

Intermediation Capital and Asset Prices

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Abstract

We introduce intermediation frictions into a Lucas (1978) asset pricing model in order to study the effects of low capital in the intermediary sector on asset prices. Our model shows that low intermediary capital can increase risk premia, Sharpe ratios, volatility and comovement among intermediated assets. Reductions in intermediary capital also lead to a flight-to-quality in which intermediaries' investors withdraw their funds and purchase bonds. We calibrate our model and find that the effects are sizable: risk premia can double when moving from states of the world where intermediary capital is plentiful to states where intermediary capital is scarce. We simulate our model to measure the average effects of intermediary capital on asset prices. In our simulations, the economy does not spend sufficient time in the low intermediary capital states, so that average effects on asset prices are small. Our model suggests that intermediation frictions are first order to understand financial crises and episodes of market illiquidity, but are second order to understand the average equity risk premium.

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1 Introduction

We study how changes in the capital of financial intermediaries affect equilibrium asset prices. In standard neoclassical models, financial intermediaries are a frictionless conduit through which households gain access to a larger investment opportunity set. If the capital of an intermediary falls in a way that shrinks the investment opportunity set of its end investors, these investors simply restore the capital of the intermediary. Asset market equilibrium is unaffected so that intermediation capital is irrelevant for asset prices.

In practice, financial intermediaries are not a frictionless “veil.”¹ Households cannot perfectly monitor and control the investment decisions of intermediaries. Suspicions of mis-management and conflicts of interest always cloud the relationship between households and intermediaries. A well capitalized intermediary allays some of these concerns.

These points are well illustrated by the LTCM crisis of the fall of 1998. Losses among hedge funds reduced their capital. Households did not restore the lost capital, and instead withdrew into safer investments such as government bonds. As a consequence, funds began to liquidate assets. In many of the markets that hedge funds specialized, prices were substantially affected by the liquidation of distressed hedge funds. Spreads and risk premia rose as prices fell; volatility increased; and market liquidity fell.²

We present a model to study the importance of intermediation capital for asset prices. Our model has an intermediation sector and a household sector. Households do not invest directly in a set of risky complex assets, but choose to delegate such investment to intermediaries that are managed by skilled “specialists”. The main friction we introduce is a capital constraint: because of an agency problem, specialists must use their own wealth to coinvest with the households in forming an intermediary. The coinvestment places some of the specialists wealth at stake in the intermediary, and thereby provides the specialist with appropriate investment incentives.

Other than the intermediation friction, our model resembles the endowment economies commonly studied in the equilibrium asset pricing literature. The risky assets are represented by a Lucas (1978) tree, which pays a dividend governed by a geometric Brownian motion process. Subject to the intermediation friction, markets are complete.

The households’ participation in the riskless asset is direct, while their participation in the risky assets is indirect because of the intermediation friction. The capital constraint determines the equilibrium supply of intermediation.

When there is a large enough amount of intermediary capital that capital constraints do not bind in

¹Allen (2001) presents a compelling case for why financial intermediaries should be incorporated into asset pricing models.

²Other important asset markets, such as the equity or housing market, were relatively unaffected by the turmoil. The dichotomous behavior of asset markets suggests that the problem was hedge fund capital specifically, and not capital more generally. Investors did not bypass the distressed hedge funds in a way as to undo any asset price impact of the hedge fund actions. They also did not restore the hedge funds’ capital.

equilibrium, the supply of intermediation is large. Households are indirectly able to hold their desired portfolio of risky assets, and the intermediation frictions play little role in determining asset prices. The economy resembles one without any agency problems. Risk is shared between specialists and households, and risk premia are small.

Starting from the level of intermediary capital where the supply of intermediation is large, if we imagine reducing intermediary capital, there comes a threshold for intermediary capital where capital constraints bind in equilibrium. As we reduce capital below this level, households withdraw funds from intermediaries, recognizing that not doing so would trigger an agency problem; i.e. specialists would not have a sufficient stake to provide investment incentives. Households reduce (indirect) participation in the risky asset. The specialists increase leverage to absorb more of this risky asset and shocks to dividends are borne predominantly by specialists.

In the latter case, shocks have a larger effect on asset prices, leading to higher conditional volatility. This reinforces the effect of capital constraints, as the asset price shocks lead to greater fluctuations in the capital levels of intermediaries, creating an amplification mechanism. Risk premia and Sharpe ratios rise. We quantify these effects and show that the degree of capital constraints can have a sizable effect on asset price measurements. For example, we find that while risk premia when capital constraints do not bind are 3% in our baseline calibration, when constraints place intermediaries near bankruptcy the risk premium rises to 6%.

The patterns that our model generates when intermediary capital is low are consistent with episodes of market illiquidity such as the events of the Fall of 1998. Indeed, our model suggest a link between market liquidity and intermediary capital. When intermediary capital is low, there are effectively less buyers of risky assets as households do not fully participate in the risky asset markets. We show that during the low capital periods, correlation across intermediated assets rises, consistent with the Fall of 1998 events. We also show that decreases in asset prices leads household to withdraw funds from intermediaries and invest in riskless bonds, consistent with a flight to quality.

We simulate the model to quantify the time series properties of asset prices in our model. In our simulations, we choose parameters such that the probability of the intermediary sector losing all of its capital and going bankrupt is low ($< 2\%$). With this restriction we find that, although the intermediary friction can have substantial effects on conditional risk premia, the average equity premium in the model is only about 0.3% higher than in a model without intermediation frictions. This result is because the risk premium in our model resembles a quadratic function of the tightness of capital constraints. Thus, risk premia are substantially higher when the intermediary sector is near bankruptcy. We also discuss how our results change if we allow for larger probabilities of bankruptcy.

In our simulations we also measure the risk premium attached to states of the world with low intermediary capital. A number of recent papers have documented that aggregate liquidity risk is a priced factor (see Amihud, 2002, Acharya and Pedersen, 2003, Pastor and Stambaugh, 2003, and Sadka, 2003), suggesting that

the marginal investor is averse to assets that pay little when aggregate liquidity is low. Interpreting our low intermediary capital states as low aggregate liquidity states, we find a natural reason why the marginal investor in our economy should be averse to low intermediary capital: the marginal investor in our model is the specialist who has low consumption and bears substantial risk when intermediary capital is low, the specialist will be averse to aggregate liquidity risk. Since the liquidity risk in our model is due to shocks to intermediary capital, we can measure the liquidity risk premium in terms of how much intermediary capital shocks raise the volatility of the specialist's consumption. We find that shocks to intermediation capital cause the risk premium on consumption growth of the specialist intermediary to rise by *45bps* relative to a baseline without intermediation capital shocks.

In our model, the specialists (intermediaries) are the marginal investor in the risky asset market. There is a growing body of papers suggesting that intermediaries are the marginal investor in many specialized asset markets. These studies include, research on the mortgage-backed securities market (Gabaix, Krishnamurthy, and Vigneron, 2005), the corporate bond market (Collin-Dufresne, Goldstein, Martin, 2001), the credit default swap market (Berndt, et. al., 2004), the catastrophe insurance market (Froot and O'Connell, 1999, 2001), and the options market (Bates, 2003; Garleanu, Pedersen, and Poteshman, 2005). These empirical studies reiterate the relevance of intermediation for asset prices. However they paint a richer picture of intermediation than is captured in our model. The studies suggest that the capital of intermediaries specialized in particular markets is relevant for those particular markets. In our model, the entire intermediary sector specializes in the same set of assets. Our model takes a broad brush approach to the relationship between intermediation capital and asset prices.

There is a large theoretical literature exploring different aspects of how frictions in intermediation may effect asset prices (see Allen and Gorton, 1993, Brennan, 1993, Dow and Gorton, 1994, Grossman and Zhou, 1996, Shleifer and Vishny, 1997, Xiong, 2001, Kyle and Xiong, 2001, Cuoco and Kaniel, 2001, Gromb and Vayanos, 2002, Liu and Longstaff, 2004, Brunnermeier and Pedersen, 2005, Dasgupta, Prat and Verardo, 2005, and Vayanos, 2005). Our model is most closely related to Xiong (2001) and Vayanos (2005). These papers also develop models to study asset price dynamics. Xiong does not explicitly model an intermediation sector, treating such a sector with a short-hand log utility preference assumption. Vayanos introduces an open-ending friction into a model of intermediaries and generates implications for the premium on liquidity and volatility risk.

There is also a literature on limited market participation and asset prices (Mankiw and Zeldes, 1991, Allen and Gale, 1994, Basak and Cuoco, 1998, Vissing-Jorgensen, 2002, Brav, Constantinides, and Geczy, 2002, Guvenen, 2005). While our model shares many features with those in the limited participation literature, it is different because the extent of participation is endogenously determined by financial intermediation.

The remainder of this paper is devoted to developing a model to assess the importance of intermediary capital for market liquidity and asset pricing.

2 Model 1: Capital Constraints and Conditional Asset Prices

We begin by developing a model that highlights the differences between an economy where there is low intermediary capital and an economy where there is high intermediary capital. We show how intermediary capital affects the sharing of risk, and the determination of asset prices.

We then turn to numerical evaluation of the model. We quantify how a given level of intermediary capital translates into asset price measurements such as risk premia, volatility, and correlations.

A drawback of the model we present below is that the region where intermediary capital is high is an absorbing state. Thus, while the model is useful for conditional asset pricing – i.e. how asset prices evolve given an initial condition of intermediary capital – it does not lend itself to simulating the average effects of the intermediary capital.

We alter the model slightly and present a second model that allows for full transiting of paths. We then simulate the model and compute the average affects in the model. These results are presented in Section 5.

2.1 Preferences and payoffs

We consider an infinite horizon, continuous time, economy populated by two classes of agents, households and specialist intermediaries. There are two assets, a riskless bond in zero net supply, and a stock that pays a risky dividend. We normalize the total supply of stocks to be one unit.

We assume that the households do not invest directly in the risky asset, but can allocate some funds to the specialists who invest in the risky asset on their behalf. We think of the risky asset not as a single stock but as corresponding to a portfolio of intermediated assets that are managed by the specialists.

The households have log preferences over date t consumption c_t^h ,

$$\int_0^\infty e^{-\rho t} \ln c_t^h dt \quad \rho > 0.$$

They indirectly hold stocks and directly hold bonds. The specialists have concave preferences over date t consumption, c_t ,

$$\int_0^\infty e^{-\rho t} u(c_t) dt \quad \rho > 0;$$

we consider a CRRA instantaneous utility function with parameter γ for the specialists. They choose portfolio shares of α^s share of stocks and $\alpha^b = 1 - \alpha^s$ bonds.

The stock pays a dividend of D_t per unit time, where $\{D_t : 0 \leq t < \infty\}$ follows a Geometric Brownian Motion,

$$\frac{dD_t}{D_t} = gdt + \sigma dZ_t \quad \text{given } D_0. \tag{1}$$

$g > 0$ and $\sigma > 0$ are constants. Throughout this paper $Z = \{Z_t : 0 \leq t < \infty\}$ is a standard Brownian motion on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with an augmented filtration $\{\mathcal{F}_t : 0 \leq t < \infty\}$ generated by the Brownian motion Z .

We denote the progressively measurable processes $\{P_t : 0 \leq t < \infty\}$ and $\{r_t : 0 \leq t < \infty\}$ as the stock price and interest rate processes, respectively.

2.2 Intermediation

The key friction we introduce is a capital constraint on intermediation. This subsection details our modeling of intermediation, which loosely follows Holmstrom and Tirole (1997). Unlike Holmstrom and Tirole we make assumptions to restrict the contract space. These assumptions are necessary in order to maintain tractability when placing the contracting problem within an asset pricing framework. But our resulting contracts still resemble those that Holmstrom and Tirole derive.

Suppose that a specialist begins an intermediary by contributing T^I and raising T^h from households. On the total funds of $T = T^I + T^h$ he makes decisions that result in a stochastic portfolio return of $\widetilde{dR}_t$.

Assumption 1 (Moral Hazard)

The specialist makes an unobserved portfolio choice decision, α_s , and an unobserved due-diligence decision, $e \in \{0, 1\}$. For any given α_s , if the specialist shirks and sets $e = 0$, the return on the portfolio falls by $x dt$, but the specialist gets a private benefit (in units of the consumption good) of $bT dt$. We assume that $x > b$ so that choosing $e = 1$ maximizes total surplus.

Specialists have to be given incentives to put in effort. These incentives are provided by writing a performance contract contingent on $\widetilde{dR}_t$. Denote $Tf(\widetilde{dR}_t)$ as the portion of the total portfolio returns that is paid to the specialist and $T(1 - f(\widetilde{dR}_t))$ as the portion that is paid to the household. We restrict attention to the following contribution-based linear sharing contracts:

Assumption 2 (Linear Sharing)

We assume that incentive contracts are linear and proportional to contributions:

$$\frac{T^I}{T^h} = \frac{f(\widetilde{dR}_t)}{1 - f(\widetilde{dR}_t)} = \frac{\beta}{1 - \beta}.$$

Where, the compensation of a specialist is:

$$C + \beta \widetilde{dR}_t T.$$

We focus on compensation contracts that have a fixed component (e.g. a fee) and a variable component that is linear in the realized portfolio return. In general, the optimal contract may be nonlinear. A realistic example of a non-linear contract is an option contract that only compensates the specialist if the return is sufficiently high. To retain tractability when embedding our intermediation model into an equilibrium asset pricing framework, we restrict attention to linear sharing contracts. Note also that in the contracts we study,

the sharing of portfolio returns links to the contributions (T^I, T^h) .³

The moral hazard and linear sharing assumption pin down β from an incentive compatibility condition: β must be sufficiently high to compensate the specialist for exerting effort.

Our final assumption pins down C based on the participation constraint of the household.

Assumption 3 (Household beliefs)

*Households believe that intermediaries, through a judicious portfolio choice, will earn higher returns than they themselves could obtain. The perceived excess return is $f dt$ per unit wealth.*⁴

A concrete story to justify this assumption is as follows. Imagine that households are ignorant regarding time-varying expected returns on the risky asset and by default invest directly only a constant portfolio share. They are aware that specialists can adjust portfolio shares and generate an excess return relative to the constant share portfolio. On average, they perceive this excess return is f . In equilibrium, as we show, specialists do generate an excess return on their portfolio. We could reconcile the story and set f to this excess return.

We envision the following market structure for intermediation. At every t , each specialist is randomly matched with a household. The specialist and household then potentially enter into an intermediation relationship. These interactions occur instantaneously and result in a continuum of (identical) one-to-one relationships. A household may choose to invest some of his wealth with the specialist, subject to the moral hazard friction. The specialist and household bargain over the terms of the contract. We assume that the specialist has all of the bargaining power in this interaction and makes a take-it or leave-it offer. If the household does not accept the offer, he must wait dt to match with another specialist. After intermediation decisions are taken, specialists trade in a Walrasian stock and bond market, and the household trades in only the bond market. At $t + dt$ the match is broken, and the intermediation market repeats itself.

The instantaneous matching structure means that all contracts are short-term. In principle, a long term contract could improve allocations. The bargaining power assumption is less important, and we could alter it, at the cost of more complicated notation.

Proposition 1 *The optimal contract between a household and specialist involves:*

1. *Both specialists and households receive the return $\widetilde{dR}_t$ on their contributions to the intermediary.*
2. *Household pays a fee (f) to the specialist on the funds that the household contributes to the intermediary:*

$$fT^h = C.$$

³In general, we may find that the specialist has to invest proportionately less funds into the intermediary, while receiving proportionately more of the portfolio returns. We can easily accomodate such a change. However, it may also be that these proportions varies as economic conditions change. If we allow for such variation we lose tractability when solving for asset prices.

⁴ x is greater than f . If not, households may prefer to have specialists shirk and set $e = 0$, but still intermediate their investments.

3. *Specialists are subject to a capital constraint. They can intermediate, at most, $m \times w_t^I$ of the funds of households, where w_t^I is the capital of a specialist at date t and $m = \frac{x}{b} - 1$ is a strictly positive constant.*

Proof. See appendix. ■

The contracts results in the specialist receiving payoffs from two sources: a fixed fraction of intermediated funds plus a fraction of the returns generated by his fund. Such a payoff structure resembles fund management contracts we observe in practice. More broadly the payoff structure captures the explicit and implicit incentives across many modes of intermediation.

