Treasury bill rates over the period when bond prices are observed.[10]

It can be shown from equation (12) that,

$$\frac{\partial S}{\partial V_t} = \bar{B}(t,T)\exp\left(\mu_{\ln(x)} + \frac{1}{2}(\sigma^2_{\ln(x)} + 2Cov_{\ln(x)}, \sigma_y)\right) e^{-\delta(T-t)} N(d_1)$$

and thus

$$\frac{\partial S}{\partial \ln V_t} = \bar{B}(t,T)\exp\left(\mu_{\ln(x)} + \frac{1}{2}(\sigma^2_{\ln(x)} + 2Cov_{\ln(x)}, \sigma_y)\right) V_t e^{-\delta(T-t)} N(d_1).$$

.

Therefore, by following the argument in Duan (1994), we are able to obtain the log-likelihood function as,

$$\begin{aligned} L(S_t, t = 1, 1+\Delta t, \cdots, n; \theta) \quad = \quad & -\frac{n-1}{2}\ln(2\pi) - \frac{n-1}{2}\ln((\frac{\sigma_{\ln(x)}}{\tau})^2 \Delta t) \\ & -\frac{1}{2\sigma_v^2}\sum_{i=2}^{n}\left[\ln\frac{V_i}{V_{i-1}} - \frac{\mu_{\ln(x)}}{\tau}\Delta t\right]^2 \\ & -\sum_{i=2}^{n}\ln \bar{B}(t_i, T) - \sum_{i=2}^{n}\left[\mu_{\ln(x)} + \frac{1}{2}(\sigma^2_{\ln(x)} + Cov_{\ln(x),y})\right] \\ & -\sum_{i=2}^{n} N(d_1(i)) - \sum_{i=2}^{n}\ln(V_i). \end{aligned}$$

with $\tau = T - t$.

4.3 The LT model

In order to calculate the default probabilities from this model we need to estimate the parameters $\theta = (\sigma_v, \pi^v, V^*)$. Since the risk-free interest rate r is assumed to be constant, the average of weekly observations of one year constant maturity Treasury note yield of each year is treated as the risk-free interest rate for the year when bond prices are observed. Asset payout ratio δ is calculated as

[10] Eom et al. (2004) use the correlation between equity returns and the changes of 3-month T-bill rates over a window of five years to proxy ρ_{rv}.

the dividend yield weighted by the leverage. Face value F, coupon rate c, and time maturity τ, which is a time varying variable, are directly observed from the sample.

Two different assumptions are made on the recovery rate $1-\omega$. The first one assumes that the recovery rate is homogeneous across industries. The mean recovery rate of more than one thousand bonds of different industries that defaulted during the period of 1987 to 2004 is calculated based on the S&P LossStats database, and a 39% recovery rate of all defaulted bonds across all industries is obtained.[11] The second assumption on the recovery rate assumes that different industries differ on their expected recovery rate. The mean recovery rate is calculated for each industry from 1987 to 2004. The marginal corporate tax rate is set to equal to 35%.[12]

Since bond prices are observed weekly for each firm, the firm asset value each week is proxied by the sum of the market value of equity and the book value of total liabilities from quarterly COMPUSTAT record. Thus a weekly time series of market value of assets is obtained. After the weekly bond prices are fit into the LT model, σ_v for each firm is estimated while V_t^* is calculated for each firm weekly. In order to predict the default probabilities under the physical measure we need to estimate the asset risk premium for each firm. From the relationship presented in (6), once the estimates of asset volatility are achieved we could infer the asset risk premium from the historical equity premium and equity volatility. The equity premium is estimated by the average of the difference of the annualized equity returns and the 3-month T-bill rate for the ten year period from 1995 to 2004. The estimates of historical equity volatility are calculated as the 10-year average annualized volatility of the stocks of each firm.

4.4 The LS model and the CDG model

For exogenous default barrier models, V^* is set to be equal to total liabilities of the firm so that the ratio of V/V^* is simply the reciprocal of the leverage ratio. The parameters in the LS model and the CDG model are

$$\theta = (\mu_v, \sigma_v, \delta_v, V^*, \kappa_r, \bar{r}, \sigma_r, r_t, \rho_{vr}, \kappa_l, \phi, \bar{\nu})$$

except that for the LS model κ_l is restricted to be zero. The parameters related to the short rate process can be estimated first by applying the MLE to the one-year constant maturity Treasury note. The correlation coefficient ρ_{vr} is estimated in the same way as in the Merton model with

[11]The recovery rate obtained from S&P LossStats database is lower than that shown in Acharya et al.(2004) due to the fact that our study covering a different time period from their study.

[12]Huang and Huang (2003) and Eom et al. (2004) assume the same marginal tax rate

stochastic interest rate. Both δ_v and V^* can be obtained from COMPUSTAT. Once σ_v is estimated π^v is achieved through $\pi^v = \pi^e \sigma_v / \sigma_e$. By assuming asset is tradable we have $\mu_v = \pi^v + r$.

From equation (32), a regression of the changes in the log-leverage ratio against lagged log-leverage ratio and the yield of one year constant maturity Treasury note will generate estimates of parameters κ_l, ϕ and $\bar{\nu}$. Suppose the estimated coefficients from the linear regression are b_0, b_1 and b_2, where b_0 is the constant and b_1 and b_2 are coefficients on lagged log-leverage and risk-free interest rate, we then have $\kappa_l = -b_1$, $\phi = -b_2/b_1$, and $\mu_v + \kappa_l \bar{\nu} = -b_0$. Since $\mu_v = \pi^v + R$, $\bar{\nu} = (b_0 - \mu_v)/\kappa_l$. The time period used for the regression is from 1995 to 2004.

5 Results and Discussions

5.1 Merton Model

The results from the maximum likelihood estimation of Merton (1974) model are consistent with the empirical findings from other studies when bond prices are used. The asset volatility estimates are unreasonably high for 52 firms out of the whole sample. The implied asset value for some of the firms reaches a value of as low as one tenth of the sum of the market value of equity and the book value of debt. One of the explanations is that firms are assumed to default only at the maturity of debt in Merton (1974) models. The implied default probabilities prior to maturity are lower than those implied by other type of models. It has been shown that with reasonable parameters Merton (1974) model and its variations are only able to generate fairly low yields for corporate bonds (see Jones et al. (1984), Kim et al. (1993), and Huang and Huang (2003)). Therefore, it is not surprising that when bond prices or yields are fit into Merton type of models either the estimates of asset volatility need to be very high or the implied asset values need to be very low in order for the model to match the market prices.

Instead, we apply the MLE on the daily equity prices observed in the same period when bond prices are obtained for each firm, with the time to maturity assumed to be one year.[13] After the estimates of μ_v and σ_v are obtained we calculate the implied asset value given the observed equity value each day. The predicted default probabilities are assessed daily for each firm correspondingly. Figure 1shows the distribution of the predicted 1-year and 4-year default probabilities for the pooled observations when the bond face value is used as proxy for the face value of debt.

With Moody's and Standard and Poor's historical default probabilities used as benchmarks,

[13]We also estimate our model with time to maturity equal to 10 years. The estimation results for μ_v and σ_v are very close to those obtained when the time to maturity is assumed to be one year for equity.

Table 1 and 2 provide the summary of the performance of Merton (1974) model at predicting 1- and 4-year default probabilities. The model performance is measured by means of mean error, mean absolute error, root mean squared error, minimum error and maximum error. When deciding the face value of debt we use three different structures. The first structure assumes the corporate bonds outstanding as the only debt that needs to be retired at the maturity date of the debt. The "KMV" measure uses the sum of short-term debt and half long-term debt as a proxy for the face value of debt. "Equal All" structure envisions that all debts are retired at the maturity of debt. In Table 1 all the mean errors except for B-rated firms are found to be negative and mean absolute errors are close to the absolute value of mean errors, which shows that most of the predicted default probabilities are lower than the historical observations. It implies that Merton (1974) model provides under-estimation for the default probabilities under the real world measure. This holds true for both pooled and per-bond basis observations. However, for B-rated firms the predicted default probabilities tend to be larger than the historical observations. This is possible due to the fewer number of observations of B-rated firms.

Table 2 shows similar results as Table 1 except for "Equal All" structure where the mean errors are found to be positive for investment-grade bonds. Merton (1974) model is found to over-predict default probabilities of longer time span for investment-grade firms when the face value of debt is set to equal to the total liability. When comparing mean errors of the three different debt structures we find that "Bond Face" implies the lowest while "Equal All" implies the highest default probabilities in Merton (1974) model, which is consistent with previous findings such as Lyden and Saraniti (2000) at explaining bond yield spreads.

5.2 Merton Model with Stochastic Interest Rate

Table 3 shows the maximum likelihood estimation results for the Vasicek (1977) process. The estimation is conducted for the monthly yields of 3-month and 6-month constant maturity Treasury bills and 1-year, 2-year and 5-year Treasury notes. Our estimates are consistent with previous findings (e.g. Duan (1994)).

In the Merton model with stochastic interest rate, interest rates either have to be very volatile or have strong positive correlation with the asset value in order to have significant effect on the credit yields and default probabilities. Since the volatility estimated for the interest rate process is not large, for stochastic interest rate to generate higher default probabilities the correlation coefficient needs to be positive. the intuitive explanation is that when asset value falls, interest rates have a tendency to fall as well, thereby decreasing the drift of asset process, which causes a higher probabilities of default for a longer time span. We find that for our sample of firms, the correlation coefficients range from -0.25 to 0.25 with most of them being positive.

The model performance of Merton (1974) with stochastic interest rate is summarized in Table 1 and Table 2. One year default probabilities predicted by the Merton model with stochastic interest rate tend to be lower than those reported by Moody's and S&P. Among the three different proposed debt structures, KMV's approach provides the best prediction. This is also the case for the predicted four-year default probabilities. Figure 2 presents the summary of the predicted default probabilities from this model, when bond face value is assumed to be equal to the total face value of debt.

5.3 The LT Model

The results for the LT model performance are reported by rating classes in Table 6 and Table 7. The first table provides the model performance at predicting one-year default probabilities while the second table shows the results of predicting four-year default probabilities. Results are reported in two panels, where the left panel reports model error statistics for the pooled time series and cross-sectional observations and the right panel reports error statistics by averaging model errors across bonds. We use historical cumulative default rates reported by Moody's and S&P independently to report our results as before. The recovery rate is assumed to be either constant or industry specific in the LT model and model performance is reported correspondingly.[14]

When predicting one year default probabilities Table 6 shows the mean error to be negative for investment-grade bonds and positive for speculative-grade bonds, which provides evidence that the LT model under-predicts the default probabilities for investment-grade bonds while over-predicts the default probabilities for non-investment-grade bonds. The mean errors estimated in the LT model are found to be much smaller than those obtained in the Merton model. Figure 3 shows the distribution of the predicted one-year default probabilities across rating classes in the LT model. We find that the default probabilities predicted by investment-grade firms tend to cluster close to zero while for speculative-grade firms they tend to spread out to the higher end of the distribution.

[14]Recent studies (Huang and Huang (2003), Leland (2004), Eom et al. (2004) etc.) treat the recovery rate or the loss given default as a constant across industries. The LossStats database provided by S&P shows that the recovery rate of corporate bonds differ significantly across industries. The value-weighted mean recovery rate for industries such as Chemicals and Petroleum can be as high as 60%. However, industries such as Real Estate only have a mean recovery rate of 24%. Based on these observations it is important to treat recovery rate differently across industries and implement the model with industry specific expected recovery rate.

When compared with Figure 1, Figure 3 provides evidence that the LT model predicts higher default probabilities on average than Merton model. In addition, by comparing the model performance with the assumption of constant recovery rate and industry specific recovery rate we do not find much difference between their model error statistics when predicting one year default probabilities.

Table 7 shows quite different results. The LT model provides higher predicted default probabilities than the historical average for all rating classes. The means errors and mean absolute error are much larger for non-investment-grade firms than for investment-grade firms. From the distribution of the predicted default probabilities shown in Figure 3 we are able to observe that the some of the predicted four-year default probabilities for BB-rated and B-rated firms are as high as 80-90%. It reflects that the LT model over-predict the default rates for a longer span of time horizon. Table 7 also shows that using industry specific recovery rate on average produces higher model errors than assuming constant recovery rates across industries.

5.4 The LS model

Table 8 and 9 provide the model performance of the LS model with constant interest rate at predicting 1-year and 4-year default probabilities respectively. Results are reported in two panels , where the left panel reports error statistics for the pooled time series and cross-sectional observations and the right panel reports the statistics by averaging model errors of each individual bonds. Historical default rates from Moody's and S&P are used to calculate model errors independently. We also report our results by treating the recovery rate as a constant of 39% across industries and using the calculated average recovery rate of each industry respectively.

In general, when the interest rate is assumed to be constant, the LS model provides reasonable prediction of 1-year default probabilities for investment-grade bonds while provides over-prediction for the non-investment-grade bonds. It's consistent with the findings from the Merton type of models. However, the LS model provides higher predicted default probabilities than the Merton type of models with the mean errors at predicting 1-year default probabilities of all rating classes in the LS model being smaller. When predicting 4-year default probabilities from the bond prices, the LS model with constant term structure provides slightly higher predictions than the historical average. When comparing the predicted 4-year default probabilities from the LS model with those from the LT model we find that the former provides more reasonable predictions.

When comparing the model performance with a constant recovery rate assumed and industry specific recovery rate assumed, we find that, on average, industry specific recovery rate assumption predicts higher default probabilities for the time horizon of both one year and four years. Since the LS model is very sensitive to the recovery rates as implied by the bond formula, our results suggest that the loss-upon-default for the sample of firms used in this study is higher than that for S&P's whole sample on average.

