Beta and Momentum

Kevin Q. Wang*

Joseph L. Rotman School of Management
University of Toronto
105 St George Street
Toronto, Ontario M5S 3E6, Canada

Phone: 416-946-5059
E-mail: kwang@rotman.utoronto.ca

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ABSTRACT

Recent empirical findings seem to suggest that none of the momentum payoff is due to risk. In particular, it is shown that the CAPM fails grossly in explaining the momentum. In this paper, I study a theory on beta uncertainty and find that this extended version of the CAPM can rationalize a number of puzzling results on momentum. The key assumption is that events that drastically produce high/low ranking-period returns on the winners/losers also induce high uncertainty about the systematic risk estimates for these stocks. In light of the unusual returns and high beta uncertainty, investors revise their beta estimates for the winners and the losers. The beta adjustment creates momentum.

1. Introduction

The capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) has long been the backbone of academic finance. In addition to numerous important applications, the model has shaped the way academics and practitioners think about risk and average return. The famous risk measure of the CAPM, the beta of a stock, is being taught in business schools for decades and widely applied in practice. Despite the preeminence of the model, however, recent empirical tests have challenged the CAPM by identifying several powerful anomalies. One of the strongest and most puzzling challenges is the momentum effect documented by Jegadeesh and Titman (1993).

Jegadeesh and Titman find that past winners continue to outperform the past losers over horizons of three to twelve months. Surprisingly, the beta estimate for the winner portfolio is even lower than that for the loser portfolio, showing that the CAPM fails grossly to explain momentum. In their tests on momentum, Fama and French (1996) find that the loadings of the size and book-to-market factors are higher for the losers. Thus, the multifactor extension does not do any better. Both the CAPM and the three factor model produce qualitatively incorrect predictions (that losers are riskier), giving rise to the conclusion that none of the momentum payoff is due to systematic risk. Furthermore, Fama and French find that among several CAPM anomalies, momentum is the only one unexplained by the three factor model. These results make momentum particularly intriguing.

Why does the CAPM fail to explain momentum? Behavioral explanations by Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Hong and Stein (1999) have gained wide attention and empirical support in recent years. The focus of these theories is on investor irrationality in processing information. The main implication is that risk does not play any significant role in capturing the momentum payoff, and thus of course, the CAPM is bound to fail. Indeed, recent tests on momentum have uncovered formidable challenges to risk-based theories and the results have generally been interpreted as strong support to the behavioral models. In this paper, I provide a counter-example. The goal is to show that many of the puzzling test results can be circumvented by an extended version of the CAPM, a theory in which investors rationally process information.

This paper is based on a simple idea. Stocks become winners/losers due to some events in the ranking period. These events are so forceful that they drastically push up/down the returns on the stocks, making them the highest/lowest among thousands of stocks. The key assumption to my argument is that such drastic events also blow away much of investors' confidence about their beta estimates for these stocks. Intuitively, new information revealed in the events may suggest a high probability that there has been a large jump in the systematic risk. In the short-run, however, investors may not have relevant information to gauge impacts of exogenous shocks on systematic risks of firms' assets including the existing cash-generating projects and potential growth options. In light of the unusual returns and high beta uncertainty, investors realize that their earlier beta estimates for the winners/losers might be too low/high. Although the event-induced uncertainty may be transitory, rational investors should respond by revising their systematic risk estimates. The beta adjustment for the winners and the losers creates momentum in stock returns.

This paper is aimed at a number of recent tests that challenge risk-based explanations.[1] First, Jegadeesh and Titman (2002) show that risk adjustment fails even when using average return to proxy for risk (see also Grundy and Martin (2001)). They show that the pre-ranking period average return on the winner portfolio is about the same as (slightly lower than) that on the loser portfolio. This suggests that for a convincing risk explanation, the focus should be on explaining variation of risk over the ranking period. That is, a successful story should explain why the winner stocks become riskier than the loser stocks over the ranking period. Second, Griffin, Ji, and Martin (2003) find that profits in both good and bad business cycle states are positive. This seems incompatible with momentum being a reward to priced business cycle risk. Third, Cooper, Gutierrez, and Hameed (2004) find that momentum profits depend on the previous overall stock market performance. They regard it as supportive evidence to over-reaction hypotheses that increases in market prices will result in greater aggregate overconfidence since investors in aggregate hold long positions in the stock market. Furthermore, Zhang (2005) find that momentum is particularly strong for stocks with high information uncertainty (also see Hong, Lim, and Stein (2000)). He uses several proxies for information uncertainty including firm size, age and analyst coverage. He concludes that the evidence supports behavioral hypotheses that psychological biases are increased when there is more uncertainty.

The simple extension of the CAPM that I propose can rationalize these puzzling findings.

[1] The following list does not include the long-term return reversal (e.g., see Jegadeesh and Titman (2001)). George and Hwang (2004, 2005) present evidence that momentum and reversal are unrelated phenomena, and that the reversal is better explained by taxes in a rational model.

At the qualitative level, the model is surprisingly successful. It explains why the risk measures of the winners and the losers change over the ranking period, generating predictions that are consistent with the test results mentioned above. My empirical results are also encouraging. I find that variation in the beta uncertainty can generate an impressive range of momentum payoffs. The observed level of the momentum payoff lies within the range. The required level of beta uncertainty for matching the momentum payoff is rather high, but it does not seem unreasonable. For instance, to match the payoff over the sample from 1934 to 2003, the average standard deviation that characterizes the beta uncertainty is 0.52 for the loser and 0.80 for the winner. After matching up with the level, I challenge the model with time-variation in the payoff to the momentum strategy. The tests reject that the variation in the winner's return is completely captured by the conditional beta (but no rejection for the loser). However, the extended CAPM is still effective in accounting for the market dependence result of Cooper, Gutierrez, and Hameed (2004). For the payoff adjusted by the model, the lagged market return no longer displays significant predictive power. These results challenge the behavioral theories to generate comparable quantitative predictions for the level and the time-variation of the momentum profits.

Several rationality-based explanations for momentum have been explored in recent years. In particular, theories proposed by Conrad and Kaul (1998), Berk, Green, and Naik (1999) and Johnson (2002) have attracted attention. Conrad and Kaul point out that cross-sectional variation in expected returns can possibly explain momentum. Berk, Green, and Naik show that momentum, along with several other anomalies, can be generated from the life-cycle variation of firms' endogenously chosen projects. Johnson shows that momentum can arise from a positive relation between expected returns and growth rates. While these theories are clearly insightful, they look pale in light of the recent empirical findings. Most notably, none of the existing risk-based theories is in a position to compete with behavioral explanations for the findings of Cooper, Gutierrez, and Hameed (2004) and Zhang (2005). The authors do not take into consideration any of the aforementioned risk explanations, reflecting how irrelevant the risk theories appear as for explaining the test results.

The results of my study also contribute to a growing literature on parameter uncertainty. Brav and Heaton (2002) and Lewellen and Shanken (2002) stress that parameter uncertainty can play an important role in explaining asset pricing anomalies. Brav and Heaton argue that due to their mathematical and predictive similarities, it is difficult to distinguish behavioral models and rational models with structural uncertainty. Lewellen and Shanken show that parameter uncertainty can drive a wedge between the distribution perceived by investors and the distribution estimated in empirical tests. Several other authors have investigated effects of parameter uncertainty on equilibrium pricing (e.g., Brown (1979), Barry and Brown (1985), Detemple (1986), Coles and Loewenstein (1988), and Coles, Loewenstein and Suay (1995)). With respect to these papers, the contribution of my paper comes from its focus on momentum. In my setting, parameter uncertainty arises from unpredictable exogenous shocks on the covariance structure of cashflows. The results suggest that this can be a useful channel to link popular factor-based pricing models to momentum.

There are a number of other related papers that center on either momentum or beta. To avoid distracting readers away from the main theme, I postpone the discussion of the papers to section 2.4. In sections 2.1 through 2.3, I discuss the motivation, present the theory, and report the empirical results. The concluding remarks are included in section 3.

2. Beta and Momentum

2.1 What if Momentum Is Rational?

There are several findings on momentum that jointly provide serious challenges to any risk-based explanation. I discuss implications of the empirical evidence under the assumption that momentum is rational. The goal is to highlight a set of constraints that one needs to keep in mind when searching for a rational explanation. I proceed with a diagrammatic approach, illustrating my discussion with Figures 1 and 2.

An obvious explanation for the failure of the factor-based pricing models is that the models are misspecified and in particular, some important risk factor is missing. However, the findings of Grundy and Martin (2001) and Jegadeesh and Titman (2002) suggest that the missing factor argument is unlikely to be able to explain the failure of the unconditional models. These authors find that the pre-ranking period average returns are about the same for the winners and the losers, indicating that the winners do not have higher unconditional risk measures than the losers. The finding is emphasized in Figure 1, where it is marked that the winners and the losers have the similar risk or risk measure before the ranking period. This is an important point, as it suggests that the key to a rational explanation should be the ranking period variation in the systematic risk measure.

$\boxed{\text{Figure 1 about here}}$

For momentum to be rational, risk would have to increase/decrease over the ranking period for the winners/losers. This is somewhat counter-intuitive, since one may feel it puzzling why risk would have to increase after good news (reflected by high positive returns of the winners). Still, it is possible that this pattern can be explained by a time-varying risk model. Such conditional models are typically motivated by co-variation between risk and business cycles of the economy. In this regard, Griffin, Ji, and Martin (2003) find that momentum profits in both good and bad business cycle states are positive. The paper is indicated in Figure 1 for the ranking period. It adds another challenge as the finding suggests that it would be hard to relate the required ranking period variation in risk to business cycles.

Moreover, it is even harder to explain Zhang's (2005) results by a standard rational expectations model, no matter whether risk is time-varying or not. Zhang finds that momentum profits depend on information uncertainty of the firms. If momentum is due to risk, this finding suggests that the difference between the risks of the winners and the losers depend on information uncertainty of these stocks. (Again, the finding is emphasized visually by Figure 1.) In a standard rational expectations model, however, information uncertainty does not play any role, as investors are assumed to know the structure and the parameters of the economy. Zhang's results suggest that rational models with parameter/structural uncertainty should be given more consideration if one looks for a rationality-based theory for momentum.