The model we have outlined has the characteristic that when the specialists' wealth falls, their ability to intermediate funds of the households also falls. This reduces the aggregate demand for stocks, requiring an increase in the equilibrium return on the stock in order to clear the stock market. On the other hand, the disintermediation results in households increasing their demand for the bond, leading to a fall in the equilibrium interest rate.

Note that the model is asymmetric, as a sufficient rise in the wealth of specialists may lead to a situation where all of the funds of the households are being intermediated, so that further changes in intermediary capital does not affect intermediation.

Finally, it is theoretically possible in the model for the capital of intermediaries to fall to zero, in which case all intermediation breaks down. In order for our model to be well specified, we must define this “bankruptcy” state.

Assumption 4 (Bankrupt intermediaries)

In bankruptcy, the specialists assets are liquidated and debt is retired, with any debts to other specialists repaid first.⁵ The specialists are given a fixed share of the risky asset, ϕ , to fund consumption for the rest of time and are restricted from trading these shares thereafter. The households receive the remainder of $1 - \phi$ shares. Households manage these investments poorly relative to the specialists, and in case of liquidation, the dividends on the risky asset fall by a fraction $1 - \delta$. We set $\delta = 1 - \phi$ which ensures that the consumption of the specialist does not jump in bankruptcy.

2.3 Decisions

At every t specialists and households interact and strike up intermediation relationships. After the intermediation decision, agents trade in the asset and goods market to achieve their desired portfolio shares and consumption rates. Given our assumption on household beliefs, the investment decision of the household is trivially to invest any remaining wealth (after giving money to the intermediary and consuming) directly in

⁵In equilibrium, there is no borrowing or lending between specialists. Requiring that debts to specialists be repaid first ensures that the debt we price is riskless.

bonds. The specialist chooses a portfolio of stock and bond and the consumption rate to maximize his lifetime utility.

The decision problem of a household is to choose his consumption rate and funds for delegation, given his wealth, $w_t^h > 0$. We denote $X_t \in [0, 1]$ as the fraction of wealth that is invested with the intermediaries. Then,

$$w_t^h X_t = \min \{mw_t^I, w_t^h\},$$

i.e. the household delegates the maximum possible funds to the intermediary given household beliefs. Then the return on the household's wealth is,

$$d\hat{R}_t = (1 - X_t)r_t dt + X_t (\widetilde{dR}_t - f dt),$$

where $\widetilde{dR}_t$ is the cumulative return process delivered by intermediaries. The optimization problem for a household is:

$$\max_{\{c_t^h\}} E \left[\int_0^\tau e^{-\rho t} \ln c_t^h dt \right] \quad (2)$$

where τ is the first time the economy reaches the bankruptcy boundary. The optimization is subject to,

$$dw_t^h = -c_t^h dt + w_t^h d\hat{R}_t$$

given some w_0^h .

The specialist chooses his consumption rate, his portfolio, as well as a quantity of intermediation services to offer to households.

$$\max_{\{c_t, \alpha_t^s, \alpha_t^b, Q_t\}} E \left[\int_0^\tau e^{-\rho t} u(c_t) dt \right] \quad (3)$$

subject to the budget constraint,

$$dw_t^I = -c_t dt + w_t^I \widetilde{dR}_t + Q_t f dt$$

and the capital constraints:

$$\begin{aligned} Q_t &\leq mw_t^I = m(\alpha_t^b + \alpha_t^s P_t); \\ \widetilde{dR}_t &= \frac{1}{w_t^I} [\alpha_t^b r_t dt + \alpha_t^s (D_t dt + dP_t)]. \end{aligned}$$

In the market for intermediation, the specialists intermediate a portion of the wealth of the households:

$$Q_t = w_t^h X_t.$$

Market clearing in the stock market is:

$$(w_t^I + w_t^h X_t) \frac{\alpha_t^s}{w_t^I} = 1.$$

This condition requires that the proportion of stocks in the specialists portfolio ($\frac{\alpha_t^s}{w_t^s}$), applied to the total funds under intermediation (specialist wealth plus intermediated funds), must add up to the total supply of stocks. For the bond market,

$$(w_t^I + w_t^h X_t) \frac{\alpha_t^b}{w_t^I} + w_t^h (1 - X_t) = 0. \quad (4)$$

The condition for the bond market is similar to that of stocks, but reflects that households may hold bonds directly, and that total bond holdings must sum to zero.

Market clearing in the goods market is,

$$c_t + c_t^h = D_t$$

which is satisfied by Walras' law.

3 Solution

We look for a stationary Markov equilibrium where the state variables are (w_t^h, D_t) , where w_t^h is the wealth of the passive household. There are two steps in deriving the solution. First, since the specialist is unconstrained in his asset choice, we can use his Euler equation to derive an equation governing the equilibrium asset prices. Second, market clearing in the goods market allows us to infer the consumption of the specialist to arrive at the Euler equation.

Define the total return on the stock as,

$$dR_t = \frac{D_t dt + dP_t}{P_t}.$$

Using the Euler equation for the specialist, we arrive at the standard consumption-based asset pricing relations:

$$-\rho dt - \gamma E_t \left[\frac{dc_t}{c_t} \right] + \frac{1}{2} \gamma (\gamma + 1) Var_t \left[\frac{dc_t}{c_t} \right] + E_t [dR_t] = \gamma Cov_t \left[\frac{dc_t}{c_t}, dR_t \right] \quad (5)$$

where c_t is the consumption rate of the specialist. The Euler equation is valid for all $t \leq \tau$ (the time the economy hits the bankruptcy boundary) since the specialist is always marginal in trading assets in the economy. Assumption 4 ensures that the Euler equation is also valid at $t = \tau$. We use $E_t[\cdot]$ as the conditional expectation operator, and $Cov_t[\cdot, \cdot]$ ($Var_t[\cdot]$) as the conditional covariance (variance) operator.

For the short-term bond, since the bond price is always one,⁶

$$r_t dt = \rho dt + \gamma E_t \left[\frac{dc_t}{c_t} \right] - \frac{\gamma(\gamma + 1)}{2} Var_t \left[\frac{dc_t}{c_t} \right]$$

To use these equations we need to derive expressions for $P(w_t^h, D_t)$ and $c(w_t^h, D_t)$. The derivation follows.

⁶The debt that household hold is riskless for $t < \tau$. But at $t = \tau$, intermediaries may default on their debt which makes household debt risky. In this case, the interest rate we describe will deviate from that of the riskless debt. We may imagine that the short-term interest rate we derive is on a repo loan between intermediaries, which is always repaid before any debts due to households. Such a repo loan will always be riskless.

3.1 Stock Price

We exploit the scale invariance property of our economy and redefine the state variable as $y_t \equiv \frac{w_t^h}{D_t}$, i.e. the dividend-scaled wealth of the households. We conjecture that the equilibrium evolution of y_t may be written as an Ito process which solves the following Stochastic Differential Equation,

$$dy_t = \mu_y dt + \sigma_y dZ_t, \quad (6)$$

where the drift μ_y and the diffusion σ_y are well-behaved functions of y (to ensure the existence and the uniqueness of solution $\{y_t : 0 \leq t \leq \tau\}$).⁷ We solve for μ_y and σ_y below.

We also conjecture that we can write the equilibrium stock price as,

$$P_t = D_t F(y_t) \quad (7)$$

where $F : \mathbb{R} \rightarrow \mathbb{R}$ is twice continuously differentiable on its relevant domain. $F(y)$ is simply the price/dividend ratio of the stock.

Applying Ito's Lemma to (7) yields the total return process for the stock:

$$dR_t = \left(g + \frac{F'}{F} \mu_y + \frac{1}{2} \frac{F''}{F} \sigma_y^2 + \frac{1}{F} + \frac{F'}{F} \sigma_y \sigma \right) dt + \left(\sigma + \frac{F'}{F} \sigma_y \right) dZ_t, \quad (8)$$

where (and in the analysis that follow) we omit the argument in $F(y)$, $F'(y)$ and $F''(y)$ for brevity.

3.2 Consumption

Consider again the household's problem, (2). It is easily verified that for a log investor, the optimal consumption rate is,

$$c_t^h = \rho w_t^h,$$

regardless of the stochastic process for $d\hat{R}_t$.⁸

Utilizing this relation and the goods market clearing condition $c_t + c_t^h = D_t$, we infer the consumption rate of the specialist to be

$$c_t = D_t - \rho w_t^h = D_t(1 - \rho y_t).$$

Then,

$$\begin{aligned} \frac{dc_t}{c_t} &= \frac{dD_t}{D_t} - \frac{\rho dy_t}{1 - \rho y_t} - \frac{\rho}{1 - \rho y} \text{Cov}_t \left[dy_t, \frac{dD_t}{D_t} \right] \\ &= \left(g - \frac{\rho}{1 - \rho y_t} (\mu_y + \sigma_y \sigma) \right) dt + \left(\sigma - \frac{\rho}{1 - \rho y_t} \sigma_y \right) dZ_t \end{aligned} \quad (9)$$

⁷After bankruptcy, or $t > \tau$, the economy stays at the point $y = \frac{\delta}{\rho}$.

⁸The Euler equation for the log investor is:

$$-\rho dt - E_t \left[\frac{dc_t^h}{c_t^h} \right] + \text{Var}_t \left[\frac{dc_t^h}{c_t^h} \right] + E_t \left[d\hat{R}_t \right] = \text{Cov}_t \left[\frac{dc_t^h}{c_t^h}, d\hat{R}_t \right]$$

The solution $c_t^h = \rho w_t^h$ satisfies the Euler equation since $dc_t^h/c_t^h = dw_t^h/w_t^h = -\rho dt + d\hat{R}_t$.

3.3 Differential Equation and Regions

We substitute the expressions for consumption (9) and asset return (8) into the specialist's pricing equation (5) to obtain the following proposition:

Proposition 2 *The equilibrium Price/Dividend ratio $F(y)$ satisfies the ordinary differential equation,*

$$\begin{aligned} g + \frac{F'}{F}\mu_y + \frac{1}{2}\frac{F''}{F}\sigma_y^2 + \frac{1}{F} + \frac{F'}{F}\sigma_y\sigma = \rho + \gamma g - \frac{\gamma\rho}{1-\rho y}(\mu_y + \sigma_y\sigma) \\ + \gamma\left(\sigma - \frac{\rho}{1-\rho y}\sigma_y\right)\left(\sigma + \frac{F'}{F}\sigma_y\right) - \frac{1}{2}\gamma(\gamma+1)\left(\sigma - \frac{\rho}{1-\rho y}\sigma_y\right)^2 \end{aligned} \quad (10)$$

Before we use this ODE to find $F(y)$, we need to derive expressions for μ_y and σ_y as y evolves along its equilibrium path. The functional forms depend on whether there is sufficient intermediary capital to intermediate all of the households wealth or not.

Let us denote $\theta_s(y)$ and $\theta_b(y)$ as the stock and bond holdings (direct plus indirect) of the household, where $w^h = \theta_b + \theta_s P$. The three cases are as follows:

1. If $mw^I \geq w^h$ then there is sufficient intermediary capital to intermediate all of the households' wealth. In this case, the households delegate all of their wealth to the intermediation sector, and hence $\theta_b(y) = 0$. From the clearing condition in the bond market, (4), we see that α^b must be equal to zero. Both specialists and households own only stocks in their portfolio. Then the households stock holdings are,

$$\theta_s(y) = \frac{w^h}{P} = \frac{y}{F(y)}.$$

The intermediation sector is capital constrained when $mw^I = w^h$. Since $w^I + w^h = P$, we can solve for this cutoff as

$$w^h \left(1 + \frac{1}{m}\right) = P$$

or, in terms of y ,

$$y^c = \frac{m}{m+1}F(y^c). \quad (11)$$

This equation implicitly defines the point when capital constraints arise, and we are sitting in the unconstrained region if $0 < y \leq y^c$.

2. If $mw^I < w^h$ (or $y > y^c$) then the intermediation sector is capital constrained. Since all stocks are held through the intermediaries,

$$\theta_s(y) = m\alpha^s$$

where α^s is the stock holdings of the specialists. But since, $m\alpha^s + \alpha^s = 1$ (i.e., all stocks have to be held through intermediaries), we find

$$\theta_s(y) = \frac{m}{1+m}.$$

Note that the stock holdings for both agents are constant in this region. For bonds, note that, $\theta_b = w^h - \theta_s P$. Defining the scaled bond holding of households as $\hat{\theta}_b(y) \equiv \frac{\theta_b}{D}$, we have,

$$\hat{\theta}_b(y) = y - \theta_s F(y).$$

3. If specialists' wealth falls to zero, the insolvent intermediaries go bankrupt and all intermediation collapses. The economy reaches this point when $w^I = P - w^h = 0$. Then, the bankruptcy threshold is implicitly defined by the equation

$$y^b = F(y^b).$$

Given our assumptions on bankruptcy, the households own the risky asset after this event but receive a lower dividend stream. Since the households have log preferences with discount rate of ρ , we have

$$F(y^b) = \frac{\delta}{\rho}.$$

As we will see, the households' stock holding $\theta_s(y)$ and their scaled bond position $\hat{\theta}_b(y)$ play a key role in our following analysis. We may omit the argument y later on to simplify our notation.

3.4 Unconstrained Intermediation: $0 < y \leq y^c$

We now derive the diffusion coefficients μ_y and σ_y in the unconstrained region.

Lemma 1 *When $y < y^c$ so that intermediation is not capital constrained, we have*

$$\sigma_y = 0,$$

and

$$\mu_y = \frac{1}{1 - \theta_s F'} (\theta_s - (\rho + f)y).$$

Proof. For households, the change in y reflects any capital gains in the stock and any changes in the asset positions, i.e.,

$$dy = \theta_s dF + F d\theta_s$$

The second term here reflects changes in the asset position. We know that,

$$DF d\theta_s = D\theta_s dt - (\rho + f)w dt.$$

That is, the asset position changes due to dividends received minus their consumption and the delegation fees. Thus,

$$dy = \theta_s dF + \theta_s dt - (\rho + f)y dt.$$

Substituting for dF we find,

$$dy(1 - \theta_s F') = \frac{1}{2} \theta_s F'' \sigma_y^2 dt + \theta_s dt - (\rho + f)y dt.$$

Notice that y is not stochastic (no dZ_t term involved), so that $\sigma_y = 0$. We can then solve for μ_y to obtain the expression given in the Lemma. ■

Proposition 3 *When $y < y^c$, the equilibrium risk premium on the stock is constant:*

$$E_t[dR_t] - r_t dt = \gamma \sigma^2 dt.$$

Proof. The risk premium on the stock is given by $\gamma Cov_t \left[\frac{dc}{c}, dR \right]$. When $\sigma_y = 0$, the diffusion terms in both (8) and (9) are σ . ■

The risk premium in the unconstrained case corresponds exactly to what we would derive in an economy with the specialist intermediary as the representative agent, but without intermediation frictions. This result provides a counterpoint to the results we derive next for the constrained region.

3.5 Capital Constrained Intermediation: $y^c < y < y^b$

Recall $\hat{\theta}_b(y) = \frac{\theta_b}{D} = y - \theta_s F(y) > 0$, where $\theta_s = \frac{m}{m+1}$ in the constrained region. Then,

$$y = \theta_s F + \hat{\theta}_b.$$

The change in y is,

$$\begin{aligned} dy &= \theta_s dF - \frac{\theta_b}{D^2} dD + \frac{\theta_b}{D^3} Var_t[dD] + F d\theta_s + d\hat{\theta}_b \\ &= \theta_s dF - \hat{\theta}_b \frac{dD}{D} + \hat{\theta}_b \sigma^2 dt + (\theta_s + r\hat{\theta}_b - \rho y - fm(F - y))dt. \end{aligned}$$

We have used the accounting identity, $F d\theta_s + d\hat{\theta}_b = (\theta_s + r\hat{\theta}_b - \rho y - fm(F - y))dt$, that governs changes in the households' assets position. Substituting for dF , we find:

Lemma 2 *When $y > y^c$ so that the intermediary sector is capital constrained, we have*

$$\sigma_y = -\frac{\hat{\theta}_b}{1 - \theta_s F'} \sigma$$

and,

$$\mu_y = \frac{1}{1 - \theta_s F'} \left(\theta_s + (r + \sigma^2 - g)\hat{\theta}_b - \rho y - fm(F - y) + \frac{1}{2} \theta_s F'' \sigma_y^2 \right)$$

where,

$$r = \rho + \gamma \left(g - \frac{\rho(\mu_y + \sigma\sigma_y)}{1 - \rho y} \right) - \frac{\gamma(\gamma + 1)}{2} \left(\sigma - \frac{\rho\sigma_y}{1 - \rho y} \right)^2.$$

Note that σ_y is less than zero in the constrained region,⁹ and increasingly so as the passive households' scaled bond holding $\hat{\theta}_b > 0$ rises. In other words, a negative innovation to D , or a negative shock to the stock market, increases the households scaled wealth y , and tightens the capital constraint.