The model performance of the LS model with the interest rate assumed stochastic is summarized in Table 10 and Table 11. Different assumptions are made on the recovery rates as the last section. Figure 5 provides the distribution of the 1-year and 4-year default probabilities of the LS model respectively. We find that the LS model with stochastic interest rate predicts lower 1-year default probabilities but higher 4-year default probabilities. Our results are consistent with Huang and Huang (2003), who find that the LS model with stochastic interest rate generates lower bond yield spread than that with constant term structure when the correlation between the asset value process and short rate process is assumed to be -0.25. As mentioned earlier, in order for a structural model to generate higher predicted default probabilities the asset value and the term structure process must be positively correlated. However, our estimation results show that the correlation coefficients range from -0.25 to 0.25 and the volatility of the short rate process is rather small. This possibly explains why when a stochastic term structure is added to the basic structure the LS model does not provide higher predicted default probabilities. In addition, the effects of a stochastic term structure on the predicted default probabilities are more relevant for a longer time span. Therefore, when the correlation coefficients between asset value process and short rate process are positive the stochastic interest rate framework generates higher predicted default probabilities for a longer time span. Our results show that the predicted 4-year default probabilities are higher under the framework of a stochastic term structure due to the correlation coefficients for most firms being positive.

5.5 The CDG model

Used as benchmark, the interest rate is first assumed to be constant in the CDG model. As described in the earlier section the CDG model assumes a mean reverting leverage ratio in order to generate higher default probabilities and yield spreads for a longer time span. This is the case only when the mean reverting rate is positive and large. In their original paper, Collin-Dufresne and Goldstein (2001) consider a mean reverting rate of 0.18 in order to simulate high yield spreads compared to the LS model. Huang and Huang (2003) also assume such high mean reverting rate. However, our regression results show that the maximum mean reverting rate of the leverage ratio can only reach as high as 0.1 while with most of the coefficients being close to zero. It explains why the default probabilities predicted by the CDG model as summarized in Table 12 and 13 are only slightly higher than those provided by the LS model.

Figure 6 presents the distribution of the predicted 1-year and 4-year default probabilities. They are very similar to those for the LS model except for B-rated bond, for which we have the least number of observations.

Next, we study the CDG model with a stochastic term structure. The results are summarized in Table 14 and 15. As has been shown by Eom et al. (2004), the CDG model generates much higher yield spreads than the observed values. It can be inferred that the risk-neutral measure of default probabilities predicted by the CDG model must be the highest among all the structural models if all the paramors are held the same. Our estimation results show that the asset volatility estimates for a number of investment-grade firms are very close to zero, which reflects the fact that in order to generate low yields for investment-grade bonds the asset volatility needs to have very low values.

Table 14 summarizes the model performance of the CDG model at predicting 1-year default probabilities when interest rates are assumed stochastic. Surprisingly, we find that the predicted values are lower than the real world observations on average. On the other hand, Table 14 shows that the predicted 4-year default probabilities are much higher than the real world observations. Our estimation results for a longer time span, which are not presented here, show that the predicted default probabilities for the CDG model with stochastic interest rate increase exponentially with the time span. It reflects that the effect of the mean reverting leverage ratios assumed in their model tend to be more pronounced in the long run.

The distribution of predicted default probabilities are shown in Figure 7.

5.6 Comparison of Model Performance

Table 16 provides the comparison of the structural models at predicting one-year and four-year default probabilities when equity and bond prices are used to obtain estimates. Merton (1974) predicts the lowest default probabilities of one year and four years for investment-grade bonds. Adding stochastic interest rate does increase model performance. However, the default probabilities predicted for B-rated bonds tend to be large from Merton type of models. One could argue that it may be due to that the six B-rated firms chosen for estimation may not be a perfect replicating group for the whole B-rated firm sample.

The performance of Merton type models are depicted in Figure 8, Figure 9, Figure 10, and

Figure 11 for different rating classes, where three different debt structures are assumed. "Bond Face" structure assumes only the corporate bond itself is retired at maturity. If the asset value falls below the bond face value at the time firm defaults. "KMV" structure follows Moody's KMV approach by setting the face value of debt equal to the short-term debt plus long-term debt. "Equal All" envisions that all debt being retired at the maturity of the bond. Not surprisingly, 'Equal All' predicts the highest default probabilities while 'Bond Face' under-predicts default probabilities for firms of all ratings except for B-rated firms. The debt structure assumed by the KMV makes the default probabilities predicted by the Merton model most attractable. Except for B-rated bonds, the default probabilities predicted by the "KMV" are very close to the real world observations for both a short and medium time span.

The LT model tends to underestimate the one year default probabilities but provides over-prediction for four year default probabilities. The LS model with constant interest rate provides quite reasonable predictions for both one year and four year default probabilities. Adding stochastic interest rates significantly increase the four year predicted default probabilities but have neglectable effect on the one year default probabilities. This can be explained as, due to the low volatility of the term structure and the low correlation coefficients between the asset value process and the interest rate process estimated from historical observations, stochastic interest rates have a major effect on whether the firm value hits a pre-specified default barrier for a longer time span. Figure 12, Figure 13, Figure 14, and Figure 15 show that the difference between the cumulative default probabilities predicted by the LS model with or without stochastic interest rates tends to increase with time. At last, we find that the CDG model predicts unreasonably high default probabilities across all rated firms. This effect is more pronounced for a longer time span.

6 Conclusions

In this paper, we study the empirical performance of structural credit risk models by examining the default probabilities calculated from these models with different time horizons.The models studied include Merton (1974), Merton model with stochastic interest rate, Longstaff and Schwartz (1995), Leland and Toft (1996) and Collin-Dufresne and Goldstein (2001).

The parameters of these models are estimated from firm's bond and equity prices. The sample firms chosen are those that have only one bond outstanding when bond prices are observed. We first find that when the Maximum Likelihood estimation, introduced in Duan (1994), is used to estimate the Merton model from bond prices the estimated volatility is unreasonable high and the estimation process does not converge for most of the firms in our sample. It shows that the Merton (1974) is not able to generate high yields to match the empirical observations. On the other hand, when equity prices are used as input we find find that the default probabilities predicted for investment-grade firms by Merton (1974) are all close to zero. When stochastic interest rates are assumed in Merton model the model performance is improved.

We find that Longstaff and Schwartz (1995) with constant interest rate as well as the Leland and Toft (1996) model provide quite reasonable predictions on real default probabilities when compared with those reported by Moody's and S&P. However, Collin-Dufresnce and Goldstein (2001) predicts unreasonably high default probabilities for longer time horizons. This is mainly due to the mean reverting leverage feature of the model, which tend to increase the default probability of a firm in the long run.

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Appendix: Derivation of the Conditional Moments and Default Probabilities in the CDG Model

Under the real world probability the $\ln(X_t)$ process is given as,

$$
\begin{aligned}
d\ln(X_t) &= \kappa_l(\overline{\ln(X_t)} - \ln(X_t))dt + \sigma_v dW_t^v \\
&= [(\mu^v + \kappa_l\bar{\nu}) - \kappa_l \ln(X_t) + \kappa_l \phi r_t]dt + \sigma_v dW_t^v
\end{aligned} \tag{32}
$$

where

$$
\begin{aligned}
\overline{\ln(X_t)} &= \frac{\pi_t - \delta - \sigma_v^2/2}{\kappa_l} + \nu - \phi\bar{r} + r_t(\frac{1}{\kappa_l} + \phi) \\
\bar{\nu} &\equiv (\nu - \phi\bar{r}) - \frac{\delta + \sigma_v^2/2}{\kappa_l}
\end{aligned} \tag{33}
$$

We can rewrite the above equation and the interest rate process as the following,

$$
e^{\kappa_l t}\ln(X_t) = \ln(X_0) + (\pi^v + \kappa_l\bar{\nu})\frac{e^{\kappa_l t} - 1}{\kappa_l} + \int_0^t (1 + \kappa_l\phi) r_u e^{\kappa_l u} du + \int_0^t \sigma_v e^{\kappa_l u} dW_u^v \tag{34}
$$

$$
r_t = r_0 e^{-\kappa_r t} + \bar{r}(1 - e^{-\kappa_r t}) + \sigma_r e^{-\kappa_r t}\int_0^t e^{\kappa_r u} dW_u^r \tag{35}
$$

From the above equations it is not hard to obtain the following results:

$$
e^{\kappa_l t}E_0[\ln(X_t)] = \ln(X_0) + [(\pi^v + \kappa_l\bar{\nu}) + (1 + \kappa_l\phi)\bar{r}]\frac{e^{\kappa_l t} - 1}{\kappa_l} + (1 + \kappa_l\phi)(r_0 - \bar{r})\frac{e^{(\kappa_l - \kappa_r)t} - 1}{\kappa_l - \kappa_r}
$$

and

$$
\begin{aligned}
Cov_0[\ln(X_t), \ln(X_u)]e^{\kappa_l(t+u)} = \; & \sigma_v^2 E_0[\int_0^t e^{\kappa_l s} dW_s^v \int_0^u e^{\kappa_l s} dW_s^v] \\
& + \sigma_v(1 + \kappa_l\phi)E_0[\int_0^t e^{\kappa_l s} dW_s^v \int_0^u e^{\kappa_l s} r_s ds] \\
& + \sigma_v(1 + \kappa_l\phi)E_0[\int_0^u e^{\kappa_l s} dW_s^v \int_0^t e^{\kappa_l s} r_s ds] \\
& + (1 + \kappa_l\phi)^2 Cov_0[\int_0^t e^{\kappa_l s} r_s, \int_0^u e^{\kappa_l s} r_s ds]
\end{aligned}
$$

In the above equation if the first, the second, the third and the fourth term are denoted as I_1, I_2, I_3, and I_4, we can show that for $t \geq u$,

$$
\begin{aligned}
I_1 &= \frac{\sigma_v^2}{2\kappa_l}(e^{2\kappa_l u} - 1) \\
I_2 &= (1 + \kappa_l\phi)\frac{\rho_{vr}\sigma_v\sigma_r}{\kappa_l + \kappa_r}[\frac{e^{2\kappa_l u} - 1}{2\kappa_l} - \frac{e^{(\kappa_l - \kappa_r)u} - 1}{\kappa_l - \kappa_r}] \\
I_3 &= (1 + \kappa_l\phi)\frac{\rho_{vr}\sigma_v\sigma_r}{\kappa_l + \kappa_r}[\frac{e^{2\kappa_l u} - 1}{2\kappa_l} + \frac{1 - e^{(\kappa_l - \kappa_r)t}}{\kappa_l - \kappa_r} + e^{(\kappa_l + \kappa_r)u}\frac{e^{(\kappa_l - \kappa_r)t} - e^{(\kappa_l - \kappa_r)u}}{\kappa_l - \kappa_r}] \\
I4 &= (1 + \kappa_l\phi)^2\frac{\sigma_r^2}{2\kappa_r}[-\frac{(e^{(\kappa_l - \kappa_r)t} - 1)(e^{(\kappa_l - \kappa_r)u} - 1)}{(\kappa_l - \kappa_r)^2} + (e^{(\kappa_l + \kappa_r)u} - 1)\frac{e^{(\kappa_l - \kappa_r)t} - e^{(\kappa_l - \kappa_r)u}}{\kappa_l^2 - \kappa_r^2} \\
&- \frac{\kappa_r}{\kappa_l^2 - \kappa_r^2}\frac{e^{2\kappa_l u} - 1}{\kappa_l} + \frac{1}{\kappa_l^2 - \kappa_r^2}(1 - 2e^{(\kappa_l - \kappa_r)u} + e^{2\kappa_l u}).
\end{aligned}
$$

By following the approach of Collin-Dufresne and Goldstein (2001) we are able to obtain the default probabilities under the real world measure in the following way. Let U be any time point between time zero to time T, the default probability in the CDG model for $U \in (0, T)$ is given as,

$$
\begin{aligned}
DP_{CDG}(X_0, r_0, U) &= \sum_{i=1}^{n} q(ti; t_0), t_i = iU/n, \\
q(t_1; t_0) &= \frac{N(a(t_1; t_0))}{N(b(t_1; t_{\frac{1}{2}}))} \\
q(t_i; t_0) &= \frac{1}{N(b(t_i; t_{i-\frac{1}{2}}))}[N(a(t_i; t_0)) - \sum_{j=1}^{i-1} q(t_{j-\frac{1}{2}}; t_0) N(b(t_i; t_{j-\frac{1}{2}}))], \\
a(t_i; t_0) &= -\frac{M(t_i, T|X_0, r_0)}{\sqrt{S(t_i, T|X_0, r_0)}} \\
b(t_1; t_j) &= -\frac{M(t_i, T|X_j)}{\sqrt{S(t_i, T|X_j)}}
\end{aligned}
$$

with

$$
\begin{aligned}
M(t, T|X_0, r_0) &\equiv E_0[\ln(X_t)] \\
S(t|X_0, r_0) &\equiv Var_0[\ln(X_t)] \\
M(t, T|X_u) &= M(t, T|X_0, r_0) - M(u, T|X_0, r_0)\frac{Cov_0[\ln(X_t), \ln(X_u)]}{S(u|X_0, r_0)}, u \in (t_0, t) \\
S(t|X_u) &= S(t|X_0, r_0) - \frac{Cov_0[\ln(X_t), \ln(X_u)]^2}{S(u|X_0, r_0)}, u \in (t_0, t).
\end{aligned}
$$