$\boxed{\text{Figure 2 about here}}$

Perhaps the most puzzling is the finding of Cooper, Gutierrez, and Hameed (2004) that momentum profits depend on the market states. This is illustrated in Figure 2. If momentum is due to risk, this result suggests that risks of the winners and the losers measured at the beginning of the holding period depend on the previous overall stock market performance. To rationalize the evidence, one needs to explain why the winners have higher/lower risk than losers when the previous market return is positive/negative. None of the existing risk-based theories seems to provide any clear intuition to understanding the dependence of momentum profits on the lagged market performance.

2.2 A Simple Theory

I present a beta-based theory for momentum. In my setting, the betas remain constant without parameter uncertainty for most of the time. For many stocks, the beta jumps occasionally, from one constant level to another.[2] Over time, the investors' perceived conditional probability of a jump in the current period is typically equal to zero or so small that the effect of a jump can be ignored. Occasionally, however, due to new information (e.g., revealed in a corporate event), investors perceive high probability that a large beta jump has occurred. At such a time, market equilibrium is achieved when investors determine the expected return or the discount rate by using the best available estimate of the beta, conditional on their information set. At a subsequent point, investors receive relevant information about the stock that resolves uncertainty about whether the beta has experienced a structural change and if so, what the new value of the beta is. Then, for the pricing of the stock, it goes back to the standard CAPM, the constant-beta version with no parameter uncertainty.

I proceed in two steps. First, I describe the storyline, which is visualized in Figure 3. Consider a stock labeled W. This is a winner stock that one would chase in momentum trading. The stock becomes a winner because of some event in the ranking period. The event is so forceful that it pushes up the price drastically, making the stock a winner among thousands of stocks. For the pre-ranking period, the stock is priced by the standard CAPM. In particular, there is no uncertainty about the beta such that the investors' beta estimate b_W is equal to the true value. In the ranking period, however, the winner-producing event induces high uncertainty about the systematic risk estimate. The event suggests a high probability that there may have been a large beta jump, but the investors do not yet have enough information to be certain about whether such a jump has happened and if so to pinpoint exactly the direction and magnitude of the jump. To the investors, the beta uncertainty is characterized by a distribution of beta, with mean b_W and variance $\sigma^2_{\beta_W}$ (that can be large). Given the combination of the high return r_W and the high beta uncertainty $\sigma^2_{\beta_W}$, in the absence of other information, the Bayesian rule suggests that there should be an adjustment of the beta estimate. Intuitively, the revised beta estimate h_W may be significantly higher than the initial estimate b_W, since b_W may seem too low in view of the high return on the winner. At the beginning of the holding period, investors use the revised beta estimate h_W

[2] Jumps in the systematic risk are due to exogenous shocks that impact the firm's existing projects and potential growth options.

to determine the expected return or the discount rate for the stock. The beta adjustment is the key to my argument.

Figure 3 about here

Next, I present details of the beta adjustment with a few equations. I start with a widely used CAPM equation for stock returns:

$$r_i = \beta_i r_m + \varepsilon_i, \tag{1}$$

for $i = 1, \cdots, n$. I focus on one time period, corresponding to the ranking period of a momentum strategy. For simplicity, I do not use time subscripts. All the notations in (1) are for the ranking period: r_i is the excess return (i.e., return in excess of the riskless rate) over the ranking period for the i-th of the n stocks, and r_m is the excess return on the market over the period. The coefficient β_i is the CAPM beta. It should be noted that without any further assumption, (1) is simply a generic expression. It always holds since one can define $\varepsilon_i \equiv r_i - \beta_i r_m$ for a given definition of the beta.

Before the ranking period, the standard CAPM holds. There is no beta jump and no parameter uncertainty such that for $i = 1, \cdots, n$, the beta estimate b_i is equal to the true beta of stock i for the pre-ranking period. Investors carry the beta estimates (b_i's) to the ranking period. Following a standard practice for discrete time models, I assume that beta adjustment and beta jumps may occur only at the beginning of each period. Investors do not know ex ante when a jump will occur. For the ranking period in particular, it is possible that the beta may jump for once at the beginning. Investors are aware of this, but at the start of the ranking period, their perceived probability of a jump is equal to zero.

The key assumption is that new information revealed around an important corporate event in the ranking period (an event that makes the stock a winner or a loser) induces high beta uncertainty. Due to the event, investors begin to suspect that there has been a large jump in the beta, but they do not yet have enough information to resolve the uncertainty. As a consequence, their confidence about their previous beta estimate is blown away. Specifically, investors summarize the beta uncertainty by a distribution

$$\beta_i \sim \mathcal{N}(b_i, \sigma^2_{\beta_i}), \tag{2}$$

where the variance $\sigma^2_{\beta_i}$ may be of a large value, capturing high uncertainty about the beta. The beta uncertainty does not have to exist for all the stocks. For explaining momentum, the uncertainty in (2) needs to be relevant for only the winners and the losers, and the standard CAPM may apply to the other stocks.

Within the context of (1), investor irrationality may be reflected by various theories on the structure of the residual ε_i. For example, if ε_i is positively serially correlated, which may arise from investor under-reaction to shocks, it is straightforward to show that the positive serial correlation can be a source of momentum profits. I aim at checking whether the beta part of equation (1) can create momentum.[3] For this purpose, I choose an extremely simplified structure for ε_i, assuming that conditional on the information set at the beginning of the ranking period, the error term is normally distributed

$$\varepsilon_i \sim \mathcal{N}(0, \sigma^2_{\varepsilon_i}), \tag{3}$$

and that ε_i is independent from r_m. Conditional on r_m, ε_i and β_i are uncorrelated. Both ε_i and β_i are cross-sectionally uncorrelated. I also assume that there is no parameter uncertainty about the variances $\sigma^2_{\varepsilon_i}$ and $\sigma^2_{\beta_i}$.

At the end of the ranking period, given the market return and the distribution of beta in (2), the conditional distribution of r_i is normal:

$$r_i|\ r_m \sim \mathcal{N}(b_i r_m, \sigma^2_{\beta_i} r^2_m + \sigma^2_{\varepsilon_i}). \tag{4}$$

Then, it is straightforward to obtain the expectation of beta conditional on r_i and r_m:

$$h_i \equiv E[\ \beta_i|\ r_m, r_i] = b_i + \frac{\sigma^2_{\beta_i} r_m}{\sigma^2_{\beta_i} r^2_m + \sigma^2_{\varepsilon_i}}(r_i - b_i r_m). \tag{5}$$

In the absence of other information, the conditional mean h_i is the best estimate of the beta (with respect to a quadratic loss function) at the beginning of the holding period.

Equation (5) is a Bayesian learning equation. It shows that one should update the beta estimate in response to the ranking period return (r_i) on the winner or the loser. On the one hand, the beta uncertainty plays an important role in determining the size of the adjustment from the prior estimate b_i to the new estimate h_i. If there exists no beta uncertainty (i.e., $\sigma^2_{\beta_i} = 0$), there is no adjustment at all. On the other hand, since the forecast of the return r_i is $b_i r_m$, the surprise in the realized return, $r_i - b_i r_m$, is also important. In times of positive market returns (i.e., r_m is positive), if $r_i - b_i r_m$ is positive and large, it is intuitive to infer that

[3]My goal is not to rule out possible theories that are based on ε_i. Rather, it is to show that it is possible to have an explanation that is not based on the residuals.

r_i and r_m are more highly correlated than thought before and thus raise the beta estimate. In contrast, given a positive observation of r_m, if the return shock $r_i - b_i r_m$ is deep in the negative zone, one would infer that r_i and r_m may be less highly correlated than thought before, and hence the systematic risk estimate should be adjusted downward.

For pricing the winners and the losers for the holding period, investors use the new beta estimates in (5). At the beginning of the holding period, the expected return or the discount rate for stock i (either a winner or a loser) is determined by $h_i \mu_m$, where μ_m is the expected excess market return. Apparently, due to the incomplete information structure, the determination of the expected holding period returns on the stocks in the momentum portfolios deviates from the standard CAPM. It may not be immediately evident to readers that the use of the estimate h_i is consistent with market equilibrium. In the Appendix, I show that this is indeed consistent with market equilibrium. Specifically, the conditional mean h_i in (5) is just equal to the beta from the conditional CAPM that is built on the investors' perceived conditional moments of stock returns.

To show that beta adjustment creates momentum, I simply use two stocks, W and L. Stock W has high return over the ranking period but stock L has low return. In other words, an event in the ranking period has made W a winner, while stock L becomes a loser due to some other event. To ease exposition and highlight the main points, I assume that both stocks have identical parameter values:[4]

$$b_W = b_L, \quad \sigma_{\beta_W} = \sigma_{\beta_L}, \quad \text{and} \quad \sigma_{\varepsilon_W} = \sigma_{\varepsilon_L}. \tag{6}$$

Even with the same parameter inputs, the two stocks have different beta estimates at the beginning of the holding period. By (5), the difference between the new beta estimates is

$$h_W - h_L = \frac{\sigma_\beta^2 r_m (r_W - r_L)}{\sigma_\beta^2 r_m^2 + \sigma_\varepsilon^2}, \tag{7}$$

where r_W and r_L are the ranking period excess returns for W and L, respectively, and σ_ε^2 and σ_β^2 are the common variance parameters for the two stocks. Let μ_m denote the conditional expected value of the excess market return at the beginning of the holding period. Then according to the revised betas, the conditional expected momentum payoff at the start of the holding period is

$$\pi_c \equiv (h_W - h_L)\mu_m = \frac{\sigma_\beta^2 r_m (r_W - r_L)\mu_m}{\sigma_\beta^2 r_m^2 + \sigma_\varepsilon^2}, \tag{8}$$

[4] As shown later in Table 2, the pre-ranking period parameters for the winner and the loser portfolios are rather similar. So the assumption (6) is not entirely unrealistic.

and the unconditional expected momentum payoff is

$$\pi_u \equiv E[\pi_c] = E\left[\frac{\sigma_\beta^2 r_m (r_W - r_L)\mu_m}{\sigma_\beta^2 r_m^2 + \sigma_\varepsilon^2}\right]. \tag{9}$$

The beta uncertainty σ_β^2 is a major determinant of the level of the momentum payoffs. On the one hand, both π_c and π_u are equal to 0 if $\sigma_\beta^2 = 0$, which corresponds to the case without beta uncertainty. On the other hand, variation in σ_β^2 can generate a range of values for the momentum payoffs (both π_c and π_u).