⁹We can show that $1 - \theta_s F'$ is always positive. To see this, note that $G(y) \equiv \frac{1}{1 - \theta_s F'}$ in (20) is positive when $y = y^c$. Suppose that G goes to positive infinity. The RHS is dominated by the fourth term which is negative, while on the LHS G' has a positive coefficient. Hence $G' < 0$, contradiction.

In the constrained region, households withdraw funds from intermediaries recognizing that not doing so will lead intermediaries to shirk on the due-diligence decision. They withdraw funds until the point that the intermediaries capital constraint is met.

But in withdrawing funds from intermediaries, households are also reducing their indirect participation in the stock market, while increasing bond holdings. Specialists absorb these changes by increasing their borrowing to buy the stock.

$$w^h = \frac{m}{m+1}P + \theta_b \text{ and } w^I = \frac{1}{m+1}P - \theta_b.$$

Intermediaries have a leveraged position in the risky asset in the constrained region. Since $\theta_b > 0$, a shock to dividends produces a muted reaction in the households' wealth, w^h , and a more amplified reaction in the specialists' wealth. As a result, $\sigma_y < 0$ and increasingly so as θ_b rises.

The increased in specialist leverage increase the volatility of the consumption rate of specialists c_t . This point are also evident from inspecting equation (9), where we derive that the instantaneous volatility of the specialist's consumption is:

$$\left(\sigma - \frac{\rho}{1 - \rho y_t} \sigma_y \right) = \sigma \left(1 + \frac{\rho}{1 - \rho y_t} \frac{\hat{\theta}_b}{1 - \theta_s F'} \right)$$

In the unconstrained region, $\hat{\theta}_b$ is equal to zero, so the consumption volatility is just σ . But, in the constrained case, consumption volatility is increasing in $\hat{\theta}_b$. Moreover as y_t rises, so that intermediaries are more capital constrained, this effect is stronger.

The increased volatility in the specialists pricing kernel leads to a larger effective risk aversion, and can induce higher risk premium. In this sense, the risk aversion of the specialist endogenously increases as capital constraints are tightened:

Proposition 4 *In the constrained region the risk premium on the stock is:*

$$E_t[dR_t] - r_t dt = \gamma \sigma^2 \left(1 + \frac{\rho}{1 - \rho y} \frac{1}{1 - \theta_s F'} \hat{\theta}_b \right) \left(1 - \frac{F'}{F(1 - \theta_s F')} \hat{\theta}_b \right) dt$$

If $F' < 0$ then the risk premium is unambiguously higher in the constrained region than the unconstrained region.

The first term in parentheses in the expression for the risk premium captures the volatility of the pricing kernel and the second term in parentheses is the loading of the stock's return on the pricing kernel. When $F' < 0$, both effects contribute to a higher risk premium.¹⁰

¹⁰In our numerical solutions, we often find that F' is positive when just entering the constrained region before turning negative over a broader range of y . Despite this non-monotonicity in F , we find that the risk premium is higher in the constrained region than the unconstrained region with virtually all of our parameter choices. This finding suggests that the increased volatility of the pricing kernel dominates the determination of the risk premium.

4 Conditional Asset Prices

We solve the ODE, (10), to numerically illustrate the workings of our model. In Lemma 1 we noted that $\sigma_y = 0$ in the unconstrained region. For most parameterizations of the model we also find that $\mu_y < 0$. As a result, given an initial condition of $y < y^c$ it is not possible for the economy to enter the constrained region. In this section we present a number of asset price measurements conditional on a value of y . The exercise gives some insight into how the degree of capital constraints translates into asset prices. In the next section, we alter the model so that $|\sigma_y| > 0$ for all y , and then simulate the model.

The solution method is detailed in Appendix B. We solve the ODE subject to boundary conditions at $y = 0$ (fully unconstrained intermediation) and $y = y^b$ (bankrupt intermediaries).

4.1 Parameters

In choosing parameters for the calibration, we envision that the intermediary is managing an investment in a risky asset that encompasses most financial claims. Our intermediary represents an amalgam of banks, mutual funds, hedge funds and insurance companies.

Table 1: Baseline Parameters

Parameter		Value
g	Dividend growth	2%
σ	Dividend volatility	10%
ρ	Time discount rate	1%
γ	RRA of specialist	3
f	Fee	1.2%
m	Intermediation multiplier	1
$F(y^b)/F(0)$	Bankruptcy boundary	60%
The household has log preferences with a time discount rate of $\rho = 1\%$.		

We set $g = 2\%$. This number is in the range of estimates for economic growth, consumption growth, and stock-market dividend growth. We choose $\sigma = 10\%$. The number matches the dividend volatility of the stock market. Volatility of consumption growth is an order of magnitude smaller. Since our model concerns the financial position of intermediaries, matching σ to financial volatility seems appropriate.¹¹

We choose $\rho = 1\%$. This choice produces a riskless interest rate around 1% in the unconstrained region. We set the risk-aversion parameter for the specialists to be $\gamma = 3$. Both the ρ and γ numbers are in the range

¹¹For example, we could imagine that there is a class of households who receive a fixed labor income each period that must be added to aggregate dividends when computing aggregate consumption. This will tend to reduce consumption volatility below 10%.

that is typical in the literature. We provide sensitivity analysis based on alternative values of γ .

The intermediation parameters in the model are f , m , and δ . Intermediation fees for mutual funds fall in the range of 0.1% to 2%. In Appendix B, we show that to ensure uniqueness of our solution, parameters must satisfy

$$f > g(\gamma - 1) + \frac{\gamma\sigma^2}{2}(1 - \gamma). \quad (12)$$

This condition says that f must be sufficiently high that specialists do not die out in the limit. Choosing a higher value of f transfers wealth from households to specialists and makes the condition easier to satisfy. Given our choices of g , γ and σ , the right hand side of the condition is equal to 1%. We set f slightly higher to give the numerical solution some room. We choose $f = 1.2\%$ in the baseline. We also provide sensitivity analysis with alternative values of f .

We choose $m = 1$ in our baseline case. We also provide sensitivity analysis to alternative values of m . Note that m determines the effective size of the intermediation sector as well as the range of the constrained region. As m rises, intermediaries can raise more household funds, which effectively increases their size. The rise in m also makes it harder for capital constraints to bind and thereby reduces the constrained region of the state space.

We justify our choice of m based on intermediation data. Allen (2001) reports that in the 1990's, 50% of total wealth was intermediated while 50% was directly invested in financial assets. In our model, specialists play two roles. First, they directly invest their wealth in the risky asset. Second, every specialist intermediates some of the funds of the passive household, and all households are only exposed to the risky asset through intermediaries.

We will interpret the specialists as the direct investors of Allen, and the households as the intermediated investors. Thus, if specialists control half of the wealth of the economy, we want to ensure that they can intermediate the wealth of the other half of the economy, suggesting a choice of $m \geq 1$.

If we choose m much larger than one the constrained region becomes very small. In the simulations in the next section, we show that a choice of m around one leads the economy to enter the constrained region about 20% of the time. We can interpret this as corresponding to a recession once every five years, which is consistent with U.S. data.

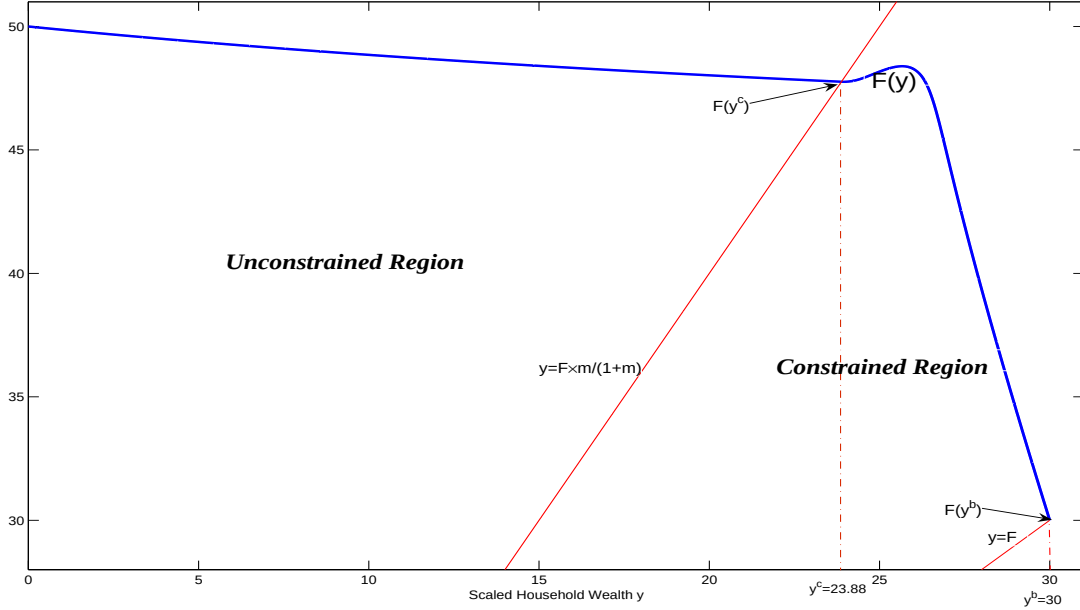
We choose δ so that the price/dividend ratio at $y = y^b$ is 60% of the price/dividend ratio at $y = 0$. In some sense, the choice of δ also defines the range of the price/dividend ratio. In the data, the price/dividend ratio on the stock market varies substantially from maximum to minimum: the minimum is roughly 30% of the maximum. We are being relatively conservative with this choice of $F(y^b)$. Since δ is exogenously specified, we always focus on parameters where, in the simulations, bankruptcy is an extremely unlikely event.¹² We

¹²For ease of interpretation, we transform the parameter δ to α , where α is defined as $\frac{F(y^b)}{F(0)}$. Since $F(0) = \frac{1}{\rho + g(\gamma - 1) + \frac{\gamma(1 - \gamma)}{2}\sigma^2}$ (see Appendix B) and $F(y^b) = \frac{\delta}{\rho}$, we have $\alpha = \delta \left(1 + \frac{g(\gamma - 1)}{\rho} + \frac{\gamma(1 - \gamma)}{2\rho}\sigma^2\right)$. Under baseline parameters, $\delta = 30\%$.

provide sensitivity analysis based on alternative values of $F(y^b)/F(0)$, which we denote as α later on.

4.2 The Equilibrium Price/Dividend Ratio $F(y)$

Figure 1: $F(y)$ for baseline case.



The equilibrium price/dividend ratio $F(y)$ is graphed for baseline case. Parameters are $\rho = 0.01$, $\gamma = 3$, $g = 2\%$, $\sigma = 10\%$, $f = 1.2\%$, $m = 1$, and $\alpha = 0.6$.

There are three effects that determine the shape of $F(y)$ in Figure 1: the bankruptcy boundary condition, an interest rate effect, and a risk premium effect. When $y = y^b$, $F(y)$ is pinned down by the bankruptcy boundary condition. For lower values of y , $F(y)$ is determined by the discounted value of the dividend stream, accounting for the possibility that at bankruptcy, the dividend stream ends and is replaced by $F(y^b)$. Thus for values of y close to y^b , the bankruptcy boundary weighs heavily in the determination of y . However, for smaller values of y , the chances of hitting bankruptcy are sufficiently small, that the discount rates applied to the dividend stream play the larger role.

In the unconstrained region for $y \in (0, y^c]$, future bankruptcy is impossible under our parameter assumptions, and only the discount rate effect determines $F(y)$. Since the risk premium is constant (see Proposition 3), variation in $F(y)$ is predominantly driven by variation in the interest rate. It is relatively easy to derive

(see Appendix B), that the sign of $F'(0)$ depends on

$$f - g(\gamma - 1) - \frac{\gamma\sigma^2}{2}(1 - \gamma) < 0,$$

under our parameter assumptions. Intuitively, when f is large, households (as a share of the economy) are slowly shrinking and transferring their wealth to specialists. Thus, specialists forecast their wealth to grow over time, and this determines the discount rate that specialists apply to financial assets. The discount rate depends on the speed at which wealth is being transferred to specialists. For y near 0, since households have little wealth, the speed is low, leading to a lower discount rate; for larger y 's, the speed increases, and the discount rate rises. This discount rate effect creates a downward sloping $F(y)$ in the unconstrained region.

In the constrained region, $y \in (y^c, y^b]$, both the interest rate and the risk premium change. As we have remarked (see Proposition 4), the risk premium is higher in the constrained region, which causes $F(y)$ to fall. However, for y just larger than y^c , $F(y)$ displays a non-monotonic behavior. The latter behavior is because a precautionary savings effect leads the interest rate to fall through the constrained region. For y close to y^c , the fall in the interest rate outweighs the rise in the risk premium, causing a reduction in the discount rate, and a rise in $F(y)$. However, for larger values of y , the risk premium rises, and the economy approaches the bankruptcy boundary, so that $F(y)$ falls.

Figure 2 illustrate the behavior of the interest rate and the risk premium as a function of y . Notice that going from y^c to y^b the risk premium increases by 240bps, while the interest rate decreases by 90bps, leading to a net increase in discount rates.

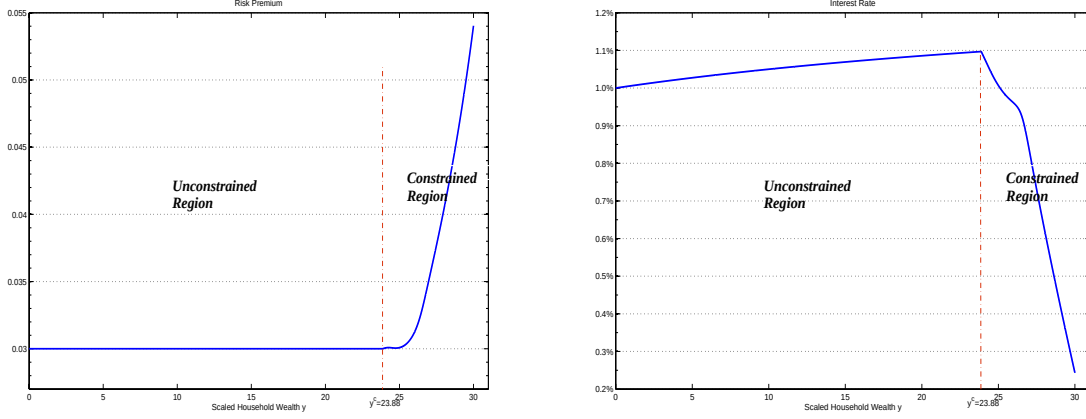
At $F(y^b)$ intermediaries become bankrupt. While the bankruptcy threshold is exogenous, the speed and probability at which the economy arrives to the bankruptcy point is endogenous, and is determined by the discount rate and risk premium effects.

4.3 Risk Premium and Interest Rate

Figure 2 (left panel) graphs the risk premium on the risky asset. As noted earlier, in the constrained region, specialists hold a leveraged position in the risky asset and their consumption rate becomes more volatile. From the figure we see that in the unconstrained region the risk premium is 3%, while through the constrained region, the risk premium rises another 2.4%.