However, in order to price corporate bond we are no longer able to use the default probability under the real probability measure but need to obtain the default probability under T forward measure. Under such measure $\ln(X_t)$ and r_t can be shown to follow,

$$
d\ln(X_t) = ((1 + \kappa_l \phi) r_t + \kappa_l \bar{\nu} - \kappa_l \ln(X_t) - \rho_{vr}\sigma_v\sigma_r C(t, T))dt + \sigma_v dW_t^{v(F_T)} \tag{36}
$$

$$
dr_t = (\kappa_r(\bar{r} - r_t) - \kappa_r^2 C(t, T))dt + \kappa_r dW_t^{r(F_T)} \tag{37}
$$

where $\bar{\nu}$ is defined in (33) and $C(t, T)$ is defined in (9). Under T-forward measure the first moment of $\ln(X_t)$ is now expressed as,

$$
\begin{aligned}
e^{\kappa_l t} E_0^{F_T}[\ln(X_t)] = \; & \ln(X_0) + \bar{\nu}(e^{\kappa_l t} - 1) + \int_0^t (1 + \kappa_l \phi) e^{\kappa_l u} E_0^{F_T}[r_u] du \\
& -\frac{\rho_{vr}\sigma_v\sigma_r}{\kappa_r}[\frac{e^{\kappa_l t} - 1}{\kappa_l} - e^{\kappa_r T}\frac{e^{(\kappa_l + \kappa_r)t} - 1}{\kappa_l + \kappa_r}]
\end{aligned} \tag{38}
$$

where

$$
E_0^{F_T}[r_u] = r_0 e^{-\kappa_r t} + (b - \frac{\sigma_r^2}{\kappa_r^2})(1 - e^{-\kappa_r t}) + \frac{\sigma_r^2}{2\kappa_r^2} e^{-\kappa_r T}(1 - e^{-2\kappa_r t}) \tag{39}
$$

Thus we obtain the expectation of $\ln(X_t)$ under the T forward measure as,

$$
\begin{aligned}
e^{\kappa_l t} E_0^{F_T}(\ln X_t) &= \ln X_0 + \bar{\nu}(e^{\kappa_l t} - 1) \\
&+ (1 + \phi\kappa_l)[(r_0 - \frac{\alpha}{\beta} + \frac{\sigma_r^2}{\beta^2} + \frac{\sigma_r^2}{2\beta^2} e^{-\beta T}) \frac{e^{(\kappa_l - \beta)t} - 1}{\kappa_l - \beta} \\
&+ (\frac{\alpha}{\beta} - \frac{\sigma_r^2}{\beta^2}) \frac{(e^{\kappa_l t} - 1)}{\kappa_l} + \frac{\sigma_r^2}{2\beta^2} e^{-\beta T} \frac{e^{(\kappa_l + \beta)t} - 1}{\kappa_l + \beta}] \\
&- \frac{\rho_{vr}\sigma_v\sigma_r}{\beta}[\frac{(e^{\kappa_l t} - 1)}{\kappa_l} - e^{-\beta T} \frac{e^{(\kappa_l + \beta)t} - 1}{\kappa_l + \beta}]
\end{aligned}
$$

For the covariance, we have $Cov_0^{F_T}[\ln(X_t), \ln(X_u)] = Cov_0[\ln(X_t), \ln(X_u)]$.

Figure 1: Distribution of predicted 1-year and 4-year default probabilities of Merton (1974) model with F=*Bond Face Value*

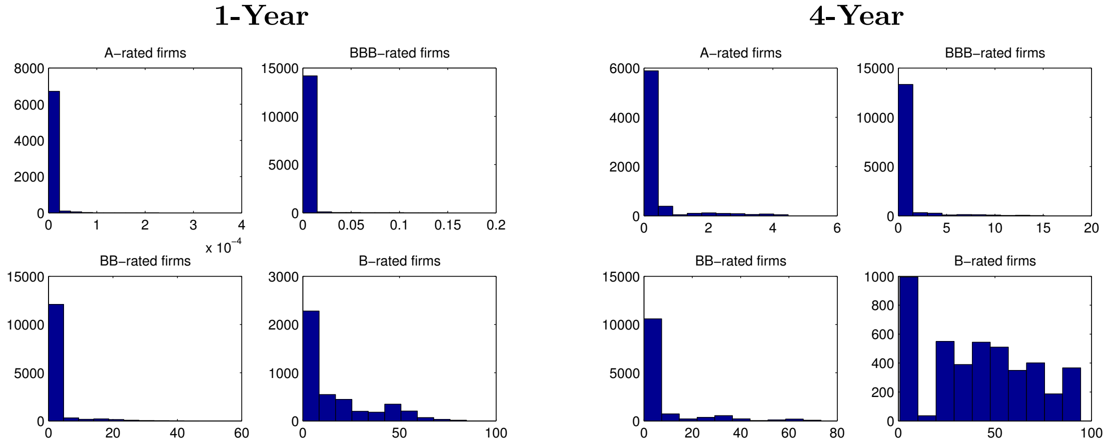

Figure 2: Distribution of predicted 1-year and 4-year default probabilities of Merton (1974) model with stochastic interest rate and F=*Bond Face Value*

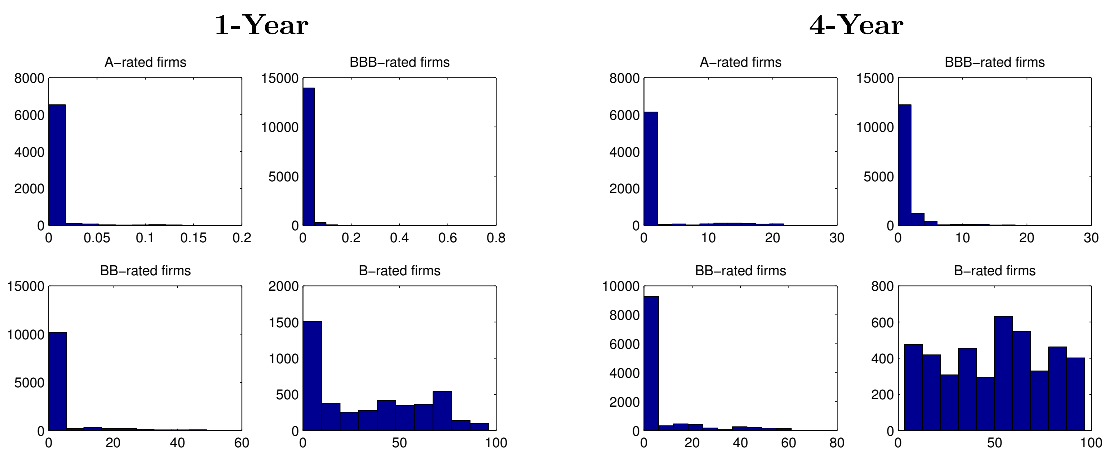

Figure 3: Distribution of predicted 1- and 4-year default probabilities of the LT model with industry recovery rates

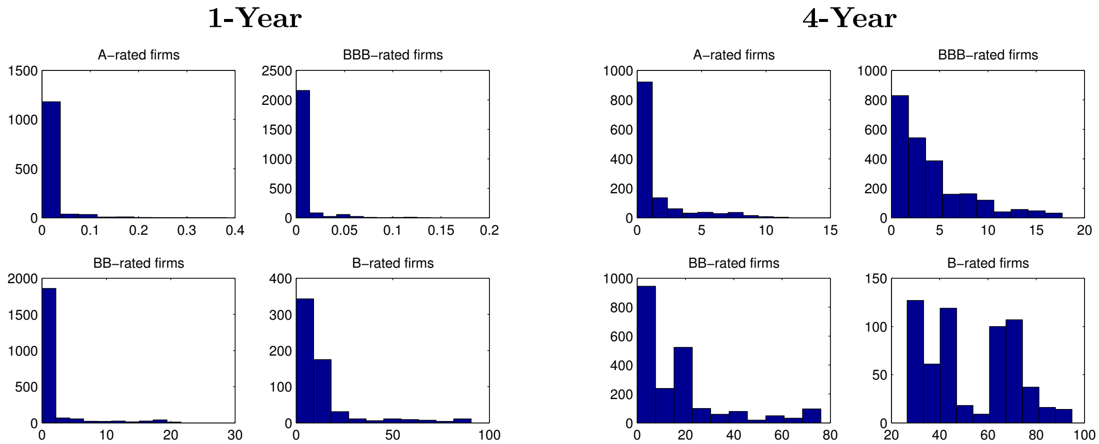

Figure 4: Distribution of predicted 1- and 4-year default probabilities of the LS model with constant interest rate and industry recovery rate

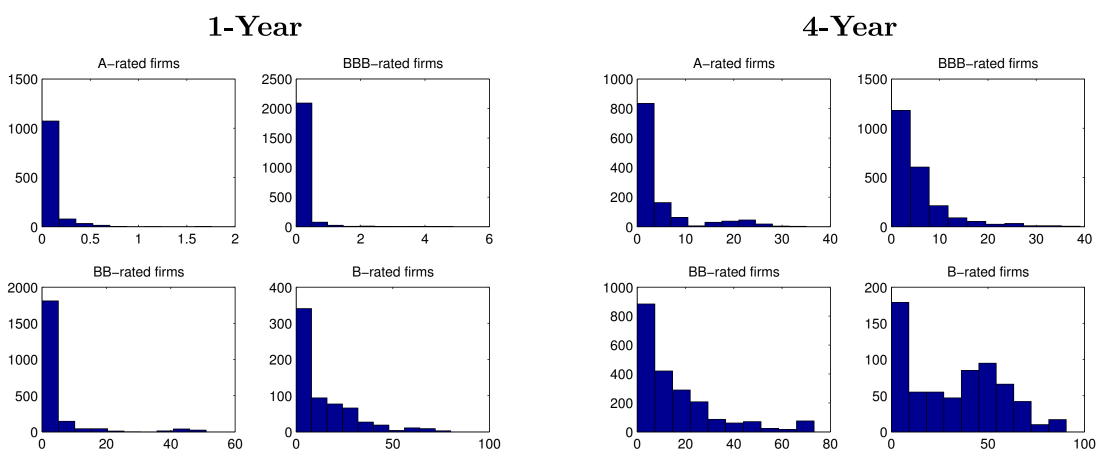

Figure 5: Distribution of predicted 1- and 4-year default probabilities of the LS model with stochastic interest rate and industry recovery rate

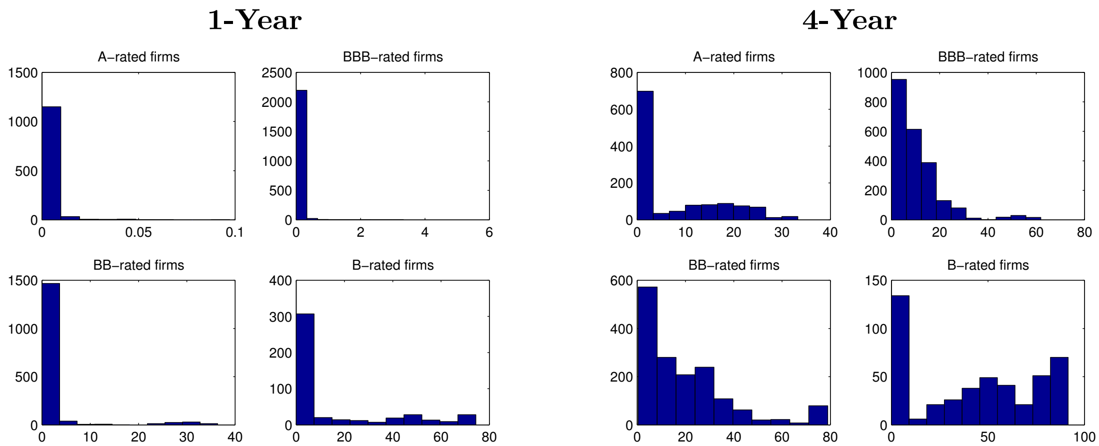

Figure 6: Distribution of predicted 1-year default probabilities of the CDG model with constant interest rate and industry recovery rate

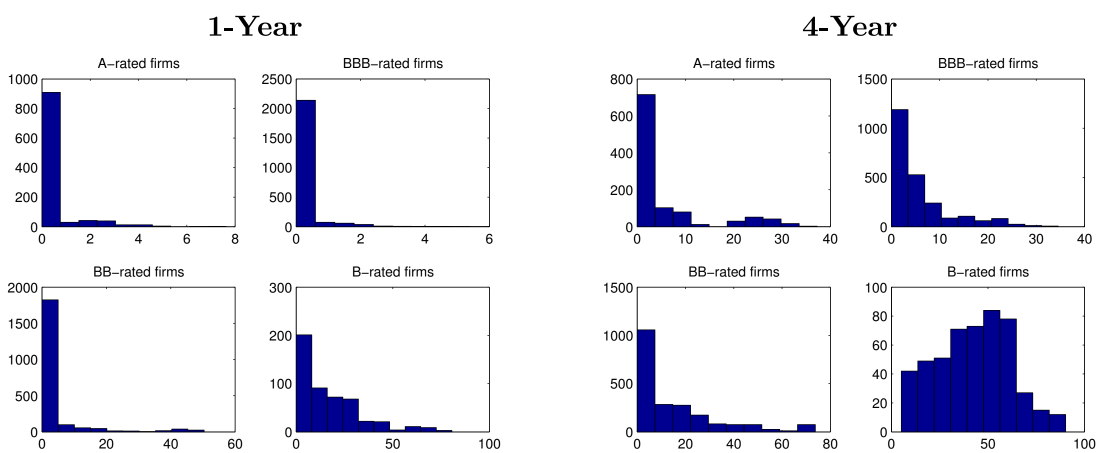

Figure 7: Distribution of predicted 1- and 4-year default probabilities of the CDG model with stochastic interest rate and industry recovery rate