The payoffs (8) and (9) are plotted in Figure 4. Following the construction of Jegadeesh and Titman (1993), I use stock returns from 1927 to 2003 to construct a winner portfolio and a loser portfolio. With identical parameters for the two portfolios (see Figure 4 for details), I plot the conditional payoff π_c as a function of the market return r_m in Panel A and the unconditional payoff π_u as a function of the standard deviation σ_β in Panel B. The plot of the conditional payoff clearly shows that it is nonlinearly related to the market return. The plot of the unconditional payoff is encouraging as well. It shows that variation in σ_β can span an impressive range of possible values for π_u, with the maximum above 25 percent on the basis of six-month returns (not in the plot). In other words, the observed levels of the unconditional payoffs are within the range.[5]

Figure 4 about here

This extension of the CAPM proposed above can qualitatively rationalize a number of puzzling aspects of momentum. First of all, with respect to the finding of Jegadeesh and Titman (2002), it is shown by (6) through (9) that momentum payoffs can be generated even if the winner and the loser have identical risk measures for the pre-ranking period. It explains why the risk measure for the winner may rise after good news (and the opposite to the loser), and hence why momentum profits can arise even if the pre-ranking period average returns are about the same. Second, it is also shown by (6) through (9) that the return momentum arises without resort in any way to business cycle risk.[6] This provides a simple

[5] The theory does not impose restrictions on the value of σ_β. In principle, if the investors completely lose their confidence about the beta estimate, then σ_β can go to infinity. However, the theory is still refutable. For example, it would be rejected by the plot in Panel B of Figure 4 if the observed level π_{obs} is above the maximum (outside the spanned range). This is discussed in more details in the next subsection.

[6] The use of identical parameter inputs in (6) through (9) also shows that the model can generate momentum without any resort to cross-sectional differences in expected returns (see Conrad and Kaul (1998) and Jegadeesh and Titman (2002)).

way to circumvent the puzzling finding of Griffin, Ji, and Martin (2003) that momentum profits are positive in bad business cycle states. Third, higher beta uncertainty leads to more beta adjustment and hence stronger momentum profits. This is clearly consistent with Zhang's (2005) finding, if beta uncertainty and information uncertainty are positively correlated. Finally, the conditional payoff π_c is dependent on the market return. It predicts that the average payoff following the positive lagged returns ($r_m > 0$) should be positive and the average payoff following the negative lagged returns ($r_m < 0$) should be negative. As equation (8) and Figure 4 show, the payoff's dependence on the lagged market return is nonlinear: π_c increases as r_m goes up within certain range but decreases with r_m when r_m is beyond certain levels. This is consistent with the pattern that Cooper, Gutierrez, and Hameed (2004) have identified empirically.

Finally, two remarks are in order. First, I have tried to keep things as simple as possible. Apparently, there are numerous potential extensions or variations that one could consider. Most notably, the assumption that no beta uncertainty is perceived at beginning of the ranking period is good for transparency of the theory, but it may be relaxed. Nonetheless, as shown in the empirical analysis below, it is the beta uncertainty at the end of the ranking period that is critical to my results. In other words, it does not matter empirically whether the initial beta uncertainty is substantial or trivial, as long as beta uncertainty is inflated to a high level by the event in the ranking period. Second, uncertainty about the systematic risk estimate is transitory in my setting. The extended model essentially deals with two periods: the ranking and the holding periods. It is assumed that at some point after the holding period, investors receive all the relevant information that resolves the beta uncertainty. Then the CAPM holds with the true beta for the stock. The theory has no prediction about the value of the beta after the holding period.

2.3 An Empirical Analysis

2.3.1 Momentum Payoffs

The data is obtained from CRSP. I use monthly return data from NYSE and AMEX to construct momentum portfolios. Following many studies, I focus on the case in which both the ranking and holding periods have the length of six months. Only NYSE and AMEX stocks contained on the CRSP monthly tape throughout the ranking periods are eligible for selection as winners or losers. Recall that the momentum strategy of Jegadeesh and Titman (1993) includes portfolios with overlapping holding periods. In any given month t, the strategy holds five portfolios that are constructed as follows. For each of the months $t-5$ through $t-1$, stocks are ranked on the basis of the past six-month returns up to the given month.[7] The stocks in the highest return decile are equally-weighted to form a portfolio. The five portfolios are then equally-weighted, each with the weight $1/5$, to form the winner portfolio of Jegadeesh and Titman. The loser portfolio is constructed in the same way, except that stocks in the lowest return decile are used in each step. The momentum strategy is the one that buys the winner portfolio and short-sells the loser portfolio.

Table 1 reports the average monthly return for the winner portfolio, the loser portfolio, and the momentum strategy. The sample period is from 1927 to 2003. Four subsamples are considered. First, the sample is divided in the middle, creating the 1927-1964 and the 1965-2003 subsamples. Then the latter subsample is divided again. The 1965-1989 interval is the sample period of Jegadeesh and Titman (1993), and thus the 1990-2003 interval is the out-of-sample period with respect to the seminal study.

$\boxed{\text{Table 1 about here}}$

Consistent with Jegadeesh and Titman (1993), the winner-loser return difference for the 1965-1989 period is impressive, at about one percent per month on average. However, the payoffs are weaker for the pre- and post-samples, which are 0.37 percent for the 1927-1964 period and 0.19 percent for the 1990-2003 period. The average payoff for the overall 1927-2003 sample is 0.51 percent per month, or about 3 percent in terms of semi-annual performance. This magnitude of the payoff for the long sample is consistent with those reported in previous studies (e.g., see Grundy and Martin (2001) and Chordia and Shivakumar (2002)). I have also considered the non-overlapping approach that all the stocks in the momentum portfolios are selected by ranking the past six-month returns from the current month. The average six-month holding period payoff of the momentum strategy is 2.90 percent.[8] In view of these numbers, I focus my following discussion on the case that the unconditional payoff level is three percent (i.e., $\pi_u = 0.03$) in terms of the six-month holding period return.

[7] The holding period starts one month after the ranking period. Skipping one month is a common practice to mitigate bid-ask bounce effects.

[8] I have replicated the CAPM and the Fama-French three factor regressions for momentum. The results are similar to those reported in the literature. The robustness-checking results are omitted.

2.3.2 Pre-ranking Statistics

It is useful to have a look at the pre-ranking period betas and other parameters of stocks in the winner and the loser portfolios. The pre-ranking period estimates also provide necessary inputs for the beta adjustment in (5). I use a five-year rolling window for estimation. At the beginning of the ranking period, monthly returns from last five years are obtained for each stock in the winner and the loser portfolios. (Stocks with fewer than 18 monthly observations are removed.) First, the mean (μ) and standard deviation (σ) of the excess return over the window are computed for each stock. Then, for each of the stocks, the excess return is regressed on the market excess return over the window. The regression estimates include the beta coefficient (b), the standard error of the beta coefficient (s_b), and the standard deviation of the regression residual (σ_ε). The means of these estimates over the 1927-2003 sample are reported in Panel A of Table 2. I also compute descriptive statistics for the cross-sectional distributions of b, σ, and ρ, where ρ is the correlation between the excess returns on the stock and the market. Panel B of Table 2 reports the time series averages of mean, standard deviation, maximum and minimum for the cross-sectional distributions.

Table 2 about here

The winner and the loser are rather similar in terms of the pre-ranking period statistics. This is perhaps the most impressive feature of Table 2. Panel A shows that both the winner and the loser have similar average monthly excess return (μ) and standard deviation (σ), both of which are rather high relative to the mean and standard deviation (0.0064 and 0.055, respectively) of the market excess return. The regression results are also rather similar between the winner and the loser. It should be emphasized that both the average return and the beta estimate are slightly lower for the winner, which is consistent with the result of Jegadeesh and Titman (2002). This suggests that there is no hope for unconditional models in which risk measures remain constant over time. Panel B shows that the winner and the loser are also very similar in terms of the cross-sectional distributions.[9]

Most of the stocks in the winner and the loser portfolios do not resemble the market. First, the stocks typically do not closely co-vary with the market. Panel B shows that the

[9] The similarity in the pre-ranking period statistics is quite a contrast to the difference in the ranking period returns. The winner's and the loser's average returns over the six-month ranking period are 72.85% and −33.80%, respectively.

correlations between their excess returns and the market excess return are below 0.5 on average. Second, most of the stocks are much more volatile than the market. The average standard deviation of their excess returns is more than twice as large as that of the market. Both the correlation and the standard deviation are disperse across stocks in the winner and loser portfolios. Consequently, the pre-ranking period betas are quite disperse across these stocks. The cross-sectional standard deviation of the beta is about 0.6, with the maximum beta around 3.5 and the minimum around -0.3.

2.3.3 Up against Momentum

To put the model up for an empirical analysis, one needs to estimate the beta uncertainty σ_{β_i}. For this purpose, one may simply assume that σ_{β_i} is constant. However, beta uncertainty is unlikely to be constant across stocks and over time. I use the following specification

$$\sigma_{\beta_i} = \lambda \left| \frac{r_i - b_i r_m}{\sigma_{\varepsilon_i}} \right|, \tag{10}$$

where λ is a constant (across stocks and over time), b_i is the pre-ranking period beta estimate, σ_{ε_i} is the standard deviation of the regression residual, r_i and r_m are the ranking period excess returns on the stock and the market, respectively.

Using (10), I aim to proxy for uncertainty induced by the events that generate the winner stocks and the loser stocks in the ranking period. Note that $(r_i - b_i r_m)/\sigma_{\varepsilon_i}$ is a "standardized abnormal return" (SAR), which is widely used in the event-study literature. Here it serves as an ex post general proxy for all kinds of surprises for the stocks. By (10), an event that gives investors a bigger surprise will induce higher beta uncertainty. Apparently, (10) allows for cross-sectional differences and time-variation in σ_{β_i}.