The higher risk premium in the constrained region is due both to increased effective risk aversion of the specialist as well as increased stock volatility. The left panel of Figure 3 graphs the volatility of the risky asset. Price volatility changes in the constrained region for two reasons. First, as we have remarked the pricing kernel is more volatile in the constrained region. Second, $F(y)$ becomes more sensitive to y in the constrained region, so that shocks to the distribution of wealth have a magnified effect on price changes. In the region around $y = y^c$, since $F'(\cdot) > 0$, the latter effect dominates the former effect, which leads to a small dip in volatility. However, over the broader range, the two effects reinforce each other causing volatility to rise.

Figure 2: Risk Premium and Interest Rate for Baseline Case.



The left panel of the figure graphs the equilibrium risk premium for the baseline parameters. Despite the non-monotonicity in $F(\cdot)$, the risk premium is higher in the constrained region than in the unconstrained region. The right panel graphs the equilibrium interest rate.

In the right panel of Figure 3 we plot the Sharpe ratio on the risky asset and show that it rises as the economy enters the constrained region. Interestingly we find that the pattern of a rising Sharpe ratio and risk premium remain true for a wide range of parameters, even if these parameters produce a more pronounced non-monotonicity in $F(\cdot)$.

In the right panel of Figure 2 we graph the equilibrium interest rate. In the constrained region, the capital of intermediaries falls, leading households to reduce the amount of funds they are willing to invest with intermediaries. These withdrawn funds are used to purchase bonds. As specialists accommodate the demand for bonds, the equilibrium interest rate falls.

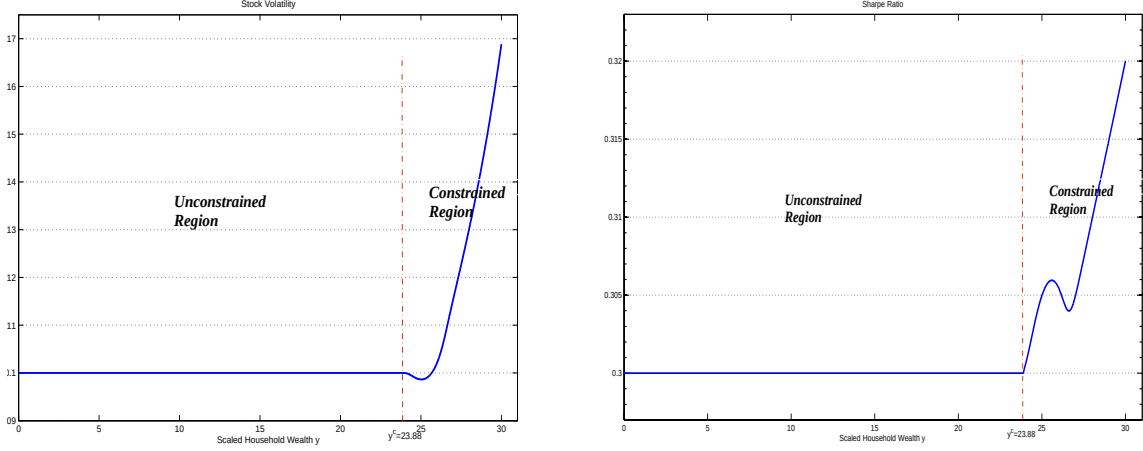
4.4 Market Liquidity

In the capital constrained region, an individual specialist who may want to sell some risky asset faces buyers with reduced capital. Additionally, since households reduce their (indirect) participation in the risky asset market, the set of buyers of the risky asset effectively shrinks in the constrained region. In this sense, the market for the risky asset “dries up”. On the other hand, if a specialist wished to sell some bonds, then the potential buyers include both specialists as well as households. Thus the bond is more liquid than the stock.

There are further connections we can draw between low intermediary capital and aggregate illiquidity periods.¹³ As we have already seen, a negative shock in the constrained region leads to a rise in risk premia,

¹³Eisfeldt (2004) presents a model in which aggregate liquidity varies across the business cycle. In Eisfeldt’s model, during bad

Figure 3: Volatility and Sharpe Ratio for Baseline Case.



The left panel of the figure graphs the equilibrium stock price volatility for the baseline parameters. The non-monotonicity around $y = y^c$ occurs because F is positively sloped in this region. The right panel graphs the equilibrium Sharpe ratio for the baseline parameters.

volatility, and fall in interest rate.

Figure 4 graphs the elasticity of the size of intermediaries with respect to the price of the risky asset. In the unconstrained region, this elasticity is one. A one percent fall in the price mechanically reduces the total assets under intermediation by one percent. In the constrained region, the elasticity rises as shocks to intermediaries have an indirect multiplier effect.¹⁴ Negative shocks reduce the specialists wealth/capital and thereby reduce the total funds that households are willing to delegate to the intermediaries. The withdrawn funds are invested in the riskless bond, in behavior consistent with the “flight to quality”.

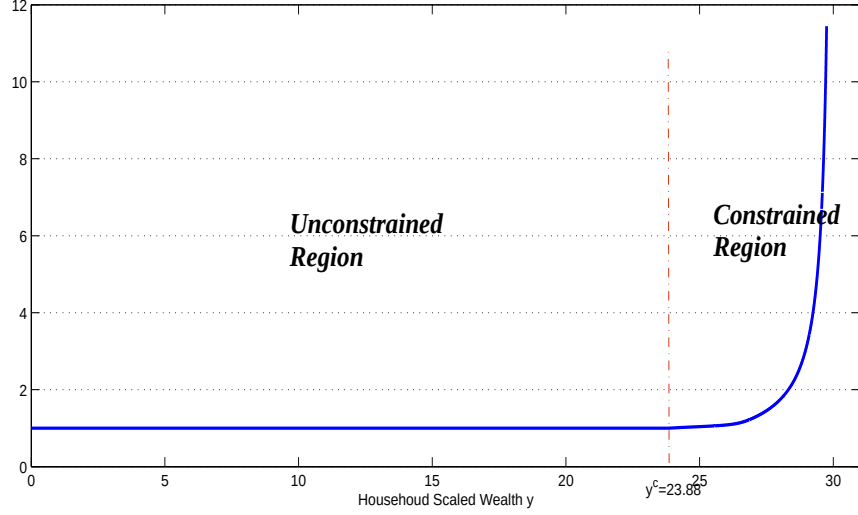
4.5 Comovement

Our model also explains the increase in comovement of assets that many papers have documented as an empirical regularity during periods of low aggregate liquidity. To show this in our model, we extend the model slightly.

We consider the pricing of a new risky asset, j , in the economy. Investment in this asset is also subject to times, adverse selection is high leading to higher illiquidity of assets.

¹⁴In the constrained region, the fund size is $D(F - y)(1 + m)$, and the elasticity is $\frac{\sigma(F - y) + (F' - 1)\sigma_y}{F - y} \cdot \frac{F}{\sigma F + F'\sigma_y}$, since the diffusion terms dominate.

Figure 4: Fund size elasticity for baseline case.



The elasticity of intermediary size with respect to the price of the risky asset is graphed for the baseline parameters. The elasticity increases sharply in the constrained region.

the delegation friction. We consider introducing an infinitesimal amount of intermediated asset- j in the sense that equilibrium is not affected by the introduction of this asset.¹⁵

We assume intermediated asset j has dividends,

$$\frac{dD^j}{D^j} = g^j(y) dt + \sigma^j dZ_t + \hat{\sigma}^j d\hat{Z}_t^j \quad \text{where} \quad \text{Cov}_t(dZ_t, d\hat{Z}_t) = 0$$

where dZ is the common factor modeled earlier, and $\hat{Z}^j$ is a second Brownian Motion, orthogonal to Z , which captures asset- j 's idiosyncratic variation. We allow the drift of its dividend growth $g^j(y)$ to be state-dependent.

As before, we guess that $P_t^j = D_t^j F^j(y_t)$, and define

$$dR_t^j = \frac{D_t^j dt + dP_t^j}{P_t^j}.$$

Note that since the asset is subject to the delegation friction, the pool of buyers for the asset also shrinks when capital constraints bind. In this sense, asset- j becomes less liquid when the market dries up. The price of asset- j must satisfy the Euler equation for the specialist:

$$E_t[d(e^{-\rho t} c_t^{-\gamma} R_t^j)] = 0.$$

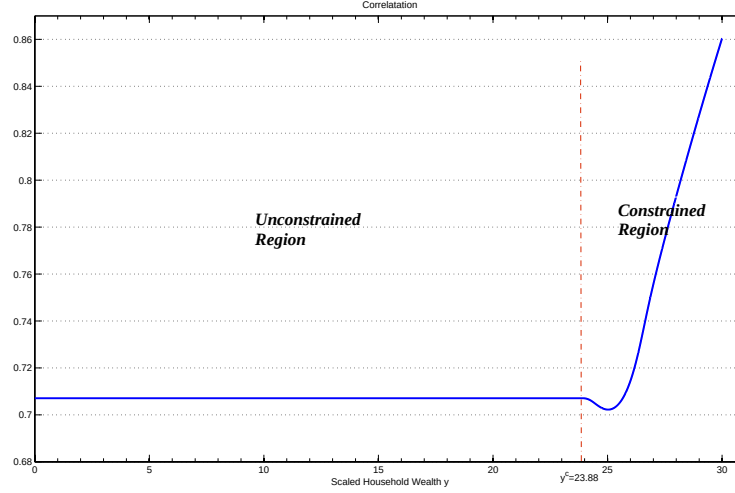
¹⁵If the asset was traded by both households and specialists then its introduction will have an effect on equilibrium, since the market is incomplete. However, introducing an intermediated asset will not alter equilibrium.

Repeating the steps as before we derive the following 2^{nd} -order ODE for asset j 's price/dividend ratio $F^j(y)$:¹⁶

$$\begin{aligned} \frac{E_t \left[dR_t^j \right]}{dt} - r_t &= g^j(y) + \frac{F^{j'}}{F^j} (\mu_y + \sigma_y \sigma^j) + \frac{1}{2} \frac{F^{j''}}{F^j} \sigma_y^2 + \frac{1}{F^j} - r_t \\ &= \gamma \left(\sigma - \frac{\rho}{1 - \rho y} \sigma_y \right) \left(\sigma^j + \frac{F^{j'}}{F^j} \sigma_y \right) \end{aligned}$$

The above equation is just the risk premium on asset j . In the unconstrained region, $\sigma_y = 0$ (since $\hat{\theta}_b = 0$), so that the risk premium is $\gamma \sigma \sigma^j$, which is positive if $\sigma^j > 0$. But in the constrained region $\sigma_y = \frac{-\hat{\theta}_b \sigma}{1 - \hat{\theta}_b F'} < 0$, the risk premium rises, and unambiguously so for the case of $F^{j'} < 0$. When the capital constraint binds and the aggregate market becomes less liquid, all intermediated assets become less liquid and yield a higher expected return. Note also that there is a spillover to asset- j from the risky asset in the constrained region. A negative innovation to D increases y and $\hat{\theta}_b$.

Figure 5: Correlation for baseline case.



The correlation between asset j and the market is graphed for the baseline parameters. When the economy enters the constrained region, the correlation falls slightly at first because of the non-monotonicity in F , but then rises as the capital constraint drives all prices.

Since the diffusion term on dR^j is $\left(\sigma^j + \frac{F^{j'}}{F^j} \sigma_y \right) dZ_t + \hat{\sigma}^j d\hat{Z}_t^j$, we can calculate the correlation between

¹⁶The boundary conditions for the ODE are, $F^j(0) = \frac{1}{\rho + \gamma g - g^j(0) - \frac{\gamma(1+\gamma)\sigma^2}{2} + \gamma\sigma\sigma^j}$, and $F^j(y^b) = \frac{\delta^j}{\rho + g - \sigma^2 - g^j(y^b) + \sigma\sigma^j}$, where δ^j is the potential loss in bankruptcy for asset- j .

asset j and the market return as¹⁷

$$Corr_t [dR^j, dR] = \left(1 + \frac{\hat{\sigma}^{j2}}{\left(\sigma^j + \frac{F^{j'}}{F^j} \sigma_y \right)^2} \right)^{-1/2} \times \text{sign} \left(\sigma^j + \frac{F^{j'}}{F^j} \sigma_y \right).$$

In the capital constrained region, the correlation between intermediated assets tends to rise. Consider the asset where $g^j = g$, $\sigma^j = \sigma$, $\hat{\sigma}^j > 0$, and $\delta^j = \delta$; this is an asset which is a noisy version of the market asset. For this case, $F^j(y) = F(y)$. In the unconstrained region, the correlation is constant, while in the constrained region the correlation rises. Figure 5 illustrates this effect for the case where $\hat{\sigma}^j = 0.1$ and our baseline parameters.

In the Fall of 1998 hedge fund crisis, the correlation of prices among intermediated asset rose. In many cases, the correlations rose so dramatically that the risk-management models of hedge funds failed. When intermediation capital falls, the investors in intermediated assets are simultaneously affected across all of their markets. In general equilibrium, the prices of these intermediated assets exhibit the comovement we observe in practice. Indeed, such comovement is symptomatic of an aggregate liquidity shortage.

4.6 Hedging asset

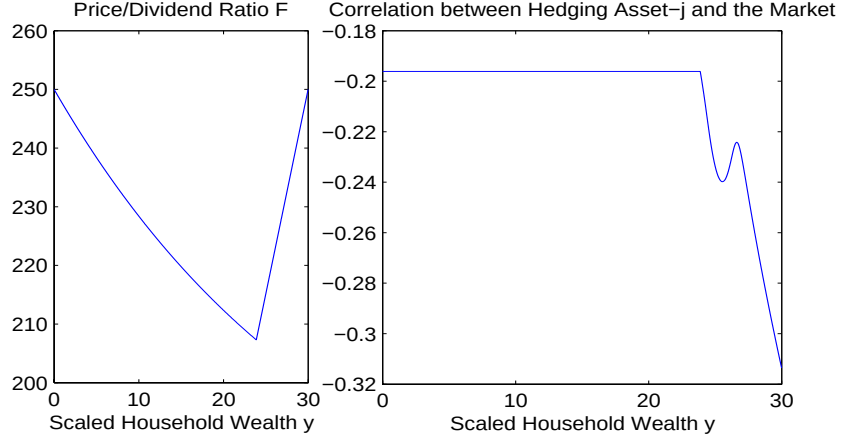
Figure 5 pertains to an asset that is similar to the market asset and loads positively on dividend shocks. We next consider a hedging asset; an asset whose payoff is negatively correlated with the market. For this asset, the negative correlation with the market increases when the economy enters the capital constrained region. In the constrained region, the specialist values the hedging asset more as liquidity falls. This leads the hedging asset to be increasingly sensitive to changes in market liquidity.

Figure 6 illustrates the case of the hedging asset. We choose $\sigma^j = -2\%$, i.e. the dividend growth of this asset has a negative loading on the common factor dZ . We also assume that $g^j(y)$ is an increasing function, so that the asset provides a hedge against reductions in market liquidity. For simplicity we choose $g^j(y)$ such that the solution of the ODE for asset- j is linear when $y \in [y^c, y^b]$. We also choose $\delta^j = 1$ (no bankruptcy loss).

The left-hand panel of 6 shows that the asset is indeed a hedge against declines in market liquidity in the constrained region. The price/dividend ratio rises for $y \in [y^c, y^b]$. The right-hand panel shows the correlation between the return on the hedging asset and the market. In the unconstrained region, y is deterministic and there are no shocks to market liquidity. Thus the correlation is constant in this region. In the constrained region, asset- j becomes increasingly sensitive to reductions in market liquidity, as does the market asset, leading to a higher (more negative) correlation.

¹⁷We compute the correlation between intermediated asset- j and the market asset. The empirical literature documents comovement among intermediated assets. It is straightforward to introduce two infinitesimal intermediated assets and compute their pairwise correlation. The correlation will follow a similar pattern to the one we have graphed.

Figure 6: Correlation for hedging asset.



The correlation between the hedging asset- j and the market is graphed for the baseline parameters. The left panel graphs the price/dividend ratio for asset- j . In the constrained region the price/dividend ratio rises because the asset is an increasingly valuable hedge against declines in market liquidity. The right panel graphs the correlation of asset- j with the market asset. The correlation becomes more negative in the constrained region.