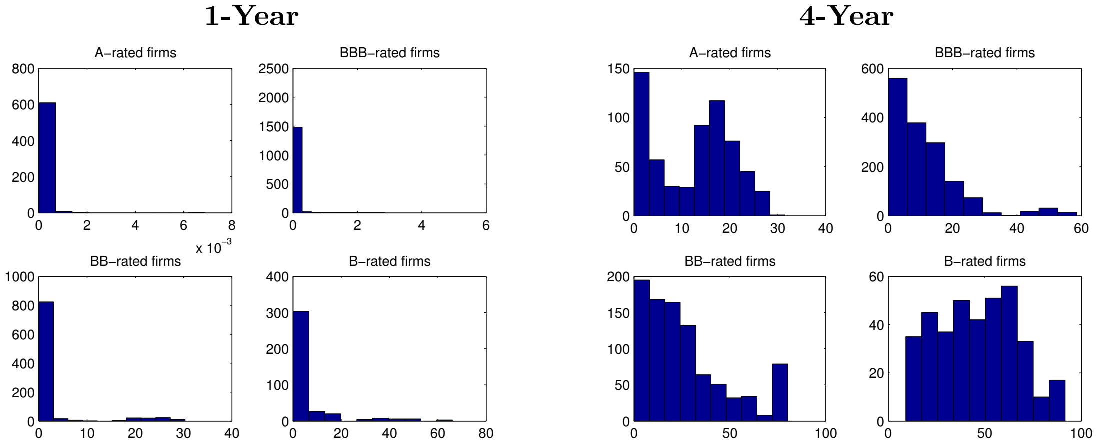

Figure 8: The Performance of Merton Models with Various Debt Structure at Predicting Default Probabilities for A-Rated Bonds

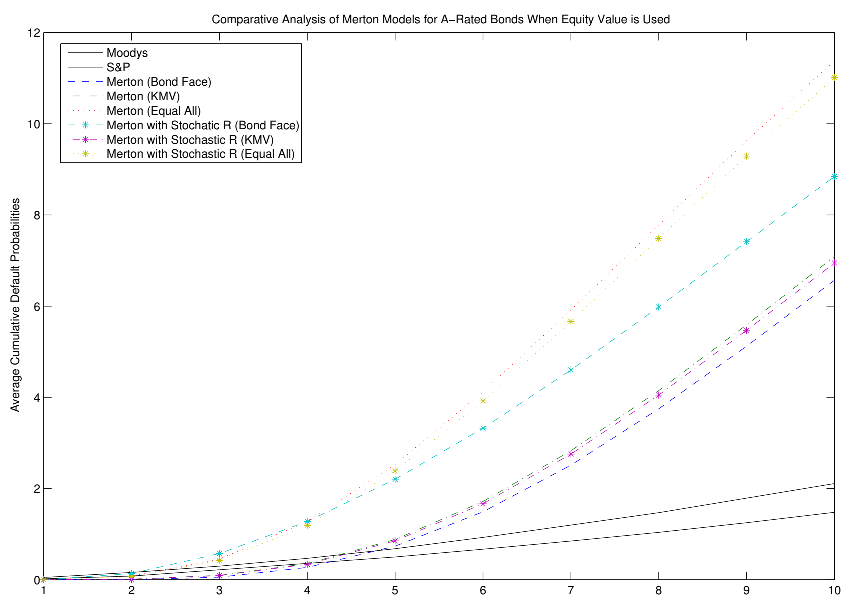

Figure 9: The Performance of Merton Models with Various Debt Structure at Predicting Default Probabilities for BBB-Rated Bonds

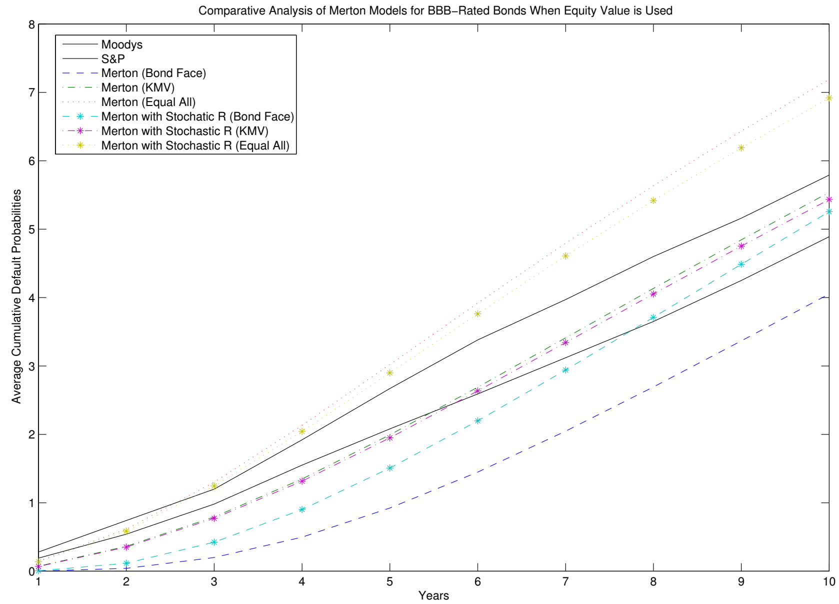

Figure 10: The Performance of Merton Models with Various Debt Structure at Predicting Default Probabilities for BB-Rated Bonds

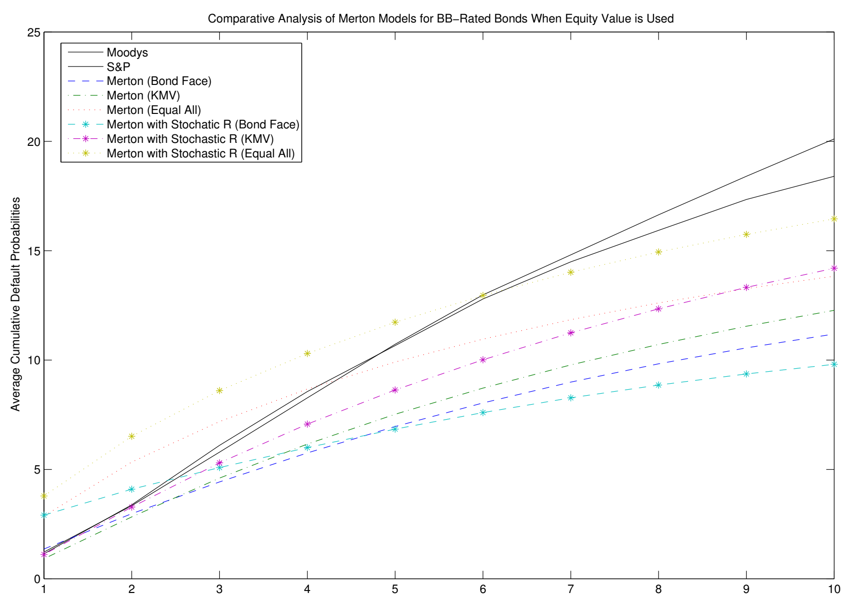

Figure 11: The Performance of Merton Models with Various Debt Structure at Predicting Default Probabilities for B-Rated Bonds

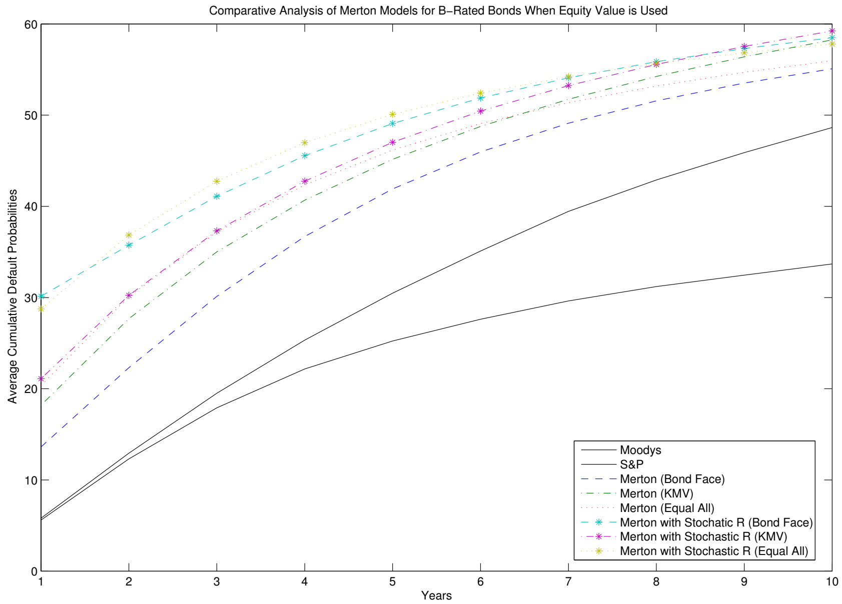

Figure 12: The Performance of Other Structural Models at Predicting Default Probabilities for A-Rated Bonds

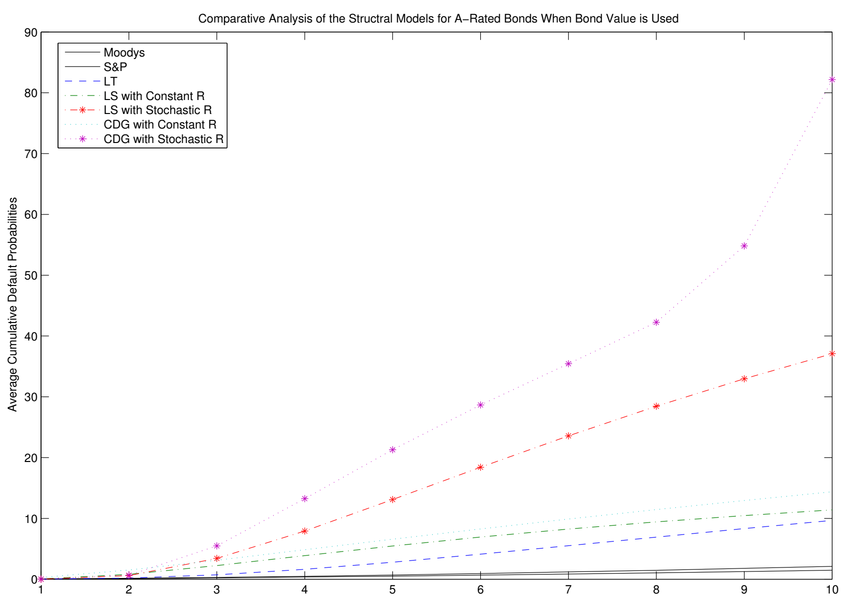

Figure 13: The Performance of Other Structural Models at Predicting Default Probabilities for BBB-Rated Bonds

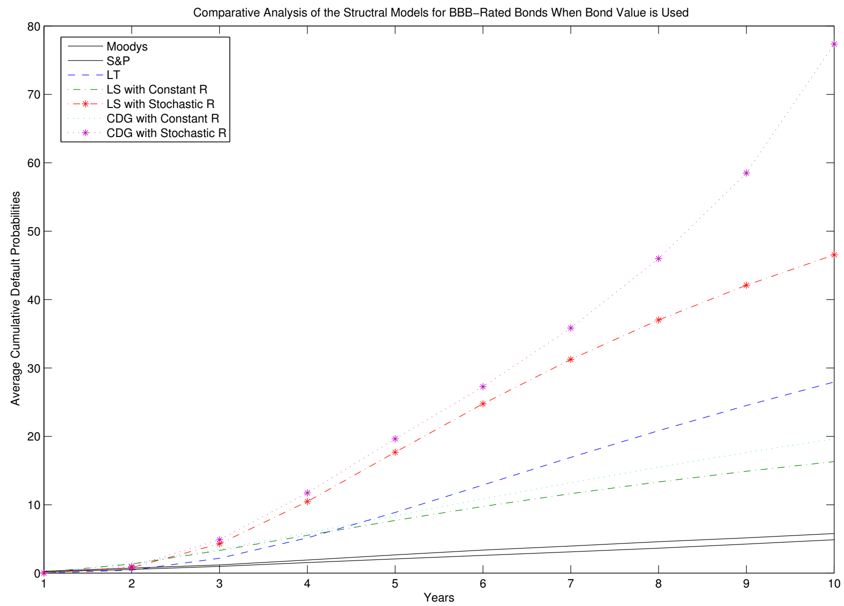

Figure 14: The Performance of Other Structural Models at Predicting Default Probabilities for BB-Rated Bonds

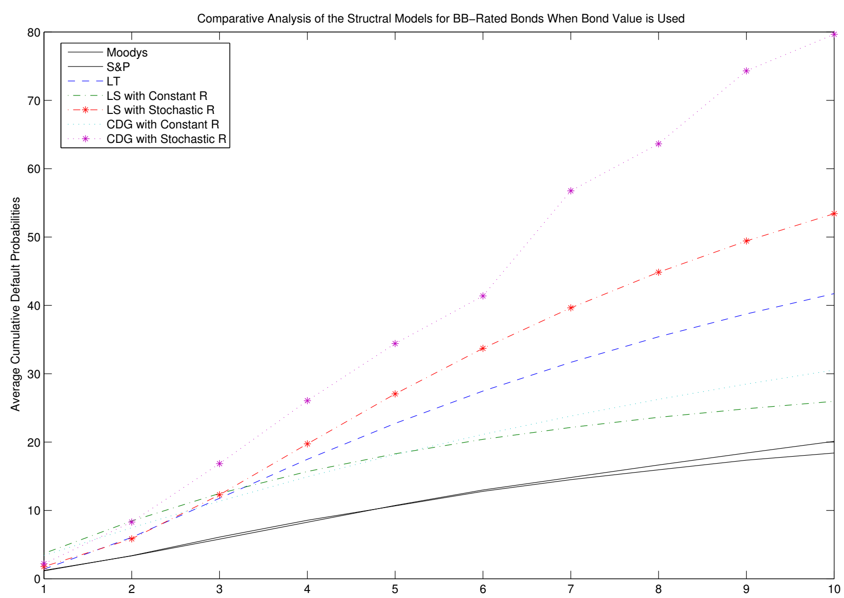

Figure 15: The Performance of Other Structural Models at Predicting Default Probabilities for B-Rated Bonds