Several alternatives to (10) have been checked, including $\sigma_{\beta_i} = \lambda|\mathrm{SRS}_i|$, $\sigma_{\beta_i} = \lambda|\mathrm{SRS}_i|s_{b_i}$, and $\sigma_{\beta_i} = \lambda|\mathrm{SRS}_i| + s_{b_i}$, where $\mathrm{SRS}_i = (r_i - \mu_i)/\sigma_i$, μ_i and σ_i are the pre-ranking period mean and standard deviation of the excess return, and s_{b_i} is the standard error of the pre-ranking period beta estimate. The variable SRS_i is a"standardized return surprise" (SRS), which is similar in spirit to SAR. I have also considered replacing s_{b_i} by σ_i and/or replacing SRS by SAR in the above cases. (Results for one of the above alternatives are reported in Panel B of Table 3.) The point for including s_{b_i} or σ_i is that after controlling for effects of the events, firms may still have different degrees of transparency in terms of information available for measuring the systematic risks. I find that in terms of the required beta uncertainty for matching the observed level of the momentum payoff, the averages of the standard deviation σ_{β_i} for the winners and the losers are similar across the specifications.

With inputs from the pre-ranking period regressions and the uncertainty estimate (10), I now challenge the extended CAPM with the momentum portfolios. I proceed in two steps. First, I check whether the model can match the observed level of the momentum payoff. The results are presented in Table 3 and Figure 5. Second, I check whether the model can explain time-variation in the momentum profits. The results are reported in Table 4.

The model could be easily rejected, if the observed level of the momentum payoff is above the range of the payoffs implied from the model. However, this is not the case. In principle, investors can be extremely uncertain about the beta estimates such that $\sigma_{\beta_i} \to \infty$. For the unconditional payoff π_u in terms of the six-month holding period returns, I find that by varying λ in (10), the maximum level of the payoff generated by the model is above 0.25 (or 25%). Recall from Table 1 that the observed level of π_u is about 0.03 (or 3%) for the sample from 1927 to 2003. Table 3 reports the required beta uncertainty for matching up with different payoff levels. As seen from Table 1, the momentum payoffs for different sample periods range between one percent and six percent on the basis of the semi-annual performance. To match each level of the unconditional payoff π_u from 1% to 6%, I identify the corresponding value of λ by grid-search. The time series average of the beta uncertainty σ_β is reported for both the winner and the loser for five different periods.

Table 3 shows that the required level of beta uncertainty is high. Panel A of the table corresponds to the SAR specification (10). For the six percent payoff level ($\pi_u = 0.06$), the average σ_{β_W} ranges from 1.14 to 2.55, which does seem extreme. However, for $\pi_u = 0.03$, the magnitude of beta uncertainty does not seem unreasonable. Recall again (see Table 1) that the payoff level is at about three percent, i.e., $\pi_u = 0.03$, for the whole 1927-2003 sample. The six percent payoff from the 1965-1989 period of Jegadeesh and Titman (1993) is the exception rather than the rule. For $\pi_u = 0.03$, we have $\sigma_{\beta_W} = 0.96$ and $\sigma_{\beta_L} = 0.61$ for the whole sample. The first few years at the beginning of the sample (from late 1920's to early 1930's) are a period of extremely volatile markets. I will focus on the 1934-2003 period in the following discussion. Omitting the first eight years, the numbers are smaller such that $\sigma_{\beta_W} = 0.80$ and $\sigma_{\beta_L} = 0.52$ on average for the 1934-2003 period.

$\boxed{\text{Table 3 about here}}$

Results in Panel B are based on an alternative to (10) that specifies $\sigma_{\beta_i} = \lambda|\text{SRS}_i|s_{b_i}$. As noted above, this specification aims to proxy for the event-induced uncertainty by SRS_i and the firm's information transparency of its risk measure by s_{b_i}. As the table clearly shows, the results are similar between the two panels. For example, we have $\sigma_{\beta_W} = 0.79$ and $\sigma_{\beta_L} = 0.55$ on average for the 1934-2003 period.

To get a sense of the magnitude of the beta uncertainty, consider a numerical example. Suppose that the initial beta estimate of a stock is 1.3. After event, investors summarize the uncertainty by a distribution such that the beta may take any of the five values, 0.1, 0.7, 1.3, 1.9, and 2.5, with equal probability. In this case, the standard deviation is equal to 0.85. Instead, if the values are 0.3, 0.8, 1.3, 1.8, and 2.3, then $\sigma_\beta = 0.71$. This example suggests that the required beta uncertainty is rather high, but they do not seem too extreme. Of course, it is a difficult issue to judge what the range of values for σ_β is reasonable since the model imposes no restriction on the level of the beta uncertainty.

Figure 5 highlights that the extended CAPM generates momentum from beta adjustment. The figure plots the revised betas h_W and h_L that correspond to the columns under the 1934-2003 period in Table 3. If there is no beta uncertainty (i.e., $\lambda = 0$), the betas are equal to the pre-ranking period estimates . As shown in the plot, the pre-ranking period betas are similar, with the winner's being a bit lower. As the uncertainty increases (or λ increases), the beta of the winner diverges from that of the loser. The figure clearly shows that most of the payoff comes from the rise in the winner's beta. The loser's beta does not change as much with respect to the pre-ranking period estimate. For example, to match the three percent payoff level (i.e., $\pi_u = 0.03$), the winner's beta h_W jumps up from 1.26 to 1.71, but the loser's beta drops only a bit, from 1.33 to 1.18. Similarly, for the case of $\pi_u = 0.06$, most of the momentum payoff comes from the winner's beta adjustment.

$\boxed{\text{Figure 5 about here}}$

A tough test of the model is to put it up against time-variation in the momentum profits. Like in Figure 5, I plug the pre-ranking period estimates (b and σ_ε) into (10) and then (5) to obtain the time series of the betas h_W and h_L for the winner and the loser. There is one free parameter, λ, in (10). I remove this degree of freedom by letting λ take the value so that $\pi_u = 0.03$. In other words, I first select λ to match the observed payoff level, and then challenge the model against time-variation in the payoff.

I perform a simple test that consists of two regressions. I regress first $\tilde{r}_W$ on $h_W \tilde{r}_m$ and then $\tilde{r}_L$ on $h_L \tilde{r}_m$, where $\tilde{r}_W$, $\tilde{r}_L$, and $\tilde{r}_m$ are the holding period excess returns on the winner, the loser, and the market, respectively.[10] I report the two regression results in Panels A1 and A2 of Table 3. The idea is that if the intercept and the slope equal to 0 and 1 in the two regressions of Panels A1 and A2, then it provides evidence that the following conditional CAPM equations are well specified

$$\tilde{r}_W = h_W \tilde{r}_m + \tilde{\varepsilon}_W, \quad (11)$$

$$\tilde{r}_L = h_L \tilde{r}_m + \tilde{\varepsilon}_L. \quad (12)$$

The results in Panel A1 are not in favor of (11), but those in A2 are supportive to (12). For the winner, the estimates $c_{W,0}$ and $c_{W,1}$ are significantly different from 0 and 1. In particular, the slope $c_{W,1}$ is around 0.68. The hypothesis that $c_{W,1} = 1$ is clearly rejected, given that the standard error for the slope is about 0.03. Thus, it is statistically rejected that time-variation in the winner's holding period return is completely captured by (11). For the loser, however, the regression estimates $c_{L,0}$ and $c_{L,1}$ are close to 0 and 1, respectively. Since the standard error for the slope is 0.08, the estimate $c_{L,1}$ is within two standard errors from 1. Thus, (12) is not rejected by the test.

In spite of the rejection, the extended CAPM may still capture important features of the time-variation in the payoff. I proceed to check if the model can account for the predictive power of the lagged market return documented by Cooper, Gutierrez, and Hameed (2004) (hereafter CGH). The results of CGH may be summarized by the following two predictive regressions:

$$\tilde{\pi} = \eta_0 + \eta_1 I(\text{LAGMKT} > 0) + \varepsilon \quad (13)$$

$$\tilde{\pi} = \eta_0 + \eta_1 \text{LAGMKT} + \eta_2 \text{LAGMKT}^2 + \varepsilon \quad (14)$$

where $\tilde{\pi}$ is either the raw payoff (i.e., $\tilde{r}_W - \tilde{r}_L$) or an adjusted payoff (adjusted by some

[10] For the regression tests in Table 4, the conditional betas for the winner and the loser are obtained as follows. Recall that for a given month t, the winner (or the loser) portfolio is a mix of five portfolios for the months $t-5$ through $t-1$. For each of the five portfolios, its beta is computed by (10) and (5). Then the beta of the winner (or the loser) is set to be the equally-weighted average of these five betas.

asset pricing model), and LAGMKT is the three-year lagged market return.[11] The first is a dummy variable regression on $I(\text{LAGMKT} > 0)$ that is equal to 1 if LAGMKT > 0 but 0 otherwise. The second is a quadratic regression on LAGMKT and LAGMKT^2.

Table 4 about here

I consider four cases and present them in Panels B1 through B4 of Table 4. The dependent variable for the regressions (13) and (14) in Panel B1 is the raw payoff, or the difference between the holding period returns on the winner and the loser.. In Panel B2, the dependent variable $\tilde{\pi}$ is the payoff adjusted by the unconditional CAPM, where the model's predicted payoff, $\tilde{\pi}_{\text{CAPM}} = b\tilde{r}_m$, is obtained from the regression $\tilde{\pi}_{\text{raw}} = a + b\tilde{r}_m + \varepsilon$. In Panel B3, the dependent variable $\tilde{\pi}$ is the payoff adjusted by the conditional CAPM. By (11) and (12), the model implied variation in the payoff is $h_W\tilde{r}_m - h_L\tilde{r}_m$. The adjusted payoff is the difference between the raw payoff and the implied payoff (which is equal to the difference between the abnormal returns on the winner and the loser). That is, the adjusted payoff is the part of the variation that is not implied by the model. In Panel B4, I modify the adjustment in B3 by using the regression estimates $c_{W,1}$ and $c_{L,1}$ from Panel A. This is to remove only the part of the return variation captured by the two regressions in A1 and A2.