We may think of the hedging asset- j as corresponding to the VIX. The VIX typically rises as market liquidity decreases. Our model explains this rise in terms of the increasing value of a hedging asset for the specialist. Our model also suggests that the VIX will become increasingly sensitive to changes in market liquidity in the constrained region, and thereby provides a rationale for why the VIX has been a meaningful indicator of market liquidity over the last 2 decades.

4.7 Liquidity factor

A number of recent papers have provided evidence of a liquidity factor governing asset returns (see Amihud, 2002, Acharya and Pedersen, 2003, Pastor and Stambaugh, 2003, and Sadka, 2003), suggesting that the marginal investor is particularly averse to times of low aggregate liquidity conditions. In our model, low liquidity corresponds to times when intermediation capital is low. The marginal investor is the specialist, and times of low intermediary capital are particularly bad times for specialists. Thus there is a natural reason to

expect that the marginal investor will be averse to assets that pay little in low aggregate liquidity states.

However, since our model is driven by a single source of uncertainty (the one-dimensional Brownian motion governing dividends), changes in both the risky asset price and intermediary capital are perfectly correlated. This makes it difficult to clarify the role of a liquidity factor for asset returns separate from the market factor.

The mechanics of our model does suggest such a factor. But the effect is multiplicative, rather than additive. In our model, shocks to dividends have an amplified effect on the economy because they affect both the cash-flows expected on the risky asset as well as the aggregate liquidity of intermediaries. In a standard model, shocks to dividends also affect the economy, but the amplification due to the change in liquidity will be absent.¹⁸

To clarify that our model does indeed have a separate liquidity effect, we develop a “additive” model. We consider the following thought experiment. We perturb our model by adding a second shock process that is orthogonal to dividends but which directly affects intermediary capital. We then trace the effects of this second factor on asset returns. The exercise gives us some understanding of the separate, additive, role of intermediary capital risk, without working out a full-blown two-factor model.

We suppose that nature randomly redistributes a small amount of wealth between intermediaries and households. The redistribution of w^h amounts to $\sigma_1 w^h dZ_{1,t}$, where $Z_{1,t}$ is orthogonal to Z_t and $\sigma_1/\sigma \rightarrow 0$. Thus this second shock process is small compared to the primary dividend process. Without loss of generality we assume $\sigma_1 > 0$.

For the households with log preferences, since this distribution only affects the actual return on their wealth (and σ_1 is small), their consumption policy remains $c^h = \rho w^h$. Also, as σ_1 is small, the equilibrium price/dividend ratio, $F(\cdot)$, remains unchanged.

Then, we can directly rewrite dy as,

$$dy = \mu_y dt + \sigma_y dZ_t - \sigma_1 y dZ_{1,t} \quad \sigma_1 > 0.$$

Recall that our derivations suggest that $\sigma_y < 0$ in the constrained region; i.e., a negative shock to dividends transfers capital away from intermediaries and towards the households. Similarly, a negative innovation in $dZ_{1,t}$ redistributes wealth away from specialists and towards households. Thus, $dZ_{1,t} < 0$ has the interpretation of an exogenous negative shock to intermediary capital, with no direct effect on the aggregate dividend process.

Also, note that the size of the intermediary shock is proportional to y . We make this assumption for two reasons. First, recall that σ_y is increasing in y . For ease of comparison, we choose the new shock to also increase in y . Second, the proportionality to y keeps the new shock in terms of a percentage so that it remains relatively “small” for all values of y .

¹⁸The amplification effect present in our model is also highlighted in the model of Xiong (2001). There is also a literature in macroeconomics that emphasizes the importance of collateral amplification effects in explaining business cycles (see Kiyotaki and Moore, 1997, or Krishnamurthy, 2003).

Our expressions for dR and dc/c remain as before, with the redefined dy . Thus we find that the risk premium on the risky asset is now,

$$E_t[dR_t] - r_t dt = \gamma \left(\sigma - \frac{\rho}{1 - \rho y} \sigma_y \right) \left(\sigma - \frac{F'}{F} \sigma_y \right) dt - \gamma \frac{F'}{F} \frac{\rho}{1 - \rho y} (\sigma_1 y)^2 dt. \quad (13)$$

The latter term on the right-hand side is new relative to our previous expressions. This term reflects the effect of shocks to intermediary capital on the aggregate market return. $dZ_{1,t}$ has no direct effect on dividends and hence on stock prices. It affects prices through a liquidity channel. Endogenously the risk aversion of the specialist is affected by y , and hence stock prices and consumption are affected by $dZ_{1,t}$.

Notice also that the new term resembles the quadratic component of the first risk premium term on the right hand side (i.e. if $\sigma_1 y$ was equal to σ_y , then the new term is exactly the quadratic component of the first term). This relation clarifies that the quadratic component is the “multiplier” that our model generates due to the liquidity channel. We will return to this point when quantifying the liquidity premium generated by our model.

As before, for $F' < 0$, the variation in intermediary capital induces an extra risk premium on the stock. Thus the extra term decreases the risk premium in the non-monotonic region around $y = y^c$, but then increases the premium as F slopes downwards.

We next consider pricing an intermediated asset, whose dividend process loads on $dZ_{1,t}$. We introduce an infinitesimal amount of asset- j with dividend process,

$$\frac{dD_t^j}{D_t^j} = g^j dt + \sigma^j dZ_t + \sigma_1^j dZ_{1,t}.$$

We consider the limiting case where $\frac{\sigma_1^j}{\sigma^j} \rightarrow 0$. We also assume that $g^j = g$ and $\sigma^j = \sigma$. As before, these assumptions imply,

$$P_t^j = D_t^j F(y_t).$$

Defining dR^j as the instantaneous return on asset- j , we find that the risk premium satisfies:

$$E_t[dR_t^j] - r_t dt = \gamma \left(\sigma - \frac{\rho}{1 - \rho y} \sigma_y \right) \left(\sigma - \frac{F'}{F} \sigma_y \right) dt + \gamma \frac{\rho}{1 - \rho y} \left(\sigma_1^j - \frac{F'}{F} \sigma_1 y \right) \sigma_1 y dt. \quad (14)$$

Combining equation (13) with (14) we find:

$$E_t[dR^j] - r_t dt = \overbrace{E_t[dR_t] - r_t dt}^{\text{Market}} + \overbrace{\gamma \frac{\rho}{1 - \rho y} \sigma_1 y \sigma_1^j dt}^{\text{Liquidity}}. \quad (15)$$

The first term on the right hand side of (15) is the return on asset- j for bearing market risk. Since $g^j = g$ and $\sigma^j = \sigma$, asset- j 's return has a β of one with the market return. As result, one component of asset- j 's risk premium is equal to the market risk premium. The second term on the right hand side of (15) is the risk premium for exposure to the new liquidity factor. The risk premium is positive and increasing in σ_1^j , the

loading on the liquidity factor. The risk premium also reflects the dependence on y that we have highlighted earlier. As y rises, the leverage of the specialist endogenously rises, and as a result his effective risk aversion increases. Thus, the risk premium also rises.

The thought experiment we have conducted makes clear that our model rationalizes a liquidity factor but through a multiplier mechanism rather than an additive mechanism. In an additive model, the risk premium is proportional to $\sigma_1 y$, the shocks to specialist consumption induced by the orthogonal shock to intermediation capital. In our multiplicative model, the volatility of specialist consumption is,

$$\sigma - \frac{\rho}{1 - \rho y} \sigma_y.$$

In the constrained region, $\sigma_y < 0$. The entire increase in consumption volatility is due to the effect of shocks to intermediation capital.

We quantify this liquidity effect in two ways. First, in our model the entire market is illiquid in the constrained region. Thus, the equity premium in our model in excess of that which would prevail if there were no intermediation friction (i.e. $\gamma\sigma^2$) can be thought of as the liquidity premium on the market asset.

Second, if we consider any asset- j , we can always write the excess return on this asset using a single- β representation:

$$E_t [dR^j] - r_t dt = \frac{Cov_t \left[dR^j, \frac{dc_t}{c_t} \right]}{Var_t \left[\frac{dc_t}{c_t} \right]} \times \gamma Var_t \left[\frac{dc_t}{c_t} \right] = \beta^{j,dc} \times \lambda^{dc}.$$

That is, projecting the asset return on the specialists' consumption growth, we can write the risk premium on consumption growth as,

$$\lambda^{dc} = \gamma Var_t \left[\frac{dc_t}{c_t} \right]$$

Thus another measure of the liquidity risk premium in our model is the difference between the risk premium on specialist consumption growth in our model, and one without intermediation frictions:

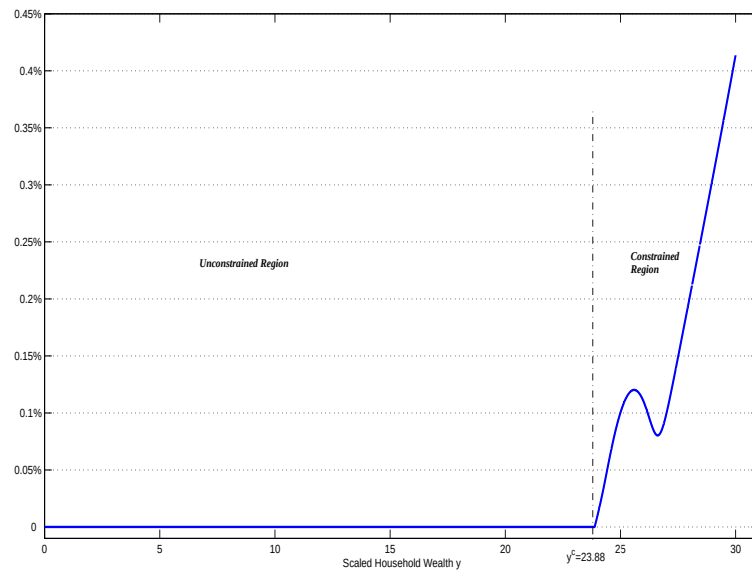
$$\text{Liquidity risk premium} = \gamma \left(\sigma - \frac{\rho}{1 - \rho y} \sigma_y \right)^2 - \gamma \sigma^2 dt, \quad (16)$$

The quadratic terms in this expression, or the terms in excess of excess of $\gamma\sigma^2$, captures the contribution of liquidity risk to the instantaneous risk premium in our economy.

Figure 7 graphs the instantaneous liquidity risk premium for the baseline case. In the unconstrained region, $\sigma_y = 0$, so that the liquidity risk premium is zero. In the constrained region, this premium rises. We will return to equation (16) in the next section of the paper when we simulate the model and measure the liquidity risk premium.

We use the above two measures to quantify the importance of liquidity in our model. These measures will not correspond to the risk premium on the liquidity factor identified in the empirical literature. In principle it is possible to match the empirically measured risk premium, but doing so necessitates taking a stand on the cross-section of assets traded in the marketplace; i.e how the asset payoffs covary with intermediation capital, and are distributed in the cross-section.

Figure 7: Instantaneous liquidity premium.



The instantaneous liquidity premium is graphed for the baseline case.

4.8 Sensitivity analysis

We present sensitivity analysis for γ , f , and α in Appendix D. The sensitivity analysis confirms that our results do not change substantively for different parameter values. In this subsection we focus on the sensitivity analysis with respect to m .

Figure 8 presents a number of asset price measurements for our baseline model as well as the case of $m = 0$ and the case of a larger m than the baseline ($m = 1.2$). The case of $m = 0$ is noteworthy because it corresponds to a pure limited participation model; one where the specialist participate in the risky asset market, and the households only participate in the riskless asset.

We see that setting $m = 0$ induces a rising risk premium through the entire y -region. This is because as y rises, the specialist's leverage increases leading to higher consumption volatility. The intermediation model which we highlight implies full participation for a substantial portion of the y -region. The risk premium only rises in a small fraction of the state space. But it is noteworthy that the risk premium rises sharply once the constraint binds, despite the fact that at the point where the constraint binds, the specialist has no leverage (compare the behavior of the risk premium from 0 to 5 for the $m = 0$ case, to the behavior of the risk premium from 25 to 30 of the baseline case). This effect is because a value of $m > 0$ implies that shocks to specialist wealth have an amplified effect on risk sharing. The households reduce participation more than one-for-one in the constrained region as specialist wealth falls and households withdraw funds from intermediaries. As a result, the risk premium rises sharply in the constrained region of the intermediation model.

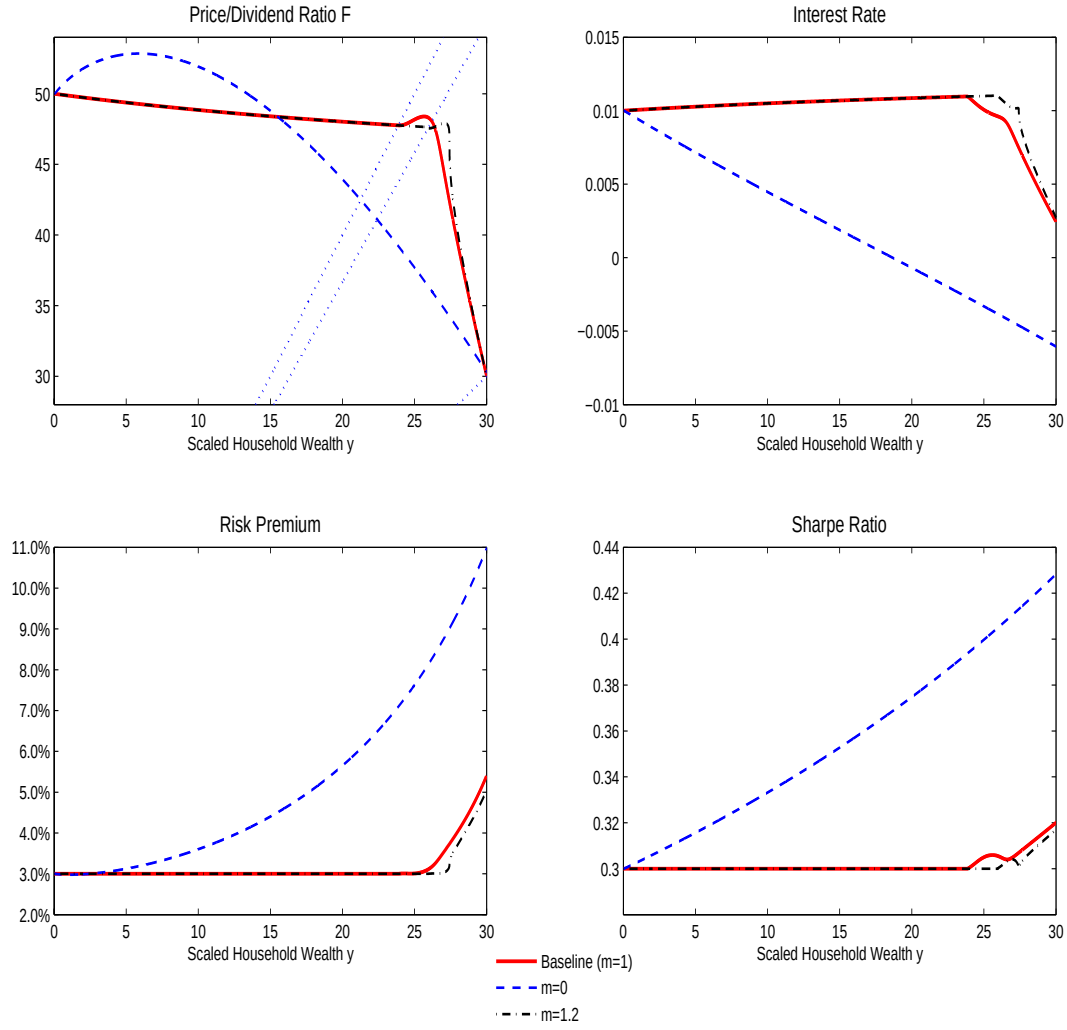
5 Model 2: Simulation

So far our model describes instantaneous asset price measurements that are conditional on the state of the economy. The average equity premium, for example, will depend on the likelihood of low intermediation capital. We turn to a simulation of our model to measure on average how asset prices are affected by the intermediation frictions.