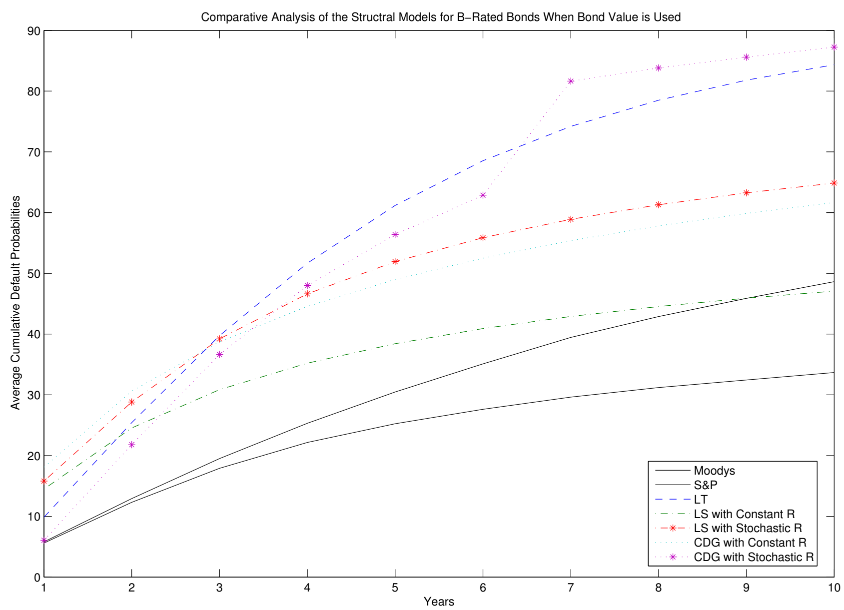

Table 1: Performance of Merton model at predicting 1-year default probability*

Panel A: All Observations Pooled						
	Using Historical DP from Moody's			Using Historical DP from S&P		
Statistics	Bond Face	KMV	Equal All	Bond Face	KMV	All Equal
	Rating Class: A					
Mean Error	-0.0200	-0.0199	-0.0191	-0.0500	-0.0499	-0.0491
Mean Abs-Err	0.0200	0.0199	0.0194	0.0500	0.0499	0.0491
Root Mean Sq-Err	0.0200	0.0200	0.0196	0.0500	0.0499	0.0493
Minimum Error	-0.0200	-0.0200	-0.0200	-0.0500	-0.0500	-0.0500
Maximum Error	-0.0198	-0.0133	0.0424	-0.0498	-0.0433	0.0124
	Rating Class: BBB					
Mean Error	-0.1892	-0.1176	-0.0446	-0.2792	-0.2076	-0.1346
Mean Abss-Err	0.1892	0.2526	0.3154	0.2792	0.3386	0.4003
Root Mean Sq-Err	0.1893	0.5154	0.9814	0.2792	0.5430	0.9896
Minimum Error	-0.1900	-0.1900	-0.1900	-0.2800	-0.2800	-0.2800
Maximum Error	-0.0407	7.6108	13.9740	-0.1307	7.5208	13.8840
	Rating Class: BB					
Mean Error	0.1416	-0.2847	1.7137	0.2216	-0.2047	1.7937
Mean Abs-Err	2.2001	1.7716	3.6544	2.1400	1.7090	3.6017
Root Mean Sq-Err	4.7731	3.0039	9.5811	4.7762	2.9974	9.5957
Minimum Error	-1.2200	-1.2200	-1.2200	-1.1400	-1.1400	-1.1400
Maximum Error	45.5912	25.9293	59.3509	45.6712	26.0093	59.4309
	Rating Class: B					
Mean Error	9.7976	14.1573	17.2791	9.9976	14.3573	17.4791
Mean Abs-Err	14.5634	17.0248	20.0143	14.5744	17.0846	20.0892
Root Mean Sq-Err	21.5413	26.4531	28.9074	21.6330	26.5607	29.0274
Minimum Error	-5.8100	-5.8082	-5.8095	-5.6100	-5.6082	-5.6095
Maximum Error	78.8910	84.8290	87.2280	79.0910	85.0290	87.4280

Panel B: Per-Bond Basis						
	Using Historical DP from Moody's			Using Historical DP from S&P		
Statistics	Bond Face	KMV	Equal All	Bond Face	KMV	All Equal
	Rating Class: A					
Mean Error	-0.0200	-0.0199	-0.0189	-0.0500	-0.0499	-0.0489
Mean Abs-Err	0.0200	0.0199	0.0189	0.0500	0.0499	0.0489
Root Mean Sq-Err	0.0200	0.0199	0.0191	0.0500	0.0499	0.0490
Minimum Error	-0.0200	-0.0200	-0.0200	-0.0500	-0.0500	-0.0500
Maximum Error	-0.0200	-0.0193	-0.0090	-0.0500	-0.0493	-0.0390
	Rating Class: BBB					
Mean Error	-0.1892	-0.1169	-0.0433	-0.2792	-0.2069	-0.1333
Mean Abs-Err	0.1892	0.2422	0.3061	0.2792	0.3227	0.3867
Root Mean Sq-Err	0.1892	0.3295	0.6004	0.2792	0.3711	0.6135
Minimum Error	-0.1900	-0.1900	-0.1900	-0.2800	-0.2800	-0.2800
Maximum Error	-0.1741	1.1900	2.4966	-0.2641	1.1000	2.4066
	Rating Class: BB					
Mean Error	0.1408	-0.3073	1.6445	0.2208	-0.2273	1.7245
Mean Abs-Err	2.1441	1.6715	3.4152	2.0907	1.6104	3.3729
Root Mean Sq-Err	3.6028	2.3022	8.6688	3.6068	2.2929	8.6843
Minimum Error	-1.2200	-1.2200	-1.2200	-1.1400	-1.1400	-1.1400
Maximum Error	11.6205	7.0928	34.6931	11.7005	7.1728	34.7731
	Rating Class: B					
Mean Error	7.8165	12.3814	14.5467	8.0165	12.5814	14.7467
Mean Abs-Err	10.8998	14.4395	17.0695	10.9665	14.5062	17.1095
Root Mean Sq-Err	12.9940	17.9129	22.7503	13.1153	18.0518	22.8787
Minimum Error	-5.8062	-5.6827	-5.4687	-5.6062	-5.4827	-5.2687
Maximum Error	24.0358	28.7307	44.1220	24.2358	28.9307	44.3220

This table reports the summary of the means and standard deviations of the difference between model prediction and the actual default probabilities (predicted-actual) for the Merton (1974) model. The performance of Merton (1974) model is performed under three different assumed debt structure. “Bond Face” structure assumes only the corporate bond itself is retired at maturity. If the asset value falls below the bond face value at the time firm defaults. ”KMV” structure follows Moody’s KMV approach by setting the face value of debt equal to the short-term debt plus long-term debt. “All Equal” envisions that all debt being retired at the maturity of the bond. The results are reported by rating classes in two panels. The first panel reports model error statistics for the pooled time series and cross-sectional observations. The second panel reports error statistics by averaging model error for each bond.

Table 2: Performance of Merton model at predicting 4-year default probability*

Panel A: All Observations Pooled

	Using Historical DP from Moody's			Using Historical DP from S&P		
Statistics	Bond Face	KMV	Equal All	Bond Face	KMV	All Equal
	Rating Class: A					
Mean Error	-0.0588	-0.0440	0.8746	-0.1688	-0.1540	0.7646
Mean Abs-Err	0.4747	0.4903	1.3589	0.5508	0.5551	1.4045
Root Mean Sq-Err	0.7723	0.7190	2.5953	0.7884	0.7340	2.5603
Minimum Error	-0.3600	-0.3600	-0.3600	-0.4700	-0.4700	-0.4700
Maximum Error	4.1192	4.5513	17.4781	4.0092	4.4413	17.3681
	Rating Class: BBB					
Mean Error	-1.0573	-0.2293	0.5536	-1.4273	-0.5993	0.1836
Mean Abs-Err	1.6726	2.3018	2.8046	1.9874	2.5903	3.0452
Root Mean Sq-Err	2.0351	5.2207	6.5579	2.2496	5.2499	6.5371
Minimum Error	-1.5500	-1.5500	-1.5500	-1.9200	-1.9200	-1.9200
Maximum Error	13.5919	40.7915	48.6232	13.2219	40.4215	48.2532
	Rating Class: BB					
Mean Error	-2.4627	-1.9687	0.5993	-2.7527	-2.2587	0.3093
Mean Abs-Err	10.4370	11.2646	13.1534	10.6256	11.4731	13.3526
Root Mean Sq-Err	13.8716	15.1526	21.1197	13.9260	15.1930	21.1135
Minimum Error	-8.2700	-8.2700	-8.2700	-8.5600	-8.5600	-8.5600
Maximum Error	65.0146	58.0085	78.6222	64.7246	57.7185	78.3322
	Rating Class: B					
Mean Error	16.1139	19.4257	22.7886	19.2739	22.5857	25.9486
Mean Abs-Err	26.3307	25.2271	30.0049	27.6754	26.7245	31.5296
Root Mean Sq-Err	32.0763	31.1179	36.0871	33.7744	33.1826	38.1614
Minimum Error	-24.5003	-21.2126	-23.4106	-21.3403	-18.0526	-20.2506
Maximum Error	69.0814	70.3604	71.4729	72.2414	73.5204	74.6329

Panel B: Per-Bond Basis						
	Using Historical DP from Moody's			Using Historical DP from S&P		
Statistics	Bond Face	KMV	Equal All	Bond Face	KMV	All Equal
	Rating Class: A					
Mean Error	-0.0890	-0.0058	0.9126	-0.1990	-0.1158	0.8026
Mean Abs-Err	0.4218	0.5000	1.3625	0.5099	0.5500	1.3925
Root Mean Sq-Err	0.6230	0.6535	2.3693	0.6479	0.6637	2.3291
Minimum Error	-0.3600	-0.3600	-0.3600	-0.4700	-0.4700	-0.4700
Maximum Error	1.8197	1.8177	5.6241	1.7097	1.7077	5.5141
	Rating Class: BBB					
Mean Error	-1.0526	-0.2029	0.5809	-1.4226	-0.5729	0.2109
Mean Abs-Err	1.6087	2.2494	2.7811	1.9080	2.5415	3.0342
Root Mean Sq-Err	1.8207	3.9832	5.3707	2.0569	4.0191	5.3434
Minimum Error	-1.5500	-1.5500	-1.5500	-1.9200	-1.9200	-1.9200
Maximum Error	4.9811	15.9737	21.3466	4.6111	15.6037	20.9766
	Rating Class: BB					
Mean Error	-2.5165	-2.1142	0.4057	-2.8065	-2.4042	0.1157
Mean Abs-Err	10.2780	10.7893	12.8481	10.4713	10.9769	13.0358
Root Mean Sq-Err	12.8905	14.0023	20.0570	12.9502	14.0490	20.0533
Minimum Error	-8.2700	-8.2699	-8.2662	-8.5600	-8.5599	-8.5562
Maximum Error	38.1973	38.9780	69.5919	37.9073	38.6880	69.3019
	Rating Class: B					
Mean Error	11.3605	15.3476	17.0144	14.5205	18.5076	20.1744
Mean Abs-Err	25.3980	22.5027	27.1856	26.4513	23.5561	27.8176
Root Mean Sq-Err	28.0297	26.7195	32.4377	29.4524	28.6516	34.2011
Minimum Error	-21.5981	-15.6592	-16.4228	-18.4381	-12.4992	-13.2628
Maximum Error	42.7849	45.6693	50.9484	45.9449	48.8293	54.1084

This table reports the summary of the means and standard deviations of the difference between model prediction and the actual default probabilities (predicted-actual) for the Merton (1974) model. The performance of Merton (1974) model is performed under three different assumed debt structure. "Bond Face" structure assumes only the corporate bond itself is retired at maturity. If the asset value falls below the bond face value at the time firm defaults. "KMV" structure follows Moody's KMV approach by setting the face value of debt equal to the short-term debt plus long-term debt. "All Equal" envisions that all debt being retired at the maturity of the bond. The results are reported by rating classes in two panels. The first panel reports model error statistics for the pooled time series and cross-sectional observations. The second panel reports error statistics by averaging model error for each bond.