The results show that the extended CAPM is effective in explaining the dependence of the payoff on the lagged market performance. On the one hand, the results in Panels B1 and B2 are consistent with those of CGH. For the raw payoff, the dummy regression shows that momentum depends on the market states. The average monthly payoff following negative lagged market performance is -0.98%. In contrast, the average monthly payoff following positive lagged market performance is 0.81%. The quadratic regression further shows that momentum profits depend in a nonlinear manner on the lagged market return. The coefficients for LAGMKT and LAGMKT^2 are significantly positive and negative, respectively. This implies that the payoff increases with LAGMKT within certain range of the lagged market return, but it decreases with LAGMKT outside the range. Panel B2 shows that the unconditional CAPM cannot explain the predictive ability of LAGMKT. The results for

[11] CGH use overlapping holding period returns. That is, the dependent variable is the six-month holding period payoff that is measured monthly. This introduces high autocorrelation into the dependent variable. Follow the common practice, I use the strategy of Jegadeesh and Titman (1993) that avoids the overlapping in the payoff. I find that the regression R^2's are lower, but otherwise my results are consistent with those of CGH.

both of the regressions remain essentially unchanged when using the adjusted payoff from the unconditional CAPM. On the other hand, it makes quite a difference to adjust the payoff by the conditional CAPM. The results in Panels B3 and B4 are in sharp contrast to those in Panels B1 and B2. The loadings on $I(\text{LAGMKT} > 0)$, LAGMKT and LAGMKT^2 are much smaller and statistically insignificant according to the t-statistics. The results in Panels B3 and B4 lead to the same conclusion that for the payoff adjusted with the conditional betas, the lagged market return no longer display significant predictive power for time-variation in the momentum profits.[12]

These results raise an interesting challenge to behavioral models. For time-variation in the momentum payoff, predictions from the existing behavioral models are rather loose even at the qualitative level. In particular, by the overconfidence and other behavioral theories, it is not very clear why the momentum profits should be nonlinearly related to the lagged overall stock market performance. CGH have discussed potential explanations. They note that overconfidence theory does not necessarily predict a fully monotonic relation between the lagged market returns and the level of overconfidence. Yet predictions from the overconfidence theory are difficult to quantify; they are still ambiguous such that we cannot put them in tests like those in Table 4. It remains to be seen whether further development on behavioral theories can give rise to quantitative predictions so that we can conduct tests to effectively compare competing models.

2.4 Related Work

A number of empirical studies have explored non-behavioral explanations for the momentum. Harvey and Siddique (2000) study skewness and asset returns. They find positive results. Although skewness may be important for asset pricing in general, it remains to be articulated why momentum arises from skewness. Pastor and Stambaugh (2003) and Sadka (2004) find that liquidity risk factors may help explain momentum. The authors are rather cautious in drawing conclusions, since it is intuitively unclear why winners should have higher liquidity risk. Chordia and Shivakumar (2002), Wu (2002) and Wang (2003) find encouraging results

[12] When the payoff is adjusted by the Fama-French three factor model, the results are similar to those in B1 and B2. For robustness-checking, I repeat all the tests in Table 4 with the specification used for Panel B of Table 3. The results based on the alternative specification are similar. In addition, I have tried different choices of LAGMKT, including six-month lagged market return and three-year lagged excess market return. The results based on these choices are even more in favor of the conditional beta model.

when they incorporate conditioning variables to allow for time-varying risk. But again, the intuition is weak, and there is concern about whether the conditioning variables have genuine predictive power. Finally, Korajczyk and Sadka (2004) and Lesmond, Schill, and Zhou (2004) study whether transaction costs can prevent arbitrage in momentum trading, but they reach different conclusions. In particular, Korajczyk and Sadka conclude that momentum remains a puzzle after taking transaction costs into account.

Recently, beta also continues to gain attention. Campbell and Vuolteenaho (2004) use a beta decomposition to shed light on the size and the value anomalies. Jostova and Philipov (2004) propose a stochastic beta process and find that it helps explain the size and book-to-market effects. Lewellen and Nagel (2005) focus on cyclical variation between beta and the market risk premium. They test the conditional CAPM by estimating beta from short-window regressions. Adrian and Franzoni (2005) focus on low frequency variation in beta over a long time horizon. They emphasize the role of learning and implement a Kalman filter to extract the factor loading. While they all provide intriguing insights, these papers differ from mine in two important ways. First, I focus on momentum and my target is on an intuitive beta-based explanation for a number of puzzles on the anomaly. Second, the key to my approach is the ranking period event-induced beta uncertainty. The other papers are aimed at the entire process (over the sample period) of time variation in beta. At any given point in time, however, most stocks do not experience drastic events that make them either winners or losers. It is the ranking period variation in the betas of the winners and the losers that is critical to my results.

Broadly speaking, the role of beta uncertainty may be motivated by the observation that betas are time-varying but difficult to estimate. Although there exists a large literature on time-varying beta, little is known about how to precisely capture time-variation in the risk measure (e.g., Ghysels (1998)). On the other hand, uncertainty about time-varying beta is important in equity valuation. For example, using industry portfolios, Fama and French (1997) find that there is strong variation in the CAPM betas and their three factor loadings such that the cost of equity estimates from their full sample (1963-1994) regressions are no more accurate than the estimates from regressions that use only the latest three years of data. In a Bayesian framework, Pastor and Stambaugh (1999) also emphasize the importance of uncertainty about betas in estimating the cost of equity for individual firms.

3. Conclusion

Anomalies against the CAPM attract great attention. Numerous researchers have defended the premier asset pricing model. For example, Jagannathan and Wang (1996) show that a conditional version of the CAPM performs much better than the unconditional CAPM. Gomes, Kogan, and Zhang (2003) recently show, in a general equilibrium setting, that a conditional version of the CAPM can explain the size and the book-to-market effects. However, momentum seems deadly. The momentum strategy is profitable and easy to implement. But tests find that the winner and the loser portfolios are rather similar in terms of various pre-ranking period statistics. In particular, the pre-ranking period beta estimate and the pre-ranking period average return of the winner are even slightly lower than those of the loser. Other tests add more challenges. It is particularly puzzling that momentum profits depend on firms' information uncertainty and the lagged overall market performance. Jointly, these test results seem to leave little room for a rationality-based explanation.

In this paper, I provide a counter example, showing that many of the test results are consistent with a beta story that does not require irrationality. I show that the puzzling features of momentum point to a "rational model with structural uncertainty," if one insists on searching for a rationality-based explanation. I propose an extension of the CAPM that incorporates an incomplete information structure. In this model, the beta jumps occasionally, but investors do not know when there is a jump and they do not have the information to immediately resolve the uncertainty after they suspect that there has been a large beta jump. At the qualitative level, the extension is fruitful, providing a theory that can rationalize various puzzling findings. In the empirical analysis, I find that the size of beta uncertainty is critical. With enough beta uncertainty, the extended CAPM can match the observed level of the momentum payoff. The results show that the required level of beta uncertainty is rather high, but it does not seem unreasonable. However, this is a gray zone, since we do not have any theoretical prediction on the level of beta uncertainty. This is a common weakness of rational structural uncertainty models that do not impose restriction on the degree of uncertainty. However, we should not be harsh on these models, as the competing behavioral models also have such ambiguity in forming testable hypotheses.

Appendix. Consistency with Equilibrium

In the setting outlined in section 2.2, investors face an incomplete information structure. After they suspect that there has been a large beta jump, they may need to wait for relevant information that resolves the uncertainty. In particular, for the winners and the losers at the beginning of the holding period, such information is not available. In the one-period setting (for the holding period), investors assume that the CAPM equation holds (i.e., $\tilde{r}_i = \beta_i \tilde{r}_m + \tilde{\varepsilon}_i$)[13] for these stocks. The beta is the only parameter that investors do not know the true value. For the pricing of the stocks at the start of the holding period, they use the best estimate of the beta, conditional on their information set at the time.

Is it consistent with market equilibrium to use h_i in (5) to determine the expected return? For a simple argument, I assume that investors have quadratic utility functions. Under this assumption, it is transparent that with and without complete information, the CAPM holds under the return distribution perceived by investors. In Bayesian terms, the subjective distribution is called the predictive distribution. Given a distribution that summarizes the beta uncertainty, investors use $\tilde{r}_i = \beta_i \tilde{r}_m + \tilde{\varepsilon}_i$ to form their perceived first and second moments of asset returns, conditional on the information set. In other words, investors hold mean-variance efficient portfolios with respect to the subjective moments, which gives rise to an asset market equilibrium that is characterized by the conditional CAPM. The goal is to show that the systematic risk measure from this conditional CAPM is equal to the conditional mean of the factor loading (i.e., h_i in section 2.2).

For the derivation below, there is no need to require that the distribution of any variable be normal. It is sufficient to assume, conditional on the information set, that β_i and $\tilde{r}_m$ are independent, and that $\tilde{\varepsilon}_i$ and $\tilde{r}_m$ are uncorrelated. Then, it follows easily that the beta measure from the conditional CAPM is

$$\begin{aligned} \frac{\operatorname{cov}(\tilde{r}_i, \tilde{r}_m)}{\operatorname{var}(\tilde{r}_m)} &= \frac{\operatorname{cov}(\beta_i \tilde{r}_m + \tilde{\varepsilon}_i, \tilde{r}_m)}{\operatorname{var}(\tilde{r}_m)} = \frac{E(\beta_i \tilde{r}_m^2) - E(\beta_i \tilde{r}_m) E(\tilde{r}_m)}{\operatorname{var}(\tilde{r}_m)} \\ &= \frac{E(\beta_i)[E[\tilde{r}_m^2] - [E(\tilde{r}_m)]^2]}{\operatorname{var}(\tilde{r}_m)} = E(\beta_i) = h_i. \end{aligned} \tag{15}$$

Results like (15) have been shown in different settings in the literature. For example, in a production economy with incomplete information, Detemple (1986) shows that standard

[13] As for notations, $\tilde{x}$ is used to denote a ranking period variable (e.g., $\tilde{r}_i$ or $\tilde{\varepsilon}_i$). In contrast, x denotes a variable known by or before the beginning of the holding period (e.g., r_i or h_i). All of the moments in this appendix are conditional upon the information set at the beginning of the holding period.

equilibrium results in asset pricing can be obtained by using a substitute state vector that is the conditional mean of the underlying unobservable state variables. See also Brown (1979) for a similar result in the unconditional setting.