5.1 Household heterogeneity

We assume that each household is made up of a pair of agents, a “stock investor” and a “debt investor”. At the beginning of date t , the households divide their wealth of w^h between the two members of the household in fractions λ (debt investor) and $1 - \lambda$ (stock investor). The debt investor uses his wealth to only purchase riskless debt. The stock investor behaves like the households we have described so far. He is matched with an intermediary and delegates a portion of his wealth subject to the intermediation constraints. We envision that at the end of each infinitesimal time interval the household aggregates the wealth from each member before making its consumption decision.

Figure 8: Effect of varying m .



The price/dividend ratio F , interest rate, risk premium, and Sharpe ratio for the case of $m = 0$ and $m = 1.2$. Baseline case result is plotted in solid line for comparison.

Our modeling of the household borrows from Lucas (1990)’s worker/shopper model. The modeling device of using a two-member household is a simple way to introduce heterogeneity among households without adding a new state variable. Since the members of the household aggregate their wealth before making their consumption decisions, w^h remains as a state variable and the problem is not substantially more complicated than the one we have described in prior sections.

Notice that with this modification, a portion of household wealth does not participate in the stock market even in the unconstrained region. Specialists, as in the constrained region, increase their leverage to absorb more of the risky asset. With this change, shocks in the unconstrained region affect specialists and households differently (as in the constrained region). Negative shocks can move the economy from the unconstrained region to the constrained region, thereby avoiding the problem with our previous model that the constrained region is an absorbing state. Our change is obviously necessary in order for our simulation to yield non-trivial results.

The second reason we make the change is because, in practice, even in the unconstrained region most intermediaries maintain some amount of debt borrowing. Although mutual funds and private equity funds typically have low leverage, banks and hedge funds always maintain some leverage. Thus, our modification also allows for a more realistic calibration of the model.

The details of the model solution are outlined in Appendix C. The introduction of λ requires us to modify $\hat{\theta}_b$ and θ_s . Subject to this change, the differential equation remains the same one as in the previous section.

5.2 Calibration

As noted earlier, Allen (2001) reports that in the 1990’s, 50% of total wealth was intermediated while 50% was directly invested in financial assets. We use this datapoint to anchor our simulation. Interpreting our specialists as the direct investors of Allen, and the households as the intermediated investors suggests centering our simulation around the point $w^I = w^h$.

The flow of funds data from the Federal reserve reports that total financial sector debt divided by total financial sector assets was 29% in 2000 (the number is 31% in 2005).

We choose $\lambda = 25\%$. With this value, households in the unconstrained region hold 25% of their wealth as debt which is supplied by the financial sector. At $w^I = w^h$, the intermediaries debt is 12.5% of the economy. Since all assets are eventually held by the intermediaries this leads to debt-to-assets ratio of 12.5%. Note that the debt-to-assets ratio rises in the constrained region (as debt rises, and the value of assets falls), so we want to be smaller than 30% of the data. As we see in our simulations, we can probably increase λ further. We are being conservative in choosing a smaller value of λ .

With $\lambda = 25\%$, households direct 37.5% of total wealth to intermediaries. We set m so that (1) the economy is not capital constrained at the initial condition, dictating a choice of $m \geq 0.75$; (2) the probability

of the economy entering the constrained region over 100 years of simulation is 20% (one in five years). As we show below, these considerations lead us to pick values of m around 0.9.

We increase the discount rate of the agents to 2% to maintain that the interest rate is around 1%. We decrease f to 0.6%. The fee is still realistic. As we noted in the previous section, the choice of f does not play an important role in determining the risk premium. It does help to ensure stability of our numerical solution. We also set the bankruptcy price/dividend ratio $\alpha = 70\%$ for the same stability reason.

Table 2: Baseline Parameters for Simulation

Parameter		Value
g	Dividend growth	2%
σ	Dividend volatility	10%
ρ	Time discount rate	2%
γ	RRA of specialist	3
f	Fee	0.6%
m	Intermediation multiplier	Near 0.9
λ	Intermediary debt	25%
$F(y^b)/F(0)$	Bankruptcy boundary	70%
The household has log preferences with a time discount rate of $\rho = 2\%$.		

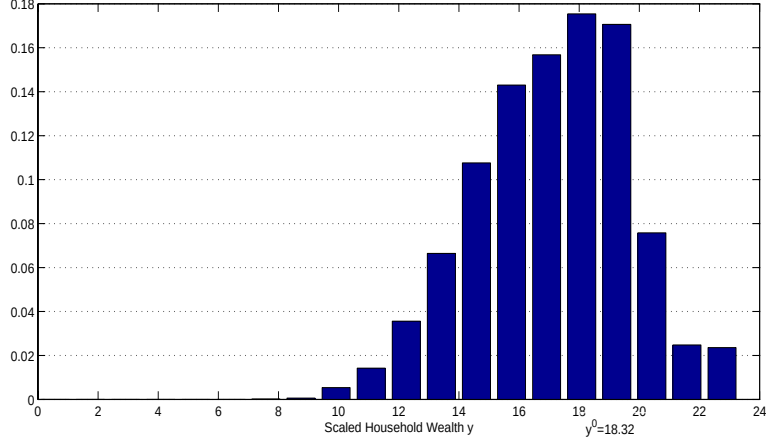
5.3 Distribution of state variable

Our economy, as with many two-agent economies, has the property that in the limit one or the other of the agents ends up with all of the wealth (at either $y = 0$ and $y = y^b$). We choose to simulate our model for 100 years, beginning with values of y in the neighborhood of y^c . As we show below, even after 100 years, the probabilities of reaching $y = 0$ or $y = y^b$ are small. The bankruptcy probability is always below 2% in the simulations.

We simulate the model 5000 times at a weekly frequency for 100 years and calculate the population values for a variety of statistics. For the return data we annualize the weekly returns and then construct annual observations by time-averaging the annualized weekly data (geometric average produces almost identical results). Hence each simulation yields 100 time-series data points, and based on these we compute a number of relevant economic variables. We also track y and classify our economy as capital constrained ($y > y^c$), unconstrained ($y < y^c$), or bankrupt (if y hits y^b).

Figure 9 plots the distribution of y after 100 years for the baseline parameters and a value of $m = 0.85$. The initial condition is denoted as $y_0 \equiv y(t = 0)$. We present an initial condition at the point where $w^h = w^l$ (denoted $y^0 = 18.32$). We divide $[0, y^b]$ evenly into 20 bins, simulate our model 5000 times, and then compute

Figure 9: Distribution of state variable.



The distribution of y after 100 years of simulating the model is graphed for the baseline parameters with $m = 0.85$. The model is simulated 5000 times using two different initial conditions. The initial condition is denoted as $y_0 \equiv y(t = 0)$. We present initial conditions at the point where $w^h = w^I$ (denoted y^0).

the sample frequency $p_{ij} = \Pr(y_{100} \text{ falls into the } j^{th} \text{ bin})$. The simulation accumulates almost 35 percent of the mass in the 16th and 17th bins.¹⁹ Note that the boundary of bin 16 and 17 is 18.67, which is close to our choices of y_0 . This distribution result is primarily driven by the almost zero μ_y around y^c .

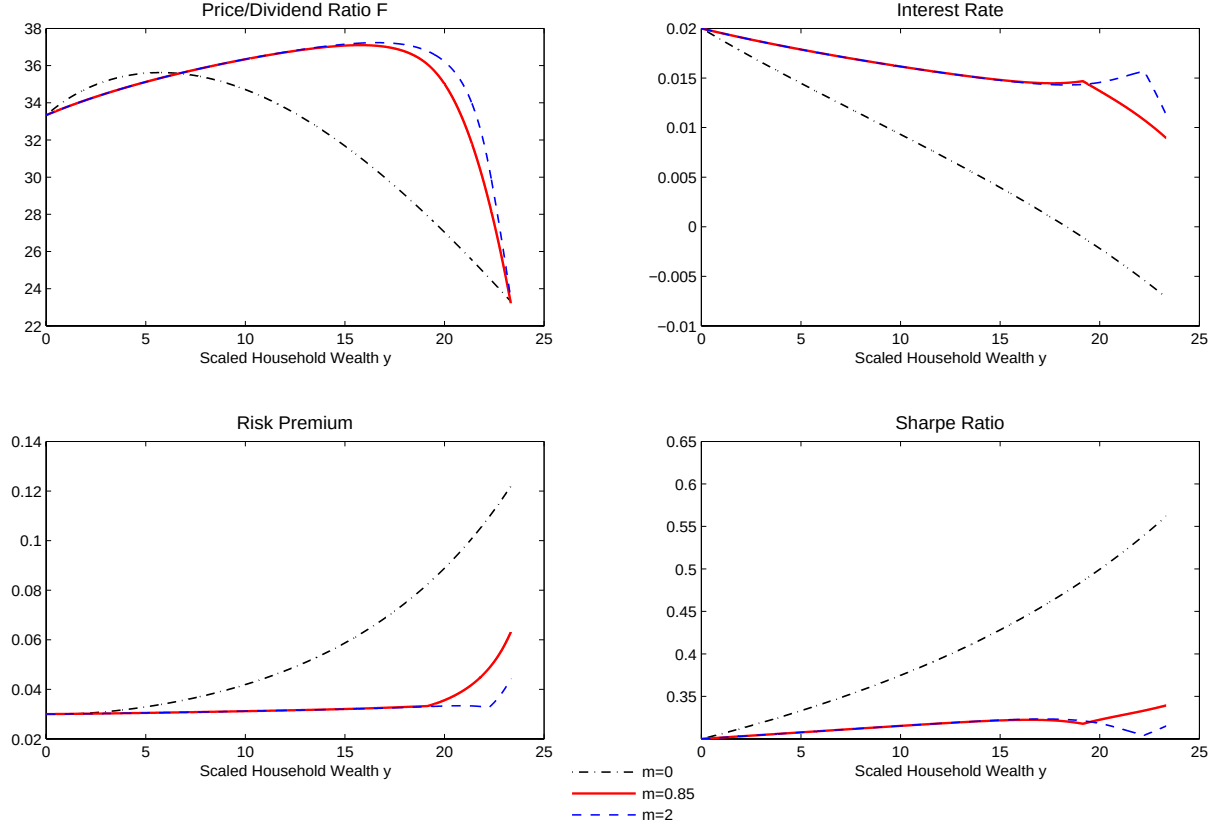
5.4 Results

Figure 10 graphs a number of relevant asset price measurements for the heterogeneous household model. The price/dividend ratio as well as many of the other measures are smoother than in the previous section, but the broad patterns for all the variables remain unchanged.

The only substantive change is that the risk premium is increasing uniformly through both the unconstrained and the constrained regions. We have now introduced some leverage into intermediaries in the unconstrained region, which results in reduced risk sharing and an increased risk premium even in the uncon-

¹⁹Note that the x -axis is our state-variable y . The length of each bin is $y^b/20 = 1.17$; hence the bin 16 is $[17.50, 18.67]$, and bin 17 is $[18.67, 19.83]$.

Figure 10: Asset Price Measurements when $\lambda = 0.25$



The price/dividend ratio, $F(y)$, interest rate, risk premium, and Sharpe ratio, are graphed for $m = 0$, $m = 0.85$, and $m = 2$, and the baseline parameters.

strained region. The range of the risk premium is similar to the previous sections. From levels around 3% in the unconstrained region, the risk premium doubles near the bankruptcy point of the constrained region.

Table 3 presents a number of statistics from the simulation. Our baseline simulation results are in the first column. The most striking result is that average risk premium is only 3.32%. In the benchmark model with $\gamma = 3$ we would find a risk premium of 3%. Thus, while the conditional risk premium in the model can rise 300bps with the tightness of capital constraints, the unconditional risk premium rises only about 32bps.

There are two reasons behind this result. First, from Figure 10 we see that the risk premium resembles a quadratic function of the capital constraints. The biggest effect is when the constraint places intermediaries close to the bankruptcy boundary. Second, we focus on simulation parameters for which the probability of bankruptcy is small, and in most of time our economy stays in the unconstrained region. In Table 3, we report that the probability of bankruptcy for the baseline simulation is less than 2%, and about 18% of time the

intermediaries are capital constrained. If we move the initial condition closer to the constrained region, the bankruptcy probability rises to 4.44%, and the average risk premium rises an additional 10**bps**.

Our failure to generate a large equity premium is mirrored in a low interest rate volatility (at most 4**bps** per year). The economy does not move between constrained and unconstrained regions enough to generate volatility.

The measured liquidity premium (see equation 16) is 44**bps** in our baseline simulations. This number is of the same magnitude as that reported in Figure 7. The average number also reflects that since $\lambda > 0$, the specialist will be sensitive to wealth shocks even in the unconstrained region.

The last column of Table 3 reports asset price measurements for the case where $m = 0$, which corresponds to a limited market participation model. The average equity premium in the model now rises to 3.92%, in keeping with Figure 10. In this model there is never any household participation in the equity market.

The last panel of Table 3 reports average statistics on the composition of wealth. Note that on average households delegate around 70% of wealth in the intermediation model. In the limited participation model, households invest none of their wealth in the stock market. We conjecture that combining our intermediation mechanism and a more sizable limited participation may generate both a larger equity premium as well as a sizable liquidity premium. We will pursue this avenue in future research.

Table 3: Simulation Results

	$m = 0.85$	$m = 0.95$	$m = 0.85$	$m = 0$
Starting Value y_0	$y^0 = 18.32$	$y^0 = 18.38$	$y^c = 19.16$	$y^0 = 15.61$
Panel A: Asset Pricing Statistics				
$E[R - r]$ (%)	3.32	3.31	3.42	3.92
$\sigma[R - r]$ (%)	10.34	10.29	10.65	10.77
$E[r]$ (%)	1.44	1.45	1.42	1.25
$\sigma[r]$ (%)	0.026	0.021	0.041	0.390
$\frac{E[R-r]}{\sigma[R-r]}$	0.32	0.32	0.32	0.35
$\rho(R, r)$ (%)	-1.45	-1.27	-0.06	-2.46
$E[P/D]$	36.44	36.50	35.83	34.31
Panel B: Constraints				
$\Pr(\text{Bankruptcy})$ (%)	1.62	1.24	4.44	6.61
$\Pr(\text{Constrained Region})$ (%)	18.05	10.69	42.07	100
$E(\text{Years in Constrained Region})$	1.04	0.88	1.72	100
$E[\text{Liquidity Premium}]$ (%)	0.44	0.43	0.44	1.23
Panel C: Wealth Composition and Intermediated Funds				
Average Debt (%) ^a	7.79	7.50	9.14	20.73
Participation in Stock Market (%) ^b	86.53	86.98	83.86	79.27
Household Delegation (%) ^c	71.17	71.66	66.85	0

^a Average debt in the economy, measured as $100 \times E[\theta_b/P]$.

^b Average total participation rate in stock market, measured as $100 \times E\left[\frac{\min(F - \lambda y, (1+m)(F-y))}{F}\right]$

^c Average fraction of households wealth that is intermediated.