Table 3: Maximum Likelihood Estimates of the Vasicek (1977) Process Using the Monthly Treasury Yield of the Constant Maturity from 1983 to 2002

Parameter	3-Month	6-Month	1-Year	2-Year	3-Year	5-Year
$\bar{r}$	0.0611	0.0637	0.0666	0.0721	0.0746	0.0783
(std)	(0.0063)	(0.0061)	(0.0054)	(0.0043)	0.0037	0.0032
κ_r	0.0629	0.0684	0.0809	0.1067	0.1235	0.1397
(std)	(0.0190)	(0.0207)	(0.0203)	(0.0201)	0.0196	0.0178
σ_r	0.0061	0.0063	0.0067	0.0076	0.0082	0.0092
(std)	(0.0003)	(0.0003)	(0.0003)	(0.0004)	0.0004	0.0006

Table 4: Performance of Merton model with stochastic interest Rate at predicting 1-year default probability*

Panel A: All Observations Pooled						
	Using Historical DP from Moody's			Using Historical DP from S&P		
Statistics	Bond Face	KMV	Equal All	Bond Face	KMV	All Equal
	Rating Class: A					
Mean Error	-0.0163	-0.0200	-0.0192	-0.0463	-0.0500	-0.0492
Mean Abs-Err	0.0210	0.0200	0.0194	0.0489	0.0500	0.0492
Root Mean Sq-Err	0.0237	0.0200	0.0196	0.0494	0.0500	0.0493
Minimum Error	-0.0200	-0.0200	-0.0200	-0.0500	-0.0500	-0.0500
Maximum Error	0.1528	-0.0137	0.0373	0.1228	-0.0437	0.0073
	Rating Class: BBB					
Mean Error	-0.1850	-0.1219	-0.0538	-0.2750	-0.2119	-0.1438
Mean Abs-Err	0.1861	0.2485	0.3072	0.2754	0.3344	0.3922
Root Mean Sq-Err	0.1869	0.4889	0.9268	0.2763	0.5187	0.9364
Minimum Error	-0.1900	-0.1900	-0.1900	-0.2800	-0.2800	-0.2800
Maximum Error	0.2892	7.3105	13.5511	0.1992	7.2205	13.4611
	Rating Class: BB					
Mean Error	1.6383	-0.0975	2.6408	1.7183	-0.0175	2.7208
Mean Abs-Err	3.6164	1.8644	4.4497	3.5613	1.8079	4.4067
Root Mean Sq-Err	8.5988	3.3480	11.2268	8.6144	3.3466	11.2459
Minimum Error	-1.2200	-1.2200	-1.2200	-1.1400	-1.1400	-1.1400
Maximum Error	53.2903	35.7548	74.5568	53.3703	35.8348	74.6368
	Rating Class: B					
Mean Error	26.9287	17.2928	25.5695	27.1287	17.4928	25.7695
Mean Abs-Err	29.8495	20.0767	27.7082	29.9203	20.1477	27.7965
Root Mean Sq-Err	39.5379	30.9186	38.9177	39.6744	31.0309	39.0494
Minimum Error	-5.8092	-5.8083	-5.7940	-5.6092	-5.6083	-5.5940
Maximum Error	90.3644	89.7998	93.4690	90.5644	89.9998	93.6690

Panel B: Per-Bond Basis						
	Using Historical DP from Moody's			Using Historical DP from S&P		
Statistics	Bond Face	KMV	Equal All	Bond Face	KMV	All Equal
	Rating Class: A					
Mean Error	-0.0169	-0.0199	-0.0190	-0.0469	-0.0499	-0.0490
Mean Abs-Err	0.0194	0.0199	0.0190	0.0469	0.0499	0.0490
Root Mean Sq-Err	0.0195	0.0199	0.0192	0.0479	0.0499	0.0491
Minimum Error	-0.0200	-0.0200	-0.0200	-0.0500	-0.0500	-0.0500
Maximum Error	0.0137	-0.0193	-0.0099	-0.0163	-0.0493	-0.0399
	Rating Class: BBB					
Mean Error	-0.1850	-0.1212	-0.0526	-0.2750	-0.2112	-0.1426
Mean Abs-Err	0.1850	0.2379	0.2979	0.2750	0.3185	0.3785
Root Mean Sq-Err	0.1856	0.3142	0.5643	0.2754	0.3587	0.5797
Minimum Error	-0.1900	-0.1900	-0.1900	-0.2800	-0.2800	-0.2800
Maximum Error	-0.1331	1.1086	2.3306	-0.2231	1.0186	2.2406
	Rating Class: BB					
Mean Error	1.6879	-0.1118	2.5657	1.7679	-0.0318	2.6457
Mean Abs-Err	3.5874	1.7605	4.1675	3.5274	1.7071	4.1408
Root Mean Sq-Err	8.0950	2.3262	9.7434	8.1121	2.3237	9.7647
Minimum Error	-1.2200	-1.2200	-1.2200	-1.1400	-1.1400	-1.1400
Maximum Error	29.4878	6.6071	38.2904	29.5678	6.6871	38.3704
	Rating Class: B					
Mean Error	24.3393	15.2977	22.9444	24.5393	15.4977	23.1444
Mean Abs-Err	26.2189	17.3748	24.4933	26.3522	17.4415	24.6266
Root Mean Sq-Err	32.0332	22.0579	30.9305	32.1855	22.1970	31.0792
Minimum Error	-5.6387	-5.6875	-4.6466	-5.4387	-5.4875	-4.4466
Maximum Error	54.2526	36.7271	51.2966	54.4526	36.9271	51.4966

This table reports the summary of the means and standard deviations of the difference between model prediction and the actual default probabilities (predicted-actual) for the Merton (1974) model with stochastic interest rate. The performance of the model is performed under three different assumed debt structure. ”Bond Face” structure assumes only the corporate bond itself is retired at maturity. If the asset value falls below the bond face value at the time firm defaults. “KMV” structure follows Moody's KMV approach by setting the face value of debt equal to the short-term debt plus long-term debt. “All Equal” envisions that all debt being retired at the maturity of the bond. The results are reported by rating classes in two panels. The first panel reports model error statistics for the pooled time series and cross-sectional observations. The second panel reports error statistics by averaging model error for each bond.

Table 5: Performance of Merton model with stochastic interest rate at predicting 4-year default probability*

Panel A: All Observations Pooled						
	Using Historical DP from Moody's			Using Historical DP from S&P		
Statistics	Bond Face	KMV	Equal All	Bond Face	KMV	All Equal
	Rating Class: A					
Mean Error	1.1404	-0.0569	0.7929	1.0304	-0.1669	0.6829
Mean Abs-Err	1.6228	0.4800	1.2824	1.6866	0.5457	1.3307
Root Mean Sq-Err	4.4246	0.6950	2.4268	4.3975	0.7125	2.3932
Minimum Error	-0.3600	-0.3600	-0.3600	-0.4700	-0.4700	-0.4700
Maximum Error	21.3248	4.3898	16.4942	21.2148	4.2798	16.3842
	Rating Class: BBB					
Mean Error	-0.6614	-0.2599	0.4660	-1.0314	-0.6299	0.0960
Mean Abs-Err	1.8166	2.2849	2.7477	2.0718	2.5762	2.9907
Root Mean Sq-Err	2.5516	5.1409	6.4203	2.6716	5.1728	6.4041
Minimum Error	-1.5500	-1.5500	-1.5500	-1.9200	-1.9200	-1.9200
Maximum Error	18.6714	40.3638	48.1882	18.3014	39.9938	47.8182
	Rating Class: BB					
Mean Error	-2.3090	-1.1191	2.1694	-2.5990	-1.4091	1.8794
Mean Abs-Err	10.3025	11.4838	14.0670	10.4883	11.6667	14.2389
Root Mean Sq-Err	13.3555	14.7178	21.1219	13.4087	14.7427	21.0941
Minimum Error	-8.2700	-8.2700	-8.2700	-8.5600	-8.5600	-8.5600
Maximum Error	52.7936	54.1271	76.7657	52.5036	53.8371	76.4757
	Rating Class: B					
Mean Error	24.9645	21.9194	26.6700	28.1245	25.0794	29.8300
Mean Abs-Err	30.4317	28.0865	32.3752	32.2380	29.7714	34.0485
Root Mean Sq-Err	36.4348	34.1429	38.6237	38.6686	36.2527	40.8697
Minimum Error	-22.0392	-21.3089	-20.7768	-18.8792	-18.1489	-17.6168
Maximum Error	71.2446	70.6713	71.6464	74.4046	73.8313	74.8064

Panel B: Per-Bond Basis						
	Using Historical DP from Moody's			Using Historical DP from S&P		
Statistics	Bond Face	KMV	Equal All	Bond Face	KMV	All Equal
			Rating Class: A			
Mean Error	0.9193	-0.0192	0.8320	0.8093	-0.1292	0.7220
Mean Abs-Err	1.3681	0.4887	1.2821	1.4241	0.5387	1.3121
Root Mean Sq-Err	3.7459	0.6311	2.2152	3.7205	0.6439	2.1763
Minimum Error	-0.3600	-0.3600	-0.3600	-0.4700	-0.4700	-0.4700
Maximum Error	12.3881	1.7443	5.1514	12.2781	1.6343	5.0414
			Rating Class: BBB			
Mean Error	-0.6485	-0.2341	0.4923	-1.0185	-0.6041	0.1223
Mean Abs-Err	1.7378	2.2374	2.7316	1.9520	2.5296	2.9847
Root Mean Sq-Err	2.2491	3.9095	5.2280	2.3823	3.9489	5.2062
Minimum Error	-1.5500	-1.5500	-1.5500	-1.9200	-1.9200	-1.9200
Maximum Error	7.6827	15.6317	20.7991	7.3127	15.2617	20.4291
			Rating Class: BB			
Mean Error	-2.2726	-1.1977	2.0307	-2.5626	-1.4877	1.7407
Mean Abs-Err	10.3185	11.0700	13.8094	10.4998	11.2311	13.9705
Root Mean Sq-Err	12.9967	13.4994	19.9768	13.0506	13.5282	19.9494
Minimum Error	-8.2700	-8.2699	-8.2673	-8.5600	-8.5599	-8.5573
Maximum Error	40.0721	29.5124	61.0245	39.7821	29.2224	60.7345
			Rating Class: B			
Mean Error	20.2219	17.4309	21.6537	23.3819	20.5909	24.8137
Mean Abs-Err	27.9194	25.9942	29.6660	28.9728	27.0475	30.7194
Root Mean Sq-Err	32.2782	29.8323	34.1674	34.3464	31.7823	36.2526
Minimum Error	-15.0481	-15.8197	-12.1336	-11.8881	-12.6597	-8.9736
Maximum Error	52.6076	45.8641	51.4773	55.7676	49.0241	54.6373

This table reports the summary of the means and standard deviations of the difference between model prediction and the actual default probabilities (predicted-actual) for the Merton (1974) model with stochastic interest rate. The performance of the model is performed under three different assumed debt structure. ”Bond Face” structure assumes only the corporate bond itself is retired at maturity. If the asset value falls below the bond face value at the time firm defaults. “KMV” structure follows Moody's KMV approach by setting the face value of debt equal to the short-term debt plus long-term debt. “All Equal” envisions that all debt being retired at the maturity of the bond. The results are reported by rating classes in two panels. The first panel reports model error statistics for the pooled time series and cross-sectional observations. The second panel reports error statistics by averaging model error for each bond.

Table 6: Performance of LT model at predicting 1-year default probability

	All Observations Pooled				Per-Bond Basis			
	Moody's		S&P		Moody's		S&P	
Statistics	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov
	Rating Class: A							
Mean Error	-0.0108	-0.0105	-0.0408	-0.0405	-0.0129	-0.0126	-0.0429	-0.0426
Mean Abs-Err	0.0245	0.0249	0.0497	0.0498	0.0233	0.0236	0.0478	0.0481
Root Mean Sq-Err	0.0361	0.0370	0.0534	0.0538	0.0257	0.0263	0.0483	0.0484
Minimum Error	-0.0200	-0.0200	-0.0500	-0.0500	-0.0200	-0.0200	-0.0500	-0.0500
Maximum Error	0.3550	0.3615	0.3250	0.3315	0.0573	0.0603	0.0273	0.0303
	Rating Class: BBB							
Mean Error	-0.1844	-0.1843	-0.2744	-0.2743	-0.1849	-0.1848	-0.2749	-0.2748
Mean Abs-Err	0.1844	0.1843	0.2744	0.2743	0.1849	0.1848	0.2749	0.2748
Root Mean Sq-Err	0.1851	0.1850	0.2749	0.2748	0.1852	0.1851	0.2751	0.2750
Minimum Error	-0.1900	-0.1900	-0.2800	-0.2800	-0.1900	-0.1900	-0.2800	-0.2800
Maximum Error	-0.0414	-0.0507	-0.1314	-0.1407	-0.1461	-0.1456	-0.2361	-0.2356
	Rating Class: BB							
Mean Error	0.1920	0.1882	0.2720	0.2682	0.1919	0.1869	0.2719	0.2669
Mean Abs-Err	2.0379	2.0362	1.9812	1.9789	2.0121	2.0076	1.9588	1.9543
Root Mean Sq-Err	3.8992	3.8813	3.9040	3.8860	3.7386	3.7153	3.7436	3.7202
Minimum Error	-1.2200	-1.2200	-1.1400	-1.1400	-1.2198	-1.2198	-1.1398	-1.1398
Maximum Error	20.4415	20.3694	20.5215	20.4494	14.8061	14.6983	14.8861	14.7783
	Rating Class:B							
Mean Error	6.1628	6.1978	6.3628	6.3978	4.0209	4.0530	4.2209	4.2530
Mean Abs-Err	9.6289	9.6502	9.6716	9.6954	8.0567	8.0812	8.0967	8.1212
Root Mean Sq-Err	18.0413	18.0695	18.1106	18.1391	10.4868	10.5148	10.5651	10.5935
Minimum Error	-5.8000	-5.8005	-5.6000	-5.6005	-5.7929	-5.7935	-5.5929	-5.5935
Maximum Error	84.6863	84.7672	84.8863	84.9672	21.1214	21.1859	21.3214	21.3859

The results provided in the columns of Moody's and S&P are those obtained by using the historical default probabilities from Moody's and S&P respectively. The columns of "Avg Recov" and "Ind Recov" refers to the results obtained by using the average recovery rate and the industry specific recovery rates provided by S&P LossStats database.