Finally, it should be noted that the assumption of quadratic preferences is sufficient but not necessary for the conditional CAPM. For example, an alternative assumption is that β_i, $\tilde{r}_m$, and $\tilde{\varepsilon}_i$ are independently normally distributed for the winners and the losers. For stocks that are neither winners nor losers, β_i is a known constant, $\tilde{r}_m$ and $\tilde{\varepsilon}_i$ are independently normally distributed. (Of course, the conditional mean of $\tilde{\varepsilon}_i$ is assumed to equal to zero.) Assume that a representative investor's utility function u is twice differentiable. Then, it follows from the first-order condition

$$E[u'(\tilde{r}_m)\tilde{r}_i] = 0 \tag{16}$$

that the conditional expected return $E(\tilde{r}_i)$ satisfies

$$E(\tilde{r}_i) = -\frac{1}{E[u'(\tilde{r}_m)]}\mathrm{cov}[u'(\tilde{r}_m), \tilde{r}_i] = -\frac{E[u''(\tilde{r}_m)]}{E[u'(\tilde{r}_m)]}\mathrm{cov}(\tilde{r}_m, \tilde{r}_i). \tag{17}$$

Thus, the conditional expected return $E(\tilde{r}_i)$ is proportional to the conditional covariance $\mathrm{cov}(\tilde{r}_i, \tilde{r}_m)$, giving rise to the conditional CAPM. The last equality in (17) can be easily derived from an extension of the Stein's lemma, which states that $\mathrm{cov}[g(x), xy] = E[g'(x)](x, xy)$ for any two independent normal variables x and y.

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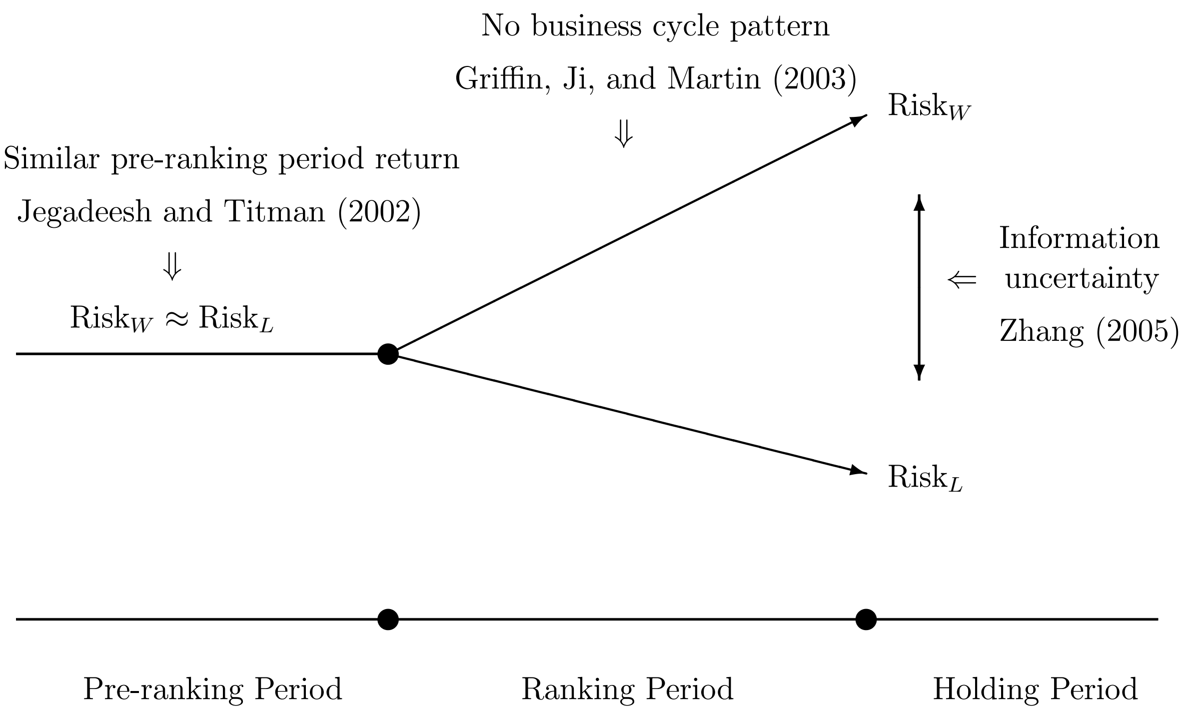

Figure 1. Empirical Challenges to a Risk Explanation

This diagram is for illustrating implications of the findings of Jegadeesh and Titman (2002), Griffin, Ji, and Martin (2003), and Zhang (2005), under the assumption that momentum is rational. Risk_W and Risk_L are the systematic risk measures of the winner and the loser portfolios, respectively. The winners or the losers are the stocks that have the highest or the lowest returns over the ranking period, respectively.

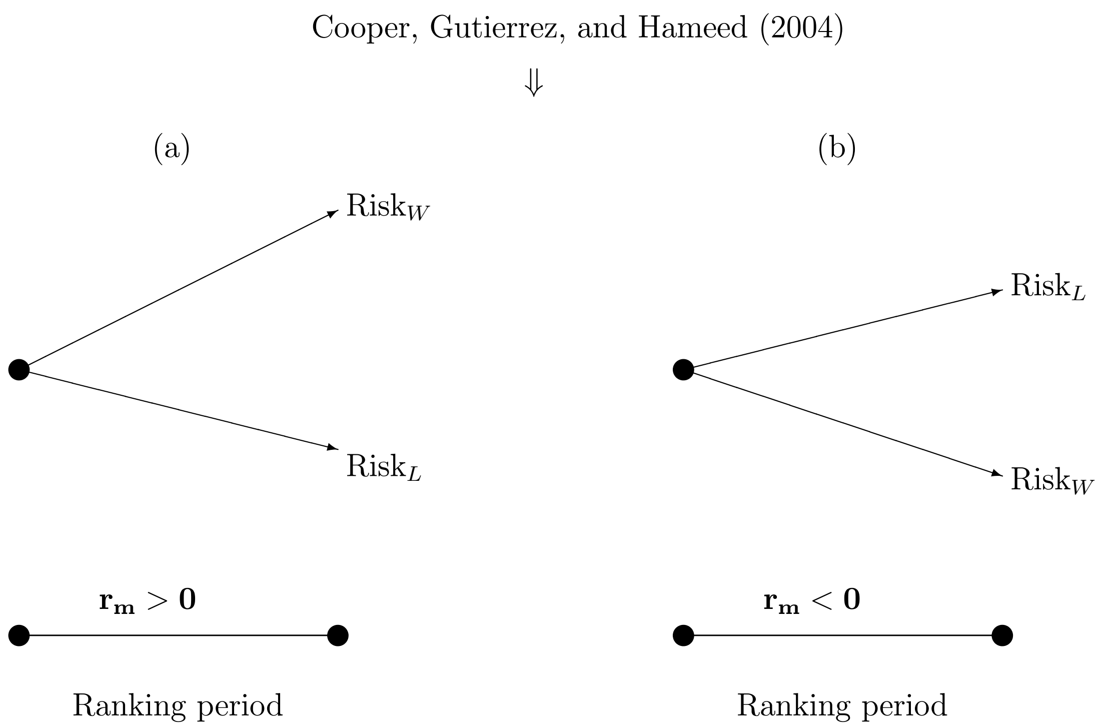

Figure 2. Market States and Momentum

This diagram is for illustrating implications of the empirical evidence documented by Cooper, Gutierrez, and Hameed (2004), under the assumption that momentum is rational. Risk$_W$ and Risk$_L$ are the systematic risk measures of the winner and the loser portfolios, respectively, and r_m is the market return over the ranking period.

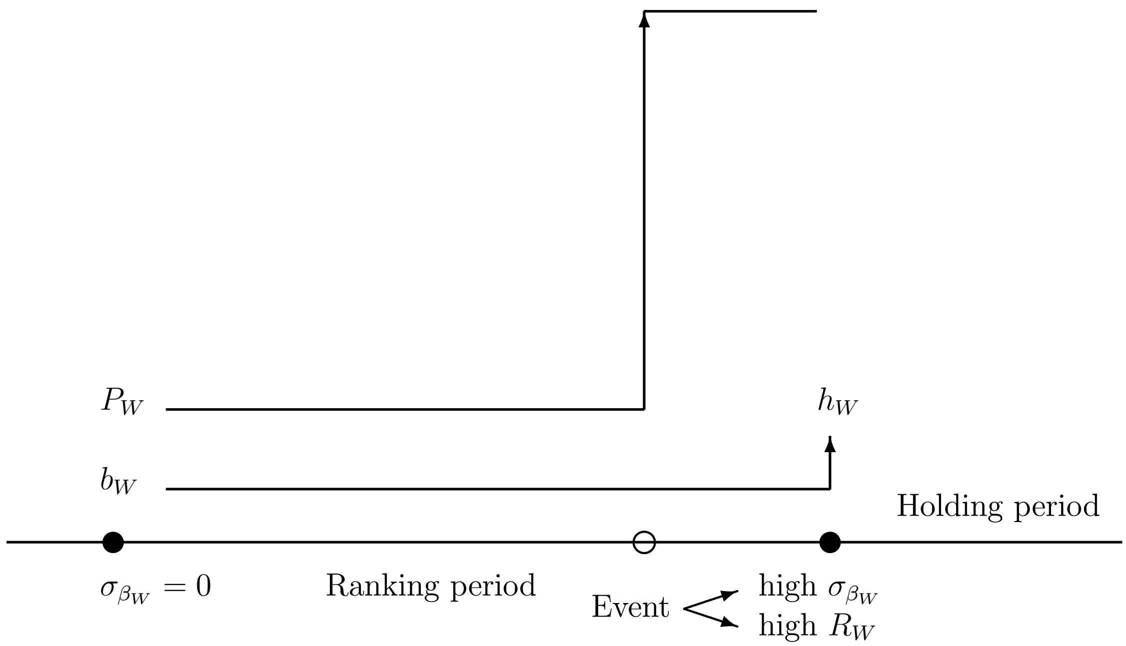

Figure 3. Beta Uncertainty for a Winner

This diagram is for illustrating the storyline. The key point is that an event that is drastically making the stock a winner may also inflate uncertainty about the systematic risk measure of the stock. P_W and R_W are the price and the return for stock W, respectively. The beta estimate of this stock at the beginning of the ranking period is b_W. At the beginning of the ranking period, the perceived probability of a beta jump is equal to zero, so that there is no uncertainty about beta ($\sigma_{\beta_W} = 0$). The event induces high beta uncertainty. Investors summarize the uncertainty by a distribution of beta, which has high standard deviation σ_β. The adjusted beta estimate at the beginning of the holding period is h_W.