References

- [1] Acharya, Viral and Lasse Pedersen, 2003, Asset Pricing with Liquidity Risk, *Journal of Financial Economics*, forthcoming.
- [2] Allen, Franklin, 2001, Do Financial Institutions Matter?, *Journal of Finance*, 2001, 56, 1165-1175.
- [3] Allen, Franklin and Douglas Gale, 1994, Limited Market Participation and Volatility of Asset Prices, *American Economic Review* 84, 933-955.
- [4] Allen, Franklin and Gary Gorton, 1993, Churning Bubbles, *Review of Economic Studies*, 60, 813-836.
- [5] Amihud, Yacov, 2002, Illiquidity and Stock Returns: Cross-Section and Time-Series Effects, *Journal of Financial Markets*, 5, 31-56.
- [6] Basak, Suleyman and Domenico Cuoco, 1998, An Equilibrium Model with Restricted Stock Market Participation, *Review of Financial Studies*, 11, 309-341.
- [7] Bates, David, 2003, Empirical Option Pricing: A Retrospection, *Journal of Econometrics*, 116, 387-404.
- [8] Berndt, Antje, Rohan Douglas, Darrell Duffie, Mark Ferguson, and David Schranz, 2004, Measuring Default Risk Premia from Default Swap Rates and EDFs, mimeo, Stanford University.
- [9] Brav, Alon, George Constantinides, and Chris Geczy, 2002, Asset Pricing with Heterogenous Consumers and Limited Participation: Empirical Evidence, *Journal of Political Economy*, 110(4), 793-824..
- [10] Brennan, Michael, 1993, Agency and Asset Pricing, working paper, UCLA.
- [11] Brunnermeier, Markus and Lasse Pedersen, 2005, Market Liquidity and Funding Liquidity, working paper, Princeton.
- [12] Cuoco, Domenico and Ron Kaniel, 2001, Equilibrium Prices in the Presence of Delegated Portfolio Management, working paper, University of Pennsylvania and Duke University.
- [13] Collin-Dufresne, Pierre, Robert Goldstein, and J. Spencer Martin, 2001, The Determinants of Credit Spread Changes, *The Journal of Finance*, 66, 2177-2207.
- [14] Dasgupta, Amil, Andrea Prat, and Michela Verardo, 2005, The Price of Conformism, working paper, LSE.
- [15] Dow, James and Gary Gorton, 1994, Arbitrage Chains, *Journal of Finance* 49, 819-849.
- [16] Eisfeldt, Andrea, 2004, Endogenous Liquidity in Asset Markets, *Journal of Finance* 59, 1-30

- [17] Froot, Kenneth, 2001. The Market for Catastrophe Risk: A Clinical Examination, *Journal of Financial Economics* 60, 529-571.
- [18] Froot, Kenneth A., and Paul G. O'Connell, 1999, The Pricing of US Catastrophe Reinsurance in *The Financing of Catastrophe Risk*, edited by Kenneth Froot, University of Chicago Press.
- [19] Gabaix, Xavier, Arvind Krishnamurthy and Olivier Vigneron, 2005, Limits of Arbitrage: Theory and Evidence from the MBS market, forthcoming, *Journal of Finance*.
- [20] Garleanu, Nicolae, Lasse Heje Pedersen, and Allen M. Poteshman, 2005, Demand-Based Option Pricing, Wharton mimeo.
- [21] Gromb, Denis and Dimitri Vayanos, 2002, Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs, *Journal of Financial Economics* 66, 361-407.
- [22] Grossman, Sanford J. and Zongquan Zhou, 1996, Equilibrium Analysis of Portfolio Insurance, *Journal of Finance* 51, 1379-1403.
- [23] Guvenen, Fatih, 2005, A Parsimonious Macroeconomic Model for Asset Pricing: Habit Formation or Cross-Sectional Heterogeneity?, working paper, University of Rochester.
- [24] Holmstrom, Bengt and Jean Tirole, 1997, Financial Intermediation, Loanable Funds, and the Real Sector, *Quarterly Journal of Economics* 112, 663-691.
- [25] Kiyotaki, N. and J. Moore, 1997, Credit Cycles, *Journal of Political Economy*, 105 (2), 211-248.
- [26] Krishnamurthy, Arvind, 2003, Collateral Constraints and the Amplification Mechanism, *Journal of Economic Theory*, 111(2), 277-292.
- [27] Kyle, Albert S. and Wei Xiong, 2001, Contagion as a Wealth Effect, *Journal of Finance* 56, 1401-1440.
- [28] Liu, Jun and Francis Longstaff, 2004, "Losing Money on Arbitrages: Optimal Dynamic Portfolio Choice in Markets with Arbitrage Opportunities" , *Review of Financial Studies* 17(3), 611-641.
- [29] Lucas, Robert E. Jr, 1990, Liquidity and interest rates, *Journal of Economic Theory* 50, 237-264.
- [30] Lucas, Robert E., Jr, 1978, Asset Prices in an Exchange Economy, *Econometrica*, 46, 1429-1445,
- [31] Mankiw, Gregory N, and Stephen Zeldes, 1991, The Consumption of Stockholders and Non-stockholders, *Journal of Financial Economics*, 29, pp. 97-112.
- [32] Pastor, Lubos and Robert Stambaugh, 2003, Liquidity Risk and Expected Stock Returns, *Journal of Political Economy*, 111, 642-685.

- [33] Sadka, Ronnie, 2003, Momentum, Liquidity Risk, and Limits to Arbitrage, working paper, University of Washington.
- [34] Shleifer, Andrei and Robert W. Vishny, 1997, The Limits of Arbitrage, *Journal of Finance* 36, 35-55.
- [35] Vayanos, Dimitri, 2004, Flight to Quality, Flight to Liquidity, and the Pricing of Risk, working paper, LSE.
- [36] Vissing-Jorgensen, Annette, 2002, Limited asset market participation and the elasticity of intertemporal substitution, *Journal of Political Economy*, 110(4), 825-853.
- [37] Xiong, Wei, 2001, Convergence Trading with Wealth Effects: An Amplification Mechanism in Financial Markets, *Journal of Financial Economics* 62, 247-292

A Appendix A: Proof of Proposition 1

Denote the value functions of household and specialist as $J^h(w^h; Y)$ and $J^I(w^I; Y)$ where w^h and w^I are household's and specialist's wealth respectively, and Y is the relevant state variable vector. Assume that they are smooth enough, and note that marginal values of wealth $J_w^h(w^h; Y)$ and $J_w^I(w^I; Y)$ must be positive.

Claim 1 follows directly from the assumption of the contract. For Claim 2, given $T^h \leq w^h$, participation constraint for household (Assumption 3) is:

$$E_t \left[J^h((w - T^h)(1 + \widetilde{dR}_t - fdt) + T^h + (1 - \beta)T\widetilde{dR}_t - Cdt; Y) \right] \geq \\ E_t \left[J^h((w - T^h)(1 + \widetilde{dR}_t - fdt) + T^h + T^h(\widetilde{dR}_t - fdt); Y) \right]$$

with the choice variable C . Since $(1 - \beta)T = T^h$, Taylor expansion immediately implies $C \leq T^h f$. Full bargaining power of the specialist and positiveness of J_w^I imply that $C = T^h f$, and given this contract the household is willing to hand over all his wealth T^h as long as the specialist's IC constraint is satisfied.

Now we have the IC constraint for specialist as:

$$E_t \left[J^I((w^I - T^I)(1 + \widetilde{dR}_t) + T^I + \beta T\widetilde{dR}_t + Cdt; Y) \right] \geq \\ E_t \left[J^I((w - T^I)(1 + \widetilde{dR}_t) + T^I + \beta T(\widetilde{dR}_t - xdt) + Tbdt + Cdt; Y) \right]$$

where the choice variable $T^I \leq w^I$. Plugging in $\beta T = T^I$ and carrying through Taylor expansion (note that 2^{nd} order terms cancel on both sides), we find that

$$J_w^I(T^I x - Tb) \geq 0$$

or,

$$T^I \left(\frac{x}{b} - 1 \right) \geq T^h.$$

Note that $T^I \leq w^I$, $T^h \leq mw^I$ where $m \equiv \frac{x}{b} - 1$. Q.E.D.

B Appendix B: ODE solution

In this appendix, we detail the solution of the ODE that characterizes the equilibrium.

B.1 ODE

Recall

$$g + \frac{F'}{F}\mu_y + \frac{1}{2}\frac{F''}{F}\sigma_y^2 + \frac{1}{F} + \frac{F'}{F}\sigma_y\sigma = \rho + \gamma g - \frac{\gamma\rho}{1 - \rho y}(\mu_y + \sigma_y\sigma) \quad (17) \\ + \gamma \left(\sigma - \frac{\rho}{1 - \rho y}\sigma_y \right) \left(\sigma + \frac{F'}{F}\sigma_y \right) - \frac{1}{2}\gamma(\gamma + 1) \left(\sigma - \frac{\rho}{1 - \rho y}\sigma_y \right)^2$$

and $\sigma_y = -\frac{\hat{\theta}_b}{1 - \theta_s F'}\sigma$, $\mu_y = \frac{1}{1 - \theta_s F'} \left(\theta_s + (r + \sigma^2 - g)\hat{\theta}_b - \rho y - f \min(m(F - y), y) + \frac{1}{2}\theta_s F''\sigma_y^2 \right)$.

B.1.1 Unconstrained Region: $y < y^c$

In this region $\theta_s = \frac{y}{F}$ and $\hat{\theta}_b = 0$; hence $\sigma_y = 0$ and $\mu_y = \frac{1}{1-\theta_s F'} (\theta_s - \rho y - f y)$, which yields the following first-order ODE to solve on the region $y \in [0, y^c]$:

$$\left(\frac{F'}{F} + \frac{\gamma \rho}{1 - \rho y} \right) \left(\frac{1}{F} - (\rho + f) \right) = \left(\frac{F'}{F} - \frac{1}{y} \right) \left(\frac{1}{F} - \left(\rho + g(\gamma - 1) + \frac{\gamma \sigma^2}{2} (1 - \gamma) \right) \right) \quad (18)$$

B.1.2 Constrained Region: $y^c \leq y \leq y^b$

In this region $\mu_y = \frac{1}{1-\theta_s F'} \left(\theta_s + (r + \sigma^2 - g) \hat{\theta}_b - \rho y - f m(F - y) + \frac{1}{2} \theta_s F'' \sigma_y^2 \right)$ and $\sigma_y = -\frac{\hat{\theta}_b}{1-\theta_s F'} \sigma$, where $\theta_s = \frac{m}{1+m}$. Substituting for μ_y , we find,

$$\begin{aligned} & \left(\frac{F'}{F} + \frac{\gamma \rho}{1 - \rho y} \right) \left(\frac{1}{1 - \theta_s F'} \right) \left(\theta_s + \hat{\theta}_b (r - g) - \rho y - f m(F - y) \right) + \frac{1}{F} \\ & \quad + \frac{1}{2} F'' \sigma_y^2 \left(\frac{1}{1 - \theta_s F'} \right) \left(\frac{1}{F} + \theta_s \frac{\gamma \rho}{1 - \rho y} \right) \\ = & \rho + g(\gamma - 1) + \gamma \left(\sigma - \frac{\rho}{1 - \rho y} \sigma_y \right) \left(\sigma + \frac{F'}{F} \sigma_y \right) \\ & \quad - \frac{1}{2} \gamma (\gamma + 1) \left(\sigma - \frac{\rho}{1 - \rho y} \sigma_y \right)^2 \end{aligned} \quad (19)$$

where,

$$\begin{aligned} r = & \rho + g\gamma - \frac{\rho \gamma}{1 - \rho y} \frac{\theta_s + (r - g) \hat{\theta}_b - \rho y - f m(F - y) + \frac{\sigma^2}{2} \theta_s F'' \frac{\hat{\theta}_b^2}{(1 - \theta_s F')^2}}{1 - \theta_s F'} \\ & - \frac{\gamma (\gamma + 1) \sigma^2}{2} \left(1 + \frac{\rho \hat{\theta}_b}{1 - \rho y} \frac{1}{1 - \theta_s F'} \right)^2 \end{aligned}$$

We define a function, $G(y) \equiv \frac{1}{1-\theta_s F'}$; with this definition, we can write $G' = \theta_s G^2 F''$, and

$$\sigma_y = -\frac{\hat{\theta}_b}{1 - \theta_s F'} \sigma = -\hat{\theta}_b \sigma G.$$

Then rewriting (19),

$$\begin{aligned} G' \frac{(\hat{\theta}_B \sigma)^2}{2} - G \left(\frac{1}{\theta_s F} + \frac{\gamma \rho}{1 - \rho y} \right) = & \rho + g(\gamma - 1) - \frac{1}{F} \\ & + \frac{1}{2} \gamma \sigma^2 \left(1 + \frac{\rho}{1 - \rho y} \hat{\theta}_B G \right) \left(\frac{2(y - G \hat{\theta}_b)}{\theta_s F} - (1 + \gamma) \frac{1 - \rho y + \rho G \hat{\theta}_b}{1 - \rho y} \right) \\ & - \left(\frac{G - 1}{\theta_s F} + \frac{\gamma \rho}{1 - \rho y} G \right) \left(\theta_s + \hat{\theta}_b (r - g) - \rho y - f m(F - y) \right) \end{aligned}$$

and

$$r = \frac{\rho + g\gamma - \frac{\rho \gamma G}{1 - \rho y} \left(\theta_s - g \hat{\theta}_b - \rho y - f m(F - y) + \frac{\sigma^2}{2} G' \hat{\theta}_b^2 \right) - \frac{\gamma (\gamma + 1) \sigma^2}{2} \left(1 + \frac{\rho \hat{\theta}_b G}{1 - \rho y} \right)^2}{1 + \frac{\rho \gamma G \hat{\theta}_b}{1 - \rho y}}.$$

We combine these two pieces, using the relation, $\hat{\theta}_b \left(\frac{G-1}{\theta_s F} + \frac{\gamma \rho G}{1-\rho y} \right) = -\frac{y-G\hat{\theta}_b}{\theta_s F} + \frac{1-\rho y+\gamma \rho G\hat{\theta}_b}{1-\rho y}$, and arrive at a final expression of the ODE:

$$\begin{aligned} & G' \frac{(\hat{\theta}_b \sigma)^2}{2} \frac{G}{\theta_s F} \left(\frac{1+\rho y(\gamma-1)}{1-\rho y+\rho \gamma G \hat{\theta}_b} \right) \\ = & \rho + g(\gamma-1) - \frac{1}{F} + \frac{\gamma(1-\gamma)\sigma^2}{2} \left(1 + \frac{\rho G \hat{\theta}_b}{1-\rho y} \right) \frac{y-G\hat{\theta}_b}{\theta_s F} \left[\frac{1-\rho y-\rho G \hat{\theta}_b}{1-\rho y+\rho \gamma G \hat{\theta}_b} \right] \\ & - \left(\frac{(1-\rho y)(G-1)}{\theta_s F} + \gamma \rho G \right) \frac{\theta_s + \hat{\theta}_b(g(\gamma-1) + \rho) - \rho y - f m(F-y)}{1-\rho y+\rho \gamma G \hat{\theta}_b}. \end{aligned} \quad (20)$$

Since,

$$F' = \frac{1}{\theta_s} \frac{G-1}{G},$$

we arrive at a system of first-order ODE's (in F and G) to solve on the region $y \in [y^c, y^b]$.

When $m = 0$, the ODE is:

$$\begin{aligned} F'' \frac{(y\sigma)^2}{2F} = & \rho + g(\gamma-1) - \frac{1}{F} + \frac{\gamma(1-\gamma)\sigma^2}{2} \left(\frac{1}{1-\rho y} \right) \left[\frac{1-2\rho y}{1+\rho y(\gamma-1)} \right] \\ & - \left(\frac{F'}{F} (1-\rho y) + \gamma \rho \right) \frac{y g(\gamma-1)}{1+\rho y(\gamma-1)}. \end{aligned}$$

B.2 Boundary Conditions and Numerical Approach

B.2.1 Unconstrained Region: $0 < y < y^c$

First, we solve F for $y \in (0, y^c]$. From (18), we can derive $F(0) = \frac{1}{\rho+g(\gamma-1)+\frac{\gamma\sigma^2}{2}(1-\gamma)}$. Note that $F(0)$ is the stock price when the economy only comprises of specialists (the households have zero wealth). Although we never reach this limit, when y is small the economy must be “close” to the limit case, in order to rule out Ponzi games (see below). Then, continuity of F at the origin requires $F(0+) = F(0)$. Utilizing this continuity, the singularity at $y = 0$ allows us to derive

$$F'(0) = \gamma \rho \left(\frac{1}{\rho+f} - F(0) \right);$$

and the sign of $F'(0)$ depends on the sign of $g(\gamma-1) + \frac{\gamma\sigma^2}{2}(1-\gamma) - f$.

The behavior of the solution to our 1st-order ODE (18) depends crucially on the above parameterization conditions. For better understanding the dynamic economic evolution behind the solution of (18), it is helpful to have the next Lemma.