Table 7: Performance of LT model at predicting 4-year default probability

	All Observations Pooled				Per-Bond Basis			
	Moody's		S&P		Moody's		S&P	
Statistics	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov
	Rating Class: A							
Mean Error	0.9952	1.0432	0.8852	0.9332	0.9245	0.9711	0.8145	0.8611
Mean Abs-Err	1.1741	1.2190	1.1452	1.1890	1.1253	1.1693	1.0953	1.1393
Root Mean Sq-Err	2.3066	2.3910	2.2613	2.3451	2.0419	2.1192	1.9945	2.0711
Minimum Error	-0.3600	-0.3600	-0.4700	-0.4700	-0.3600	-0.3600	-0.4700	-0.47
Maximum Error	11.1301	11.4033	11.0201	11.2933	6.0641	6.3034	5.9541	6.1934
	Rating Class: BBB							
Mean Error	2.2879	2.3604	1.9179	1.9904	2.1049	2.1755	1.7349	1.8055
Mean Abs-Err	3.0965	3.1644	2.9866	3.0517	3.0161	3.0847	2.8964	2.9643
Root Mean Sq-Err	4.5231	4.6228	4.3476	4.4453	4.0714	4.1788	3.8931	3.9987
Minimum Error	-1.5500	-1.5500	-1.9200	-1.9200	-1.5500	-1.5500	-1.9200	-1.92
Maximum Error	16.1227	16.1441	15.7527	15.7741	9.2398	9.7108	8.8698	9.3408
	Rating Class: BB							
Mean Error	9.3197	9.2934	9.0297	9.0034	9.2656	9.2055	8.9756	8.9155
Mean Abs-Err	13.1287	13.1195	13.0999	13.0910	13.0780	13.0370	13.0458	13.0047
Root Mean Sq-Err	20.6423	20.5076	20.5130	20.3779	20.3254	20.1479	20.1949	20.0171
Minimum Error	-8.2463	-8.2464	-8.5363	-8.5364	-7.8186	-7.8182	-8.1085	-8.1082
Maximum Error	68.0688	67.8315	67.7788	67.5415	63.9234	63.6200	63.6334	63.33
	Rating Class: B							
Mean Error	27.0864	27.1548	30.2464	30.3148	26.2783	26.3499	29.4383	29.5099
Mean Abs-Err	27.0864	27.1548	30.2464	30.3148	26.2783	26.3499	29.4383	29.5099
Root Mean Sq-Err	32.5727	32.6724	35.2442	35.3425	30.4582	30.5764	33.2229	33.3382
Minimum Error	1.0869	1.1802	4.2469	4.3402	4.0786	3.7622	7.2386	6.9222
Maximum Error	69.4273	69.4758	72.5873	72.6358	46.8452	46.8599	50.0052	50.0199

The results provided in the columns of Moody's and S&P are those obtained by using the historical default probabilities from Moody's and S&P respectively. The columns of "Avg Recov" and "Ind Recov" refers to the results obtained by using the average recovery rate and the industry specific recovery rates provided by S&P LossStats database.

Table 8: Performance of LS model with constant term structure at predicting 1-year default probability

	All Observations Pooled				Per-Bond Basis			
	Moody's		S&P		Moody's		S&P	
Statistics	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov
	Rating Class: A							
Mean Error	0.0194	0.0380	-0.0106	0.0080	0.0185	0.0347	-0.0115	0.0047
Mean Abs-Err	0.0472	0.0646	0.0623	0.0784	0.0415	0.0565	0.0523	0.0662
Root Mean Sq-Err	0.0983	0.1457	0.0969	0.1409	0.0613	0.0917	0.0595	0.0850
Minimum Error	-0.0200	-0.0200	-0.0500	-0.0500	-0.0198	-0.0197	-0.0498	-0.0497
Maximum Error	1.2047	1.7393	1.1747	1.7093	0.1388	0.2435	0.1088	0.2135
	Rating Class: BBB							
Mean Error	-0.0755	-0.0548	-0.1655	-0.1448	-0.0771	-0.0560	-0.1671	-0.1460
Mean Abs-Err	0.1985	0.2078	0.2690	0.2747	0.1282	0.1322	0.2005	0.1929
Root Mean Sq-Err	0.3426	0.3896	0.3729	0.4120	0.1555	0.1627	0.2148	0.2113
Minimum Error	-0.1900	-0.1900	-0.2800	-0.2800	-0.1890	-0.1888	-0.2790	-0.2788
Maximum Error	4.3667	4.6958	4.2767	4.6058	0.3737	0.4298	0.2837	0.3398
	Rating Class: BB							
Mean Error	2.6177	2.4846	2.6977	2.5646	2.6698	2.4838	2.7498	2.5638
Mean Abs-Err	3.8069	3.6492	3.7878	3.6328	3.6478	3.4104	3.6300	3.4015
Root Mean Sq-Err	9.6912	9.2142	9.7131	9.2361	8.9636	8.4628	8.9878	8.4866
Minimum Error	-1.2200	-1.2200	-1.1400	-1.1400	-1.1760	-1.1762	-1.0960	-1.0962
Maximum Error	51.4296	49.7846	51.5096	49.8646	36.2076	34.5219	36.2876	34.6019
	Rating Class:B							
Mean Error	7.0928	7.5025	7.2928	7.7025	8.1549	8.6745	8.3549	8.8745
Mean Abs-Err	11.3633	11.7074	11.3759	11.7237	10.4486	10.9592	10.5686	11.0792
Root Mean Sq-Err	17.1433	17.6441	17.2270	17.7301	14.3908	15.1295	14.5051	15.2450
Minimum Error	-5.8100	-5.8100	-5.6100	-5.6100	-5.7344	-5.7116	-5.5344	-5.5116
Maximum Error	73.3357	74.2067	73.5357	74.4067	29.6690	31.3890	29.8690	31.5890

The results provided in the columns of Moody's and S&P are those obtained by using the historical default probabilities from Moody's and S&P respectively. The columns of "Avg Recov" and "Ind Recov" refers to the results obtained by using the average recovery rate and the industry specific recovery rates provided by S&P LossStats database.

Table 9: Performance of LS model with constant term structure at predicting 4-year default probability

	All Observations Pooled				Per-Bond Basis			
	Moody's		S&P		Moody's		S&P	
Statistics	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov
	Rating Class: A							
Mean Error	3.0082	3.5983	2.8982	3.4883	2.9440	3.5151	2.8340	3.4051
Mean Abs-Err	3.1786	3.7447	3.1785	3.7387	3.0605	3.6166	3.0605	3.6114
Root Mean Sq-Err	6.5678	7.6455	6.5182	7.5944	5.9501	6.9432	5.8965	6.8881
Minimum Error	-0.3327	-0.3388	-0.4427	-0.4488	-0.2197	-0.2411	-0.3297	-0.3511
Maximum Error	31.1940	34.6744	31.0840	34.5644	17.4138	20.4289	17.3038	20.3189
	Rating Class: BBB							
Mean Error	3.3990	3.7833	3.0290	3.4133	3.5712	3.9836	3.2012	3.6136
Mean Abs-Err	3.8501	4.1992	3.7173	4.0534	3.7080	4.1151	3.4640	3.8679
Root Mean Sq-Err	6.4541	6.8836	6.2671	6.6874	4.9406	5.3507	4.6802	5.0813
Minimum Error	-1.5362	-1.5339	-1.9062	-1.9039	-0.9671	-0.9184	-1.3371	-1.2884
Maximum Error	36.6659	37.5704	36.2959	37.2004	10.4375	10.8953	10.0675	10.5253
	Rating Class: BB							
Mean Error	7.7360	7.7446	7.4460	7.4546	7.5568	7.4262	7.2668	7.1362
Mean Abs-Err	12.0506	12.0482	12.0217	12.0146	11.6375	11.5363	11.6053	11.5041
Root Mean Sq-Err	18.9095	18.4982	18.7927	18.3787	17.9411	17.3676	17.8209	17.2456
Minimum Error	-8.2572	-8.2573	-8.5472	-8.5473	-7.3700	-7.3721	-7.6600	-7.6621
Maximum Error	66.2175	65.0224	65.9275	64.7324	55.3964	53.8698	55.1064	53.5798
	Rating Class: B							
Mean Error	6.2896	6.8353	9.4496	9.9953	9.1775	9.8777	12.3375	13.0377
Mean Abs-Err	21.8294	22.2303	22.3795	22.8398	19.5725	20.2519	21.0239	21.6870
Root Mean Sq-Err	25.2049	25.6788	26.1729	26.6943	23.5540	24.2354	24.9555	25.6860
Minimum Error	-25.3300	-25.3300	-22.1700	-22.1700	-24.8760	-24.7833	-21.7160	-21.6233
Maximum Error	64.6457	65.1838	67.8057	68.3438	38.8640	40.4801	42.0240	43.6401

The results provided in the columns of Moody's and S&P are those obtained by using the historical default probabilities from Moody's and S&P respectively. The columns of "Avg Recov" and "Ind Recov" refers to the results obtained by using the average recovery rate and the industry specific recovery rates provided by S&P LossStats database.

Table 10: Performance of the LS model at predicting 1-year default probability

	All Observations Pooled				Per-Bond Basis			
	Moody's		S&P		Moody's		S&P	
Statistics	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov
	Rating Class: A							
Mean Error	-0.0186	-0.0184	-0.0486	-0.0484	-0.0180	-0.0182	-0.0480	-0.0483
Mean Abs-Err	0.0191	0.0193	0.0487	0.0486	0.0180	0.0182	0.0480	0.0483
Root Mean Sq-Err	0.0197	0.0196	0.0490	0.0489	0.0187	0.0185	0.0483	0.0484
Minimum Error	-0.0200	-0.0200	-0.0500	-0.0500	-0.0200	-0.0200	-0.0500	-0.0500
Maximum Error	0.1391	0.0773	0.1091	0.0473	-0.0023	-0.0107	-0.0323	-0.0407
	Rating Class: BBB							
Mean Error	-0.1689	-0.1657	-0.2589	-0.2557	-0.1716	-0.1687	-0.2616	-0.2587
Mean Abs-Err	0.2000	0.2025	0.2860	0.2881	0.1855	0.1875	0.2649	0.2669
Root Mean Sq-Err	0.2348	0.2468	0.3060	0.3144	0.1862	0.1876	0.2714	0.2714
Minimum Error	-0.1900	-0.1900	-0.2800	-0.2800	-0.1900	-0.1900	-0.2800	-0.2800
Maximum Error	2.8459	3.1458	2.7559	3.0558	0.1183	0.1598	0.0283	0.0698
	Rating Class: BB							
Mean Error	0.6196	0.5680	0.6996	0.6480	0.6132	0.5603	0.6932	0.6403
Mean Abs-Err	2.7099	2.6229	2.6482	2.5615	2.6068	2.4987	2.5481	2.4416
Root Mean Sq-Err	6.8261	6.5153	6.8338	6.5228	6.0264	5.7114	6.0350	5.7198
Minimum Error	-1.2200	-1.2200	-1.1400	-1.1400	-1.2200	-1.2200	-1.1400	-1.1400
Maximum Error	37.4180	35.1609	37.4980	35.2409	22.9313	20.9968	23.0113	21.0768
	Rating Class:B							
Mean Error	8.4548	8.8572	8.6548	9.0572	9.5702	10.0070	9.7702	10.2070
Mean Abs-Err	15.2389	15.4094	15.1711	15.3488	15.1038	15.4399	15.1038	15.4399
Root Mean Sq-Err	24.3422	24.5417	24.4124	24.6145	19.8727	20.0420	19.9698	20.1426
Minimum Error	-5.8100	-5.8100	-5.6100	-5.6100	-5.7471	-5.7341	-5.5471	-5.5341
Maximum Error	68.7936	68.5997	68.9936	68.7997	36.9368	36.6812	37.1368	36.8812

The results provided in the columns of Moody's and S&P are those obtained by using the historical default probabilities from Moody's and S&P respectively. The columns of "Avg Recov" and "Ind Recov" refers to the results obtained by using the average recovery rate and the industry specific recovery rates provided by S&P LossStats database.

Table 11: Performance of LS model at predicting 4-year default probability

	All Observations Pooled				Per-Bond Basis			
	Moody's		S&P		Moody's		S&P	
Statistics	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov
	Rating Class: A							
Mean Error	6.3770	7.2487	6.2670	7.1387	6.9169	7.5725	6.8069	7.4625
Mean Abs-Err	6.4847	7.3266	6.4447	7.2738	6.9742	7.6085	6.9413	7.5471
Root Mean Sq-Err	10.6438	11.6431	10.5783	11.5749	10.9544	11.4804	10.8853	11.4082
Minimum Error	-0.3399	-0.3480	-0.4499	-0.4580	-0.1842	-0.1741	-0.2942	-0.2841
Maximum Error	33.3412	32.9123	33.2312	32.8023	24.1673	21.7370	24.0573	21.6270
	Rating Class: BBB							
Mean Error	7.8825	8.8585	7.5125	8.4885	7.8762	8.9132	7.5062	8.5432
Mean Abs-Err	7.9990	8.9529	7.7131	8.6558	7.8762	8.9132	7.5245	8.5432
Root Mean Sq-Err	12.3725	13.3890	12.1401	13.1472	10.7178	11.8715	10.4489	11.5962
Minimum Error	-1.4958	-1.4798	-1.8658	-1.8498	0.2150	0.4487	-0.1550	0.0787
Maximum Error	59.6602	60.3472	59.2902	59.9772	27.7588	28.5154	27.3888	28.1454
	Rating Class: BB							
Mean Error	10.6557	11.5622	10.3657	11.2722	10.7985	11.4640	10.5085	11.1740
Mean Abs-Err	13.4621	14.5145	13.3842	14.4367	12.7529	13.6080	12.5789	13.4423
Root Mean Sq-Err	21.4892	21.9303	21.3469	21.7788	20.1435	20.2992	19.9896	20.1369
Minimum Error	-7.9880	-7.8564	-8.2780	-8.1464	-5.5982	-5.6120	-5.8882	-5.9020
Maximum Error	71.8375	70.6447	71.5475	70.3547	63.3919	61.7486	63.1019	61.4586
	Rating Class:B							
Mean Error	16.2768	17.2630	19.4368	20.4230	20.1999	21.2941	23.3599	24.4541
Mean Abs-Err	31.5649	32.3187	32.6177	33.4976	31.7845	32.7521	33.3645	34.3321
Root Mean Sq-Err	36.3663	36.9657	37.8862	38.5428	34.8903	35.5993	36.8101	37.5749
Minimum Error	-25.3300	-25.3300	-22.1700	-22.1700	-23.1692	-22.9158	-20.0092	-19.7558
Maximum Error	66.2602	66.1691	69.4202	69.3291	49.0663	48.7793	52.2263	51.9393

The results provided in the columns of Moody's and S&P are those obtained by using the historical default probabilities from Moody's and S&P respectively. The columns of "Avg Recov" and "Ind Recov" refers to the results obtained by using the average recovery rate and the industry specific recovery rates provided by S&P LossStats database.