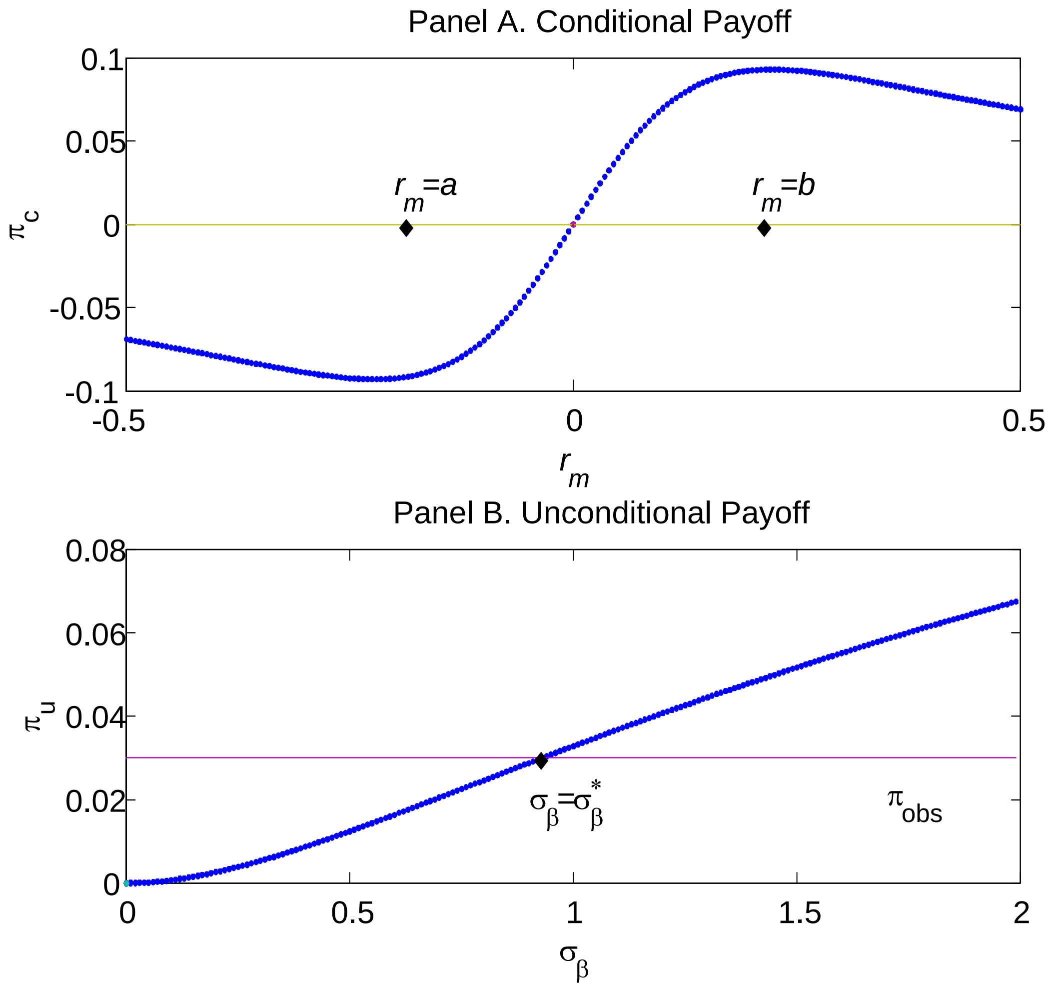

Figure 4. Momentum Payoffs from the Extended CAPM

Panels A and B provide plots for illustrating equations (8) and (9), respectively. Both the conditional and the unconditional payoffs, π_c and π_u, are computed from the momentum strategy with both the ranking and the holding periods being of six months in length. The NYSE and AMEX monthly return data from 1927 to 2003 are used. The standard deviation σ_ε is set to equal to 0.2. The market excess return forecast μ_m is replaced by the time series mean over the sample. For Panel A, $r_W - r_L$ is set to be its time series mean. The observed unconditional payoff level π_{obs} is set to be 0.03 (or 3 percent) in terms of the six month holding period performance. The required beta uncertainty σ_β^* is around 0.9 in this example.

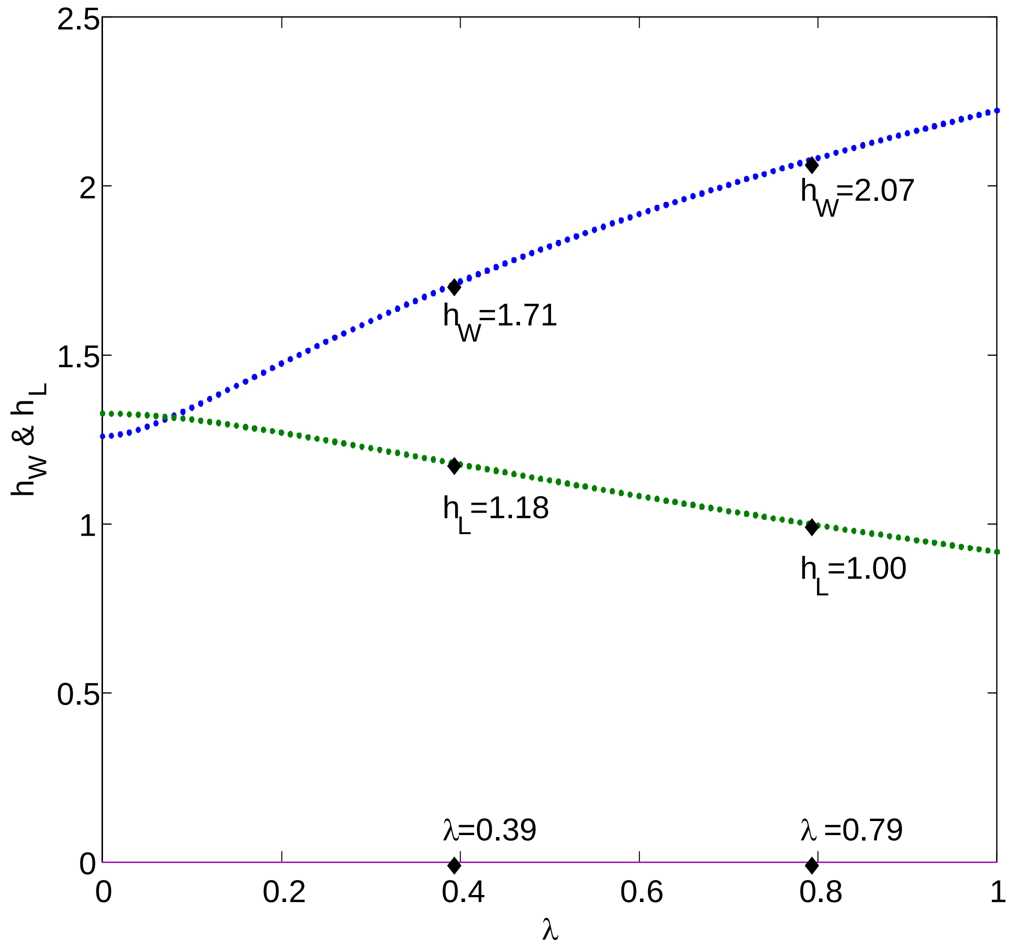

Figure 5. Beta Adjustment

This figure provides a plot for the revised betas h_W and h_L that correspond to the columns under the 1934-2003 period in Table 3. The betas of the winner and the loser vary as functions of the constant λ in (10). If $\lambda = 0$, there is no beta adjustment such that h_W and h_L are equal to their pre-ranking period estimates. When $\lambda = 0.39$, the average momentum payoff $\pi_u = 0.03$ (or 3 percent) over the six month holding period. When $\lambda = 0.79$, $\pi_u = 0.06$. The betas in these cases are marked in the plot.

TABLE 1

Momentum Payoffs

	1927-2003	1927-1964	1965-2003	1965-1989	1990-2003
W−L	**0.0051**	**0.0037**	**0.0070**	**0.0099**	**0.0019**
	(2.10)	(1.02)	(2.12)	(3.50)	(0.24)
Winner	0.0147	0.0159	0.0132	0.0108	0.0181
	(5.70)	(4.06)	(3.94)	(2.68)	(2.93)
Loser	0.0096	0.0123	0.0062	0.0009	0.0162
	(2.52)	(2.05)	(1.30)	(0.19)	(1.55)

Momentum portfolios are constructed with monthly return data on NYSE and AMEX stocks. Both the ranking and the holding periods for the momentum strategy consist of six months. Following Jegadeesh and Titman (1993), either the winner or the loser portfolio includes five portfolios with overlapping holding periods. In any given month t, the winner portfolio contains five portfolios that are constructed in the month $t-5$ through $t-1$. To mitigate bid-ask bounce effects, the holding period is assumed to start one month after the ranking period. For a given month $t-k$, stocks are ranked on the basis of the past six-month returns up to the month. Then, the stocks in the highest return decile are equally-weighted to form a portfolio. The winner portfolio is the equally-weighted mix of the five portfolios, each given the weight $1/5$ for $k = 1, \cdots, 5$. The loser portfolio is constructed in the same way, except that stocks in the lowest return decile are used in each step. The momentum strategy is the one that buys the winner portfolio and short-sells the loser portfolio, denoted by W−L. The average monthly excess returns of the portfolios are reported, and t-statistics are provided in the parentheses. The full sample is from 1927 to 2003.