Table 12: Performance of the CDG model with constant term structure at predicting 1-year default probability

	All Observations Pooled				Per-Bond Basis			
	Moody's		S&P		Moody's		S&P	
Statistics	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov
	Rating Class: A							
Mean Error	0.2200	0.3308	0.1900	0.3009	0.1781	0.2688	0.1481	0.2388
Mean Abs-Err	0.2488	0.3590	0.2662	0.3749	0.2049	0.2932	0.2215	0.3098
Root Mean Sq-Err	0.7116	1.0204	0.7029	1.0111	0.5606	0.8267	0.5518	0.8174
Minimum Error	-0.0200	-0.0200	-0.0500	-0.0500	-0.0200	-0.0200	-0.0500	-0.0500
Maximum Error	5.8319	7.6054	5.8019	7.5754	1.6807	2.4791	1.6507	2.4491
	Rating Class: BBB							
Mean Error	-0.0460	-0.0141	-0.1360	-0.1041	-0.0425	-0.0083	-0.1325	-0.0983
Mean Abs-Err	0.2601	0.2854	0.3284	0.3510	0.2255	0.2495	0.2855	0.3012
Root Mean Sq-Err	0.4704	0.5381	0.4875	0.5479	0.2801	0.3259	0.3069	0.3403
Minimum Error	-0.1900	-0.1900	-0.2800	-0.2800	-0.1900	-0.1900	-0.2800	-0.2800
Maximum Error	5.4143	5.8148	5.3243	5.7248	0.7305	0.8531	0.6405	0.7631
	Rating Class: BB							
Mean Error	2.4061	2.2115	2.4861	2.2915	2.4130	2.1782	2.4930	2.2582
Mean Abs-Err	4.0281	3.7933	3.9891	3.7564	3.8783	3.5556	3.8338	3.5201
Root Mean Sq-Err	9.9114	9.4053	9.9311	9.4244	9.1047	8.5850	9.1262	8.6057
Minimum Error	-1.2200	-1.2200	-1.1400	-1.1400	-1.2196	-1.2197	-1.1396	-1.1397
Maximum Error	51.0283	49.1430	51.1083	49.2230	35.6728	33.7181	35.7528	33.7981
	Rating Class:B							
Mean Error	10.4147	11.0712	10.6147	11.2712	11.4956	12.3005	11.6956	12.5005
Mean Abs-Err	13.2002	13.7597	13.2652	13.8283	11.4956	12.3005	11.6956	12.5005
Root Mean Sq-Err	19.4297	20.1606	19.5377	20.2712	16.6579	17.6954	16.7965	17.8350
Minimum Error	-5.7999	-5.8001	-5.5999	-5.6001	1.4794	2.1858	1.6794	2.3858
Maximum Error	73.8393	74.8082	74.0393	75.0082	31.3557	33.4784	31.5557	33.6784

The results provided in the columns of Moody's and S&P are those obtained by using the historical default probabilities from Moody's and S&P respectively. The columns of "Avg Recov" and "Ind Recov" refers to the results obtained by using the average recovery rate and the industry specific recovery rates provided by S&P LossStats database.

Table 13: Performance of the CDG model with constant term structure at predicting 4-year default probability

	All Observations Pooled				Per-Bond Basis			
	Moody's		S&P		Moody's		S&P	
Statistics	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov
	Rating Class: A							
Mean Error	4.1582	4.8147	4.0482	4.7047	3.8744	4.4911	3.7644	4.3811
Mean Abs-Err	4.4619	5.0969	4.4691	5.0977	4.1274	4.7350	4.1152	4.7227
Root Mean Sq-Err	8.7773	9.9193	8.7257	9.8664	7.9317	8.9622	7.8786	8.9076
Minimum Error	-0.3599	-0.3599	-0.4699	-0.4699	-0.3589	-0.3592	-0.4689	-0.4692
Maximum Error	33.7620	36.9166	33.6520	36.8066	22.0879	25.1346	21.9779	25.0246
	Rating Class: BBB							
Mean Error	3.5508	4.0230	3.1808	3.6530	3.8188	4.3317	3.4488	3.9617
Mean Abs-Err	4.2856	4.7196	4.1913	4.6134	4.2073	4.7051	4.0182	4.4996
Root Mean Sq-Err	7.0357	7.6167	6.8564	7.4279	6.0986	6.6973	5.8740	6.4641
Minimum Error	-1.5500	-1.5500	-1.9200	-1.9200	-1.3829	-1.3594	-1.7529	-1.7294
Maximum Error	32.2416	33.0143	31.8716	32.6443	15.1525	16.1500	14.7825	15.7800
	Rating Class: BB							
Mean Error	6.7559	6.7723	6.4659	6.4823	6.7825	6.6457	6.4925	6.3557
Mean Abs-Err	12.3549	12.3591	12.3663	12.3643	11.5567	11.4715	11.5567	11.4715
Root Mean Sq-Err	19.0690	18.6647	18.9682	18.5614	17.9842	17.4147	17.8769	17.3061
Minimum Error	-8.2525	-8.2526	-8.5425	-8.5426	-7.1862	-7.2779	-7.4762	-7.5679
Maximum Error	66.6363	65.5300	66.3463	65.2400	55.7878	54.3145	55.4978	54.0245
	Rating Class: B							
Mean Error	16.6978	17.3898	19.8578	20.5498	18.3604	19.2298	21.5204	22.3898
Mean Abs-Err	21.2370	21.7761	23.0449	23.6656	18.3604	19.2298	21.5204	22.3898
Root Mean Sq-Err	25.5561	26.1695	27.7242	28.3678	22.7600	23.6530	25.3780	26.2866
Minimum Error	-20.1619	-20.1862	-17.0019	-17.0262	3.8415	3.7990	7.0015	6.9590
Maximum Error	64.4213	64.9182	67.5813	68.0782	38.8733	40.4590	42.0333	43.6190

The results provided in the columns of Moody's and S&P are those obtained by using the historical default probabilities from Moody's and S&P respectively. The columns of "Avg Recov" and "Ind Recov" refers to the results obtained by using the average recovery rate and the industry specific recovery rates provided by S&P LossStats database.

Table 14: Performance of the CDG model with stochastic term structure at predicting 1-year default probability

	All Observations Pooled				Per-Bond Basis			
	Moody's		S&P		Moody's		S&P	
Statistics	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov
	Rating Class: A							
Mean Error	-0.0199	-0.0199	-0.0499	-0.0499	-0.0198	-0.0199	-0.0498	-0.0499
Mean Abs-Err	0.0199	0.0199	0.0499	0.0499	0.0198	0.0199	0.0498	0.0499
Root Mean Sq-Err	0.0199	0.0199	0.0499	0.0499	0.0198	0.0199	0.0498	0.0499
Minimum Error	-0.02	-0.02	-0.05	-0.05	-0.02	-0.02	-0.05	-0.05
Maximum Error	-0.0046	-0.0131	-0.0346	-0.0431	-0.0193	-0.0197	-0.0493	-0.0497
	Rating Class: BBB							
Mean Error	-0.1579	-0.1588	-0.2479	-0.2488	-0.1623	-0.1637	-0.2523	-0.2537
Mean Abs-Err	0.2039	0.205	0.287	0.2889	0.1796	0.1843	0.2523	0.2593
Root Mean Sq-Err	0.2439	0.2497	0.3098	0.3147	0.1822	0.1852	0.2656	0.2681
Minimum Error	-0.19	-0.19	-0.28	-0.28	-0.19	-0.19	-0.28	-0.28
Maximum Error	2.4089	2.6687	2.3189	2.5787	0.0861	0.1238	-0.0039	0.0338
	Rating Class: BB							
Mean Error	1.308	0.9785	1.388	1.0585	1.3119	0.9799	1.3919	1.0599
Mean Abs-Err	3.193	3.013	3.1439	2.9546	3.0772	2.9639	3.0327	2.9039
Root Mean Sq-Err	7.0955	6.621	7.1107	6.6333	6.1098	5.6805	6.1275	5.6948
Minimum Error	-1.22	-1.22	-1.14	-1.14	-1.22	-1.22	-1.14	-1.14
Maximum Error	31.89	29.0552	31.9703	29.1352	17.9875	15.775	18.0675	15.855
	Rating Class:B							
Mean Error	-4.8247	-0.4288	-4.6247	-0.2288	-5.0082	0.2557	-4.8082	0.4557
Mean Abs-Err	5.1529	7.3362	4.9766	7.2221	5.0082	6.8688	4.8082	6.8021
Root Mean Sq-Err	5.3304	11.4405	5.1501	11.4348	5.0484	7.3954	4.85	7.405
Minimum Error	-5.81	-5.81	-5.61	-5.61	-5.6437	-5.5387	-5.4437	-5.3387
Maximum Error	6.6981	60.2265	6.8981	60.4265	-4.3728	10.6867	-4.1728	10.8867

Table 15: Performance of the CDG model with stochastic term structure at predicting 4-year default probability

	All Observations Pooled				Per-Bond Basis			
	Moody's		S&P		Moody's		S&P	
Statistics	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov	Avg Recov	Ind Recov
	Rating Class: A							
Mean Error	9.7024	11.8245	9.5924	11.7145	11.6916	12.8945	11.5816	12.7845
Mean Abs-Err	9.8286	11.9091	9.7754	11.841	11.7718	12.9381	11.7168	12.8721
Root Mean Sq-Err	13.4093	14.5592	13.3299	14.47	14.5615	14.7412	14.4733	14.6451
Minimum Error	-0.3435	-0.3364	-0.4535	-0.4464	-0.1603	-0.109	-0.2703	-0.219
Maximum Error	33.6211	31.1166	33.5111	31.0066	23.5244	21.1298	23.4144	21.0198
	Rating Class: BBB							
Mean Error	10.0061	9.9633	9.6361	9.5933	10.4214	10.1796	10.0514	9.8096
Mean Abs-Err	10.1166	10.0986	9.8263	9.8139	10.4214	10.1796	10.0514	9.8096
Root Mean Sq-Err	14.5695	14.6648	14.3179	14.416	12.8456	13.0312	12.5473	12.7442
Minimum Error	-1.1259	-1.5066	-1.4959	-1.8766	0.5346	0.5225	0.1646	0.1525
Maximum Error	56.3731	57.0137	56.0031	56.6437	27.1473	27.84	26.7773	27.47
	Rating Class: BB							
Mean Error	20.5935	18.0142	20.3035	17.7242	20.724	17.7747	20.434	17.4847
Mean Abs-Err	22.623	20.1618	22.4498	19.9978	21.5397	18.6951	21.3141	18.4776
Root Mean Sq-Err	30.9756	28.3088	30.7835	28.1252	29.6928	26.8474	29.4912	26.6563
Minimum Error	-8.1777	-8.1786	-8.4677	-8.4686	-3.6708	-3.6816	-3.9608	-3.9716
Maximum Error	73.0057	71.89	72.7157	71.6	64.1321	62.531	63.8421	62.241
	Rating Class: B							
Mean Error	10.7537	20.6833	13.9137	23.8433	10.8786	22.6718	14.0386	25.8318
Mean Abs-Err	15.3221	23.7637	16.6094	25.7625	10.8786	22.6718	14.0386	25.8318
Root Mean Sq-Err	19.337	29.2623	21.2572	31.5751	10.8872	27.1482	14.0453	29.8378
Minimum Error	-16.182	-16.2246	-13.022	-13.0646	10.4464	10.3915	13.6064	13.5515
Maximum Error	44.9042	66.2994	48.0642	69.4594	11.3108	43.692	14.4708	46.852

The results provided in the columns of Moody's and S&P are those obtained by using the historical default probabilities from Moody's and S&P respectively. The columns of "Avg Recov" and "Ind Recov" refers to the results obtained by using the average recovery rate and the industry specific recovery rates provided by S&P LossStats database.

Table 16: Comparison of the model performance

Predicting 1-Year Default Probabilities				
Rating classes	A	BBB	BB	B
Moody's Historical	0.0200	0.1900	1.2200	5.8100
S&P Historical	0.0500	0.2800	1.1400	5.6100
Merton	0.0000	0.0008	1.3616	15.6076
Merton with Stochastic Interest Rate	0.0037	0.4926	2.8583	32.7387
LT	0.0092	0.0056	1.4120	11.9728
LS with Constant Interests Rate	0.0394	0.1145	3.8377	12.9028
LS with Stochastic Interest Rate	0.0014	0.0211	1.8396	14.2648
CDG with Constant Interest Rate	0.2400	0.1440	3.6261	16.2247
CDG with Stochastic Interest Rate	0.0001	0.0264	2.1991	6.0669

Predicting 4-Year Default Probabilities				
Rating classes	A	BBB	BB	B
Moody's Historical	0.3600	1.5500	8.2700	25.3300
S&P Historical	0.4700	1.9200	8.5600	22.1700
Merton	0.3012	0.4926	5.8073	41.4439
Merton with Stochastic Interest Rate	1.5004	0.8886	5.9610	50.2945
LT	1.3552	3.8379	17.5897	52.4164
LS with Constant Interests Rate	3.3682	4.9490	16.0060	31.6196
LS with Stochastic Interest Rate	6.7370	9.4325	18.9257	41.6068
CDG with Constant Interest Rate	4.5182	5.1008	15.0260	42.0278
CDG with Stochastic Interest Rate	13.2539	11.7302	26.0447	47.9995