TABLE 2

Pre-ranking Period Statistics

Panel A. Rolling Window Regressions

	b	s_b	σ_ε	μ	σ
Winner	1.264	0.357	0.115	0.011	0.136
Loser	1.327	0.386	0.121	0.012	0.143

Panel B. Cross-Sectional Distributions

		mean	stddev	max	min
Winner	b	1.264	0.593	3.447	−0.263
Loser	b	1.327	0.625	3.652	−0.302
Winner	σ	0.136	0.053	0.377	0.054
Loser	σ	0.143	0.056	0.402	0.055
Winner	ρ	0.480	0.153	0.787	0.011
Loser	ρ	0.478	0.154	0.788	−0.008

At the beginning of the ranking period, monthly returns of the last five years are obtained for stocks in the winner and the loser portfolios. The winner and the loser portfolios include stocks in the highest and the lowest return deciles on the basis of six-month returns over the ranking period. This table reports statistics based on the five-year rolling estimation window before the ranking period. Panel A includes the mean (μ) and standard deviation (σ) of the monthly excess returns. It also reports results from the regressions of the excess returns on the market excess returns, including the regression slope or the beta estimate (b), the standard error of the beta estimate (s_b), and the standard deviation of the regression residual (σ_ε). For each of the estimates, Panel A reports the time series average over the sample from 1927 to 2003. Statistics for the cross-sectional distributions of b, σ, and ρ are included in Panel B, where ρ is the correlation between the excess returns on the stock and the market. For each of estimates b, σ, and ρ, Panel B reports the time series averages of mean, standard deviation, maximum and minimum for the cross-sectional distribution.

TABLE 3

Beta Uncertainty and Payoff Levels

	1927 – 2003		1934 – 2003		1934 – 1964		1965 – 1989		1990 – 2003	
π_u	σ_{β_W}	σ_{β_L}	σ_{β_W}	σ_{β_L}	σ_{β_W}	σ_{β_L}	σ_{β_W}	σ_{β_L}	σ_{β_W}	σ_{β_L}
Panel A. Results from the specification (10)										
0.01	0.44	0.28	0.35	0.23	0.27	0.19	0.49	0.28	0.40	0.26
0.02	0.69	0.44	0.57	0.37	0.42	0.29	0.80	0.47	0.64	0.42
0.03	**0.96**	**0.61**	**0.80**	**0.52**	**0.57**	**0.40**	**1.15**	**0.67**	**0.88**	**0.58**
0.04	1.25	0.80	1.05	0.68	0.75	0.52	1.58	0.92	1.15	0.76
0.05	1.59	1.02	1.31	0.85	0.93	0.65	2.02	1.18	1.41	0.93
0.06	1.99	1.27	1.62	1.05	1.14	0.79	2.55	1.49	1.72	1.14
Panel B. Results from an alternative specification										
0.01	0.43	0.29	0.34	0.24	0.25	0.19	0.42	0.28	0.37	0.25
0.02	0.68	0.46	0.56	0.39	0.41	0.31	0.73	0.48	0.59	0.41
0.03	**0.95**	**0.64**	**0.79**	**0.55**	**0.57**	**0.44**	**1.08**	**0.72**	**0.80**	**0.55**
0.04	1.25	0.85	1.04	0.73	0.75	0.57	1.47	0.98	1.02	0.70
0.05	1.58	1.07	1.32	0.92	0.95	0.72	1.92	1.27	1.26	0.87
0.06	1.96	1.33	1.64	1.14	1.18	0.94	2.43	1.61	1.52	1.04

Parameter estimates for the pre-ranking period are used to compute the standard deviation σ_β in (10) for each stock in the winner and the loser portfolios. These inputs are then used to compute the revised betas h_W and h_L by (5). The unconditional momentum payoff is given by $\pi_u = E[(h_W - h_L)\mu_m]$, where μ_m is estimated by the (semi-annualized) average excess return on the market portfolio over the past five years. Then π_u is estimated by the time series average of the product $(h_W - h_L)\mu_m$. Each level of the momentum payoff, $\pi_u = 1\%, \cdots, \text{or } 6\%$, is achieved by selecting a value of the constant λ in (10). The time series average of the mean standard deviation σ_{β_W} for the winner portfolio and the time series average of the mean standard deviation σ_{β_L} for the loser portfolio are reported. Panel A is based on (10). Panel B is based on an alternative that specifies $\sigma_{\beta_i} = \lambda|\text{SRS}_i|s_{b_i}$, where λ is a constant over time and across stocks, $\text{SRS}_i = (r_i - \mu_i)/\sigma_i$, r_i is the ranking period excess return on the stock, μ_i and σ_i are the pre-ranking period mean and standard deviation of the excess return, and s_{b_i} is the standard error of the pre-ranking period beta estimate.

TABLE 4

Time-Variation in the Momentum Profits

Panel A. Return Variation and Conditional CAPM

	A1. $\tilde{r}_W = c_{W,0} + c_{W,1}(h_W\tilde{r}_m) + \tilde{\varepsilon}_W$				A2. $\tilde{r}_L = c_{L,0} + c_{L,1}(h_L\tilde{r}_m) + \tilde{\varepsilon}_L$			
	$c_{W,0}$	$c_{W,1}$		R^2	$c_{L,0}$	$c_{L,1}$		R^2
$\hat{c}$	0.0074	0.6763		63.59	−0.0003	1.1054		50.89
t	4.22	21.93			−0.14	13.97		

Panel B. Predictive Regressions with LAGMKT

$$\tilde{\pi} = \eta_0 + \eta_1 I(\text{LAGMKT} > 0) + \varepsilon$$
$$\tilde{\pi} = \eta_0 + \eta_1 \text{LAGMKT} + \eta_2 \text{LAGMKT}^2 + \varepsilon$$

	B1. $\tilde{\pi} = \tilde{\pi}_{\text{raw}} \equiv \tilde{r}_W - \tilde{r}_L$				B2. $\tilde{\pi} = \tilde{\pi}_{\text{raw}} - \tilde{\pi}_{\text{CAPM}}$			
	η_0	η_1	η_2	R^2	η_0	η_1	η_2	R^2
$\hat{\eta}$	−0.0098	0.0179		0.80	−0.0114	0.0178		0.72
t	−1.49	2.60			−1.57	2.33		
$\hat{\eta}$	−0.0013	0.0402	−0.0325	1.12	−0.0033	0.0408	−0.0327	1.05
t	−0.39	3.36	−3.54		−0.89	3.11	−3.20	

	B3. $\tilde{\pi} = \tilde{\pi}_{\text{raw}} - (h_W\tilde{r}_m - h_L\tilde{r}_m)$				B4. $\tilde{\pi} = \tilde{\pi}_{\text{raw}} - (c_{W,1}h_W\tilde{r}_m - c_{L,1}h_L\tilde{r}_m)$			
	η_0	η_1	η_2	R^2	η_0	η_1	η_2	R^2
$\hat{\eta}$	−0.0003	0.0044		0.04	0.0020	0.0065		0.12
t	−0.04	0.60			0.29	0.91		
$\hat{\eta}$	0.0025	0.0122	−0.0133	0.15	0.0063	0.0144	−0.0151	0.23
t	0.60	0.85	−1.12		1.49	1.09	−1.57	

Panel A includes results from two regressions. Panel A1 reports the regression of $\tilde{r}_W$ on $h_W\tilde{r}_m$ and Panel A2 reports the regression of $\tilde{r}_L$ on $h_L\tilde{r}_m$, where $\tilde{r}_W$, $\tilde{r}_L$, and $\tilde{r}_m$ are the holding period excess returns on the winner, the loser, and the market, respectively. The construction of the winner and the loser portfolios is described in Table 1. The conditional betas are computed by (5), where the value of λ in (10) is selected to match the three percent unconditional payoff level (i.e., $\pi_u = 0.03$). For any given month, the betas h_W and h_L of the winner and the loser are the equally-weighted averages of the betas of the five portfolios used in construction of the winner and the loser portfolios, respectively. For all cases throughout the table, the t-statistics are adjusted for serial correlation and heteroskedasticity, the regression R^2's are expressed in percentage, and the sample period is from 1934 to 2003. Panel B reports results from predictive regressions using LAGMKT, where LAGMKT is the three-year lagged market return. In each case, the dependent variable $\tilde{\pi}$ is either the raw payoff or the payoff adjusted by some model. Two regressions are reported in each panel. The first is on a dummy variable $I(\text{LAGMKT} > 0)$ that is equal to 1 if LAGMKT > 0 but 0 otherwise. The second is a quadratic regression on LAGMKT and LAGMKT^2. In Panel B1, the dependent variable $\tilde{\pi}$ is the holding period raw payoff (i.e., $\tilde{r}_W - \tilde{r}_L$). In Panel B2, the dependent variable $\tilde{\pi}$ is the payoff adjusted by the unconditional CAPM, where $\tilde{\pi}_{\text{CAPM}} = b\tilde{r}_m$, which is obtained from the regression $\tilde{\pi}_{\text{raw}} = a + b\tilde{r}_m + \varepsilon$. In Panel B3, the payoff is adjusted by the conditional CAPM, i.e., $\tilde{\pi} = \tilde{\pi}_{\text{raw}} - (h_W\tilde{r}_m - h_L\tilde{r}_m) = (\tilde{r}_W - h_W\tilde{r}_m) - (\tilde{r}_L - h_L\tilde{r}_m)$. In Panel B4, the payoff is adjusted by the return variation captured by the conditional CAPM in the regressions of Panel A. That is, $c_{W,1}$ and $c_{L,1}$ are the slope estimates from Panel A.