Lemma 3 *Suppose there is no delegation friction in this economy and $f \neq g(\gamma-1) + \frac{\gamma\sigma^2}{2}(1-\gamma)$. Then there does not exist $y^* \in \left(0, \frac{1}{\rho}\right)$ to have $F(y^*) = \frac{1}{\rho+f}$.*

Proof. Note that the (scaled) wealth change of passive investors, μ_y , is proportional to $\theta_s - (\rho+f)y = y(\frac{1}{F} - (\rho+f))$ (the dividend income less the consumption and delegation fee; no bond here). Suppose there

exists y^* so that $F(y^*) = \frac{1}{\rho+f}$, which implies that $\mu_y = 0$ and the economy stays at (or approaches to) y^* with $F(y^*) = \frac{1}{\rho+f}$. But we know in the absorbing state the equilibrium price/dividend ratio implied by the active specialist is $F(0)$ (note that then specialist just consumes $1 - \rho y^*$ fraction of total endowment, or converges to this limit), which leads to a contradiction. ■

Now we are ready to discuss our three cases, where the idea is similar to the “saddle path” argument. Assume away delegation frictions; so the relevant domain for ODE (18) is $\left(0, \frac{1}{\rho}\right)$. Note that the ending point $\frac{1}{\rho}$ represents the world where households are wealthy enough to consume everything.

1. $f > g(\gamma - 1) + \frac{\gamma\sigma^2}{2}(1 - \gamma)$. In this case there is a unique solution for F satisfying the continuity condition $F(0+) = F(0)$, $F'(y) < 0$, and $F\left(\frac{1}{\rho}\right) = \frac{1}{\rho+f}$. Since $F(0) > \frac{1}{\rho+f}$ it is not difficult to check that in this solution we have $\frac{dy}{dt} = \mu_y < 0$, which implies that the households' wealth diminishes with time. Intuitively, we need a large management fee to ensure that the wealth is slowly transferred from households to the intermediary sector. Now for any $y > 0$, $F(y)$ is determined by the future price, which is $F(0)$.

We find other solutions diverges ($F(0+) = +\infty$ or $-\infty$) on the singular point $y = 0$. $F(0+) = -\infty$ is easily ruled out by no-arbitrage, or the result that F must stay above $\frac{1}{\rho+f}$ (Lemma 3). On the other hand, since y_t converges to 0 deterministically when $t \rightarrow \infty$, this implies that in those solutions with $F(0+) = +\infty$ the specialists are playing Ponzi game with each other.

2. $f < g(\gamma - 1) + \frac{\gamma\sigma^2}{2}(1 - \gamma)$. Due to relatively small management fee the households become wealthier over time, or $\frac{dy}{dt} = \mu_y > 0$.²⁰ It is because $F(0) < \frac{1}{\rho+f}$ and there is no $F(y)$ to be above $\frac{1}{\rho+f}$. In this case there are continuum of solutions for F satisfying the continuity condition $F(0+) = F(0)$, and $F'(0) > 0$. However, under this parameter assumption, $y = \frac{1}{\rho}$ plays the same role as $y = 0$ in the first case. As a result, we find that among these solutions only one of them will land at $F\left(\frac{1}{\rho}\right) = \frac{1}{\rho+f}$, and other solutions diverges at this ending point. The result in Lemma 3, combined with the similar “no-arbitrage” argument, show that this is the only equilibrium solution in this case (without capital constraints).

3. $f = g(\gamma - 1) + \frac{\gamma\sigma^2}{2}(1 - \gamma)$. In this case constant function $F(y) = \frac{1}{\rho+g(\gamma-1)+\frac{\gamma\sigma^2}{2}(1-\gamma)} = \frac{1}{\rho+f}$ solves (18), and $\mu_y = 0$. The appropriate management fee just balances the passive households' dividend income and their cash outflow (consumption and management fee), so any wealth point y works as an absorbing state.

Throughout this paper we choose the first parameter case, not only because this is consistent with most of existing calibration specifications, but because it also gives rise to a unique equilibrium once delegation friction is introduced (see below). It is less appealing to have specialists “die out” slowly in the economy.

²⁰Even though $F' > 0$, it is easy to check that in solution to (18) $1 - \frac{y}{F}F'$ stays bounded below from 0.

In sum, under the condition $f > g(\gamma - 1) + \frac{\gamma\sigma^2}{2}(1 - \gamma)$, in the unconstrained region $y_t \rightarrow 0$ deterministically with time, and each $F(y)$ for $y \in (0, y^c]$ is pinned down by the future stock price $F(0)$ through (18).

B.2.2 Constrained Region: $y^c \leq y \leq y^b$

From the unconstrained region, we obtain a boundary at $y^c = \frac{m}{m+1}F(y^c)$. We require continuity of F , $F(y^c+) = F(y^c)$. At the other boundary, $y^b = F(y^b) = \frac{\delta}{\rho}$. Hence, our problem is second-order ODE with two boundary conditions, and the shooting method is the natural solution method to employ. Specifically, starting from $(\frac{\delta}{\rho}, \frac{\delta}{\rho})$, we attempt different values for $\phi = \tilde{F}'(y^b)$ and integrate equation (19) down to y^c ; when $\tilde{F}(y^c)$ hits $F(y^c)$ (in fact, until the solution pastes smoothly to the unconstrained region solution), we pin down the desired solution. We use *Matlab_7's* built-in solver *ode15s* with tolerance level 10^{-12} to numerically solve our problem; other solvers, such as *ode45*, *ode15i* etc., deliver almost identical results once we find the correct ϕ .

There is one theoretical caveat, though. Notice that (y^c, F^c) is a singular point for (19) (at this point $\hat{\theta}_b = 0$ so $\sigma_y = 0$). Theoretically we cannot rule out the situation where the solutions are too stable to be unique (i.e., they always land at the same point regardless of the initial condition). Fortunately, the concern does not seem serious for any of the parameters we study. We find that the ending value $\tilde{F}(y^c)$ is very sensitive to our starting trial ϕ and usually diverges away from (y^c, F^c) , suggesting that our solution is unique.

What happens when $f < g(\gamma - 1) + \frac{\gamma\sigma^2}{2}(1 - \gamma)$? Because in this case y tends to grow with time, (y^c, F^c) is no longer pinned down by $F(0)$. When there is no capital constraint, it is determined by $y = \frac{1}{\rho} > y^c$ (see Section B.2.1). However, in the presence of capital constraint, (y^c, F^c) is indeterminate. Intuitively, now the bankruptcy state $(\frac{\delta}{\rho}, \frac{\delta}{\rho})$ or the capital constraint threshold point (y^c, F^c) becomes the remote future for any y between y^c and y^b , and the endogenous boundary (y^c, F^c) gives rise to a multiplicity of solutions. More specifically, there could be continuum of equilibrium price/dividend ratio F 's for $y \in [y^c, y^b]$: for any (y^c, F^c) with $y^c = \frac{m}{1+m}F^c$ and $F^c < \frac{1}{\rho+f}$, combining with the bankruptcy state $(\frac{\delta}{\rho}, \frac{\delta}{\rho})$ we could determine an equilibrium evolution path for F . In any of these solutions, if we start from $y < y^c$, then the households' wealth shrinks and the economy slowly slides into the capital constrained region $[y^c, y^b]$. Once the capital constrained region is reached, the economy stays inside until it hits the bankruptcy state.

C Appendix C: Simulation

C.1 Modified ODE

The only difference between this model and our previous one is in the household's assets positions. We arrive at the same second order ODE after simply substituting the new scaled bond position $\hat{\theta}_b^{sb}$ and the new stock position θ_s^{sb} into those expressions in Lemma 2.

1. If $mw^I \geq (1 - \lambda)w^h$ then all of the stock investor's wealth $(1 - \lambda)w^h$ is intermediated, while the debt

investor purchases $\lambda w^h = \lambda y D$ of the riskless bond. The specialist's positions are α_b and α_s . The relations, $w^I = P - w^h = D(F - y)$ along with the trust constraint and market clearing in the short-term bond imply that,

$$\frac{F - y + (1 - \lambda)y}{F - y} \alpha_b + \lambda y D = 0,$$

where the first term is the total bond position of mutual fund (including stock investors holdings), and the second term $\lambda y D$ is the bond position directly held by the debt investor. So $\alpha_b = -\frac{\lambda y(F - y)}{F - \lambda y} D$, and as a result the stock investor's indirect bond holding is $\frac{(1 - \lambda)y}{F - y} \alpha_b = -\frac{\lambda(1 - \lambda)y^2}{F - \lambda y} D$. Therefore the household's total scaled bond holding is

$$\hat{\theta}_b^{sb} = -\frac{\lambda(1 - \lambda)y^2}{F - \lambda y} + \lambda y = \lambda y \frac{F - y}{F - \lambda y}.$$

Applying a similar argument to α_s , we have $\theta_s^{sb} = \frac{(1 - \lambda)y}{F - y} \alpha_s = \frac{(1 - \lambda)y}{F - \lambda y}$. The scaled delegation fee is $(1 - \lambda)fy$.

The intermediation sector is capital constrained when $mw^I = (1 - \lambda)w^h$, or $m(F - y) = (1 - \lambda)y$. We can solve for this cutoff as

$$y^c = \frac{m}{1 - \lambda + m} F(y^c).$$

and the economy is in the unconstrained region if $0 < y \leq y^c$.

2. If $mw^I < (1 - \lambda)w^h$ (or $y > y^c$) then the intermediation sector is capital constrained. The stock investor will invest mw^I into the intermediary, and put the rest of his wealth $(1 - \lambda)w^h - mw^I$ into the riskless bond; the debt investor still invests λw^h into the short-term bond. Since all stocks are held through intermediaries, and the ratio of intermediated funds to the specialist's capital is always m , the trust constraint tells us that $\theta_s^{sb} = \frac{m}{1 + m}$. Hence $\hat{\theta}_b^{sb} = y - \theta_s^{sb}F(y)$, and the scaled delegation fee is $fm(F - y)$.
3. Intermediaries go bankrupt when $w^I = P - w^h = 0$. The bankruptcy threshold is implicitly defined by the equation $y^b = F(y^b) = \frac{\delta}{\rho}$.

C.2 Solution Method

Note that the ODE, (20), and the boundary conditions $F(0) = \frac{1}{\rho + g(\gamma - 1) + \frac{\gamma\sigma^2}{2}(1 - \gamma)}$ and $F(y^b) = \alpha F(0)$ remain unchanged. Similar to the previous method, we could attempt different $\phi = F'(y^b)$ so that the solution to (20) lands at $F(0)$. It turns out that, due to the high nonlinearity of (20) and the singularity at $y = 0$, the *ode15s* in this backward-shooting scheme diverges easily for y close to 2. Note that this observation implies the uniqueness of our solution, and this uniqueness holds even when $f < g(\gamma - 1) + \frac{\gamma\sigma^2}{2}(1 - \gamma)$ because of the non-dying 2^{nd} -order term (households have some positive bond position always).

To overcome this issue, we adopt a “forward-shooting and line-connecting” method. More specifically, starting from $\epsilon > 0$, for each trial $\phi \equiv \tilde{F}'(\epsilon)$ we set $\tilde{F}(\epsilon) = F(0) + \phi\epsilon$. Since $(\epsilon, \tilde{F}(\epsilon))$ is off from the singularity, via attempting different ϕ 's we could apply the standard shooting method to obtain the desired

solution F that lands at $(y^b, F(y^b))$. For $y < \epsilon$, we simply approximate the solution by a line connecting $(0, F(0))$ and $(\epsilon, F(\epsilon))$, and by construction this line meets the solution F tangentially for $\epsilon \leq y \leq y^b$. For Model 2 simulation parameters, we use $\epsilon = 0.15$ which gives error bounded by 3×10^{-7} for $y < \epsilon$, and different ϵ 's deliver almost identical solutions for $y \geq 1$. Because we are mainly interested in the solution behavior near y^c and onwards, our main calibration results are free of the approximation errors due to the choice of ϵ . Finally, we find that, in fact, these errors are at the same magnitude as those generated by the capital constraint around y^c (3.5×10^{-7}).

D Appendix D: Sensitivity

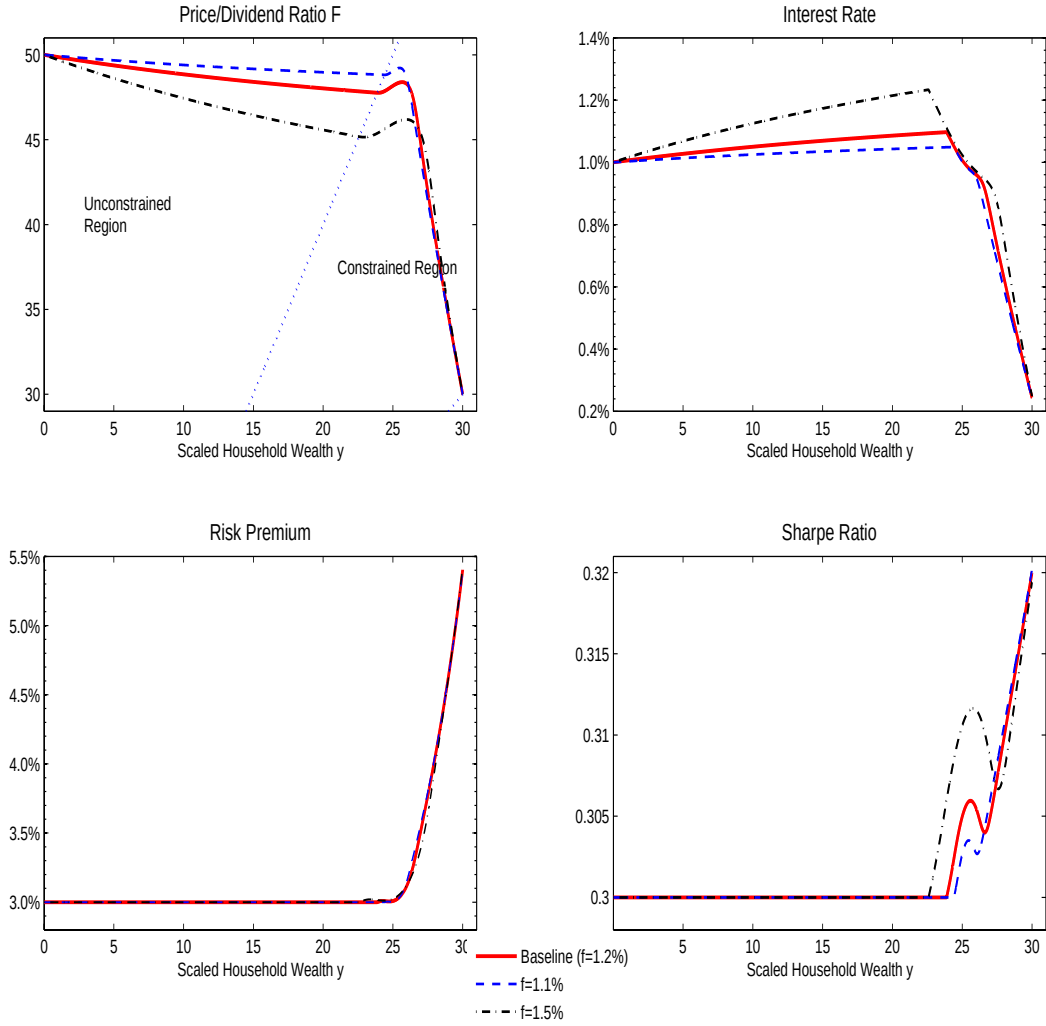
In Figure 11 we plot the price/dividend ratio F , interest rate, risk premium, and Sharpe ratio for the case of $f = 1.1\%$, $f = 1.5\%$, as well as our baseline case of $f = 1.2\%$. The lower value of f reduces the slope of $F(y)$, consistent with the intuition we provided earlier: wealth is transferred more slowly from households to specialists when f is lower, leading to a smaller discount rate effect. The risk premium effect continues to dominate the shape of F in the constrained region. In the panel for the risk premium, we see that the risk premium behaves almost identically across these different values of f .

In Figure 12 we illustrate the behavior of the economy for different values of γ . We consider $\gamma = 1$ and $\gamma = 2$ in addition to the baseline case of $\gamma = 3$. Higher values of γ raise the risk premium and Sharpe ratio, as one would expect. Roughly speaking, the risk premium doubles in all cases when moving from the unconstrained region to the point $y = y^b$ when intermediaries are bankrupt.

We also note that when $\gamma = 1$, the hump in $F(\cdot)$ around $y = y^c$ disappears. This behavior is in keeping with the earlier intuition we offered regarding the precautionary savings effect on the interest rate. For the smaller γ , the interest rate does not fall as fast for y larger than y^c , and the hump vanishes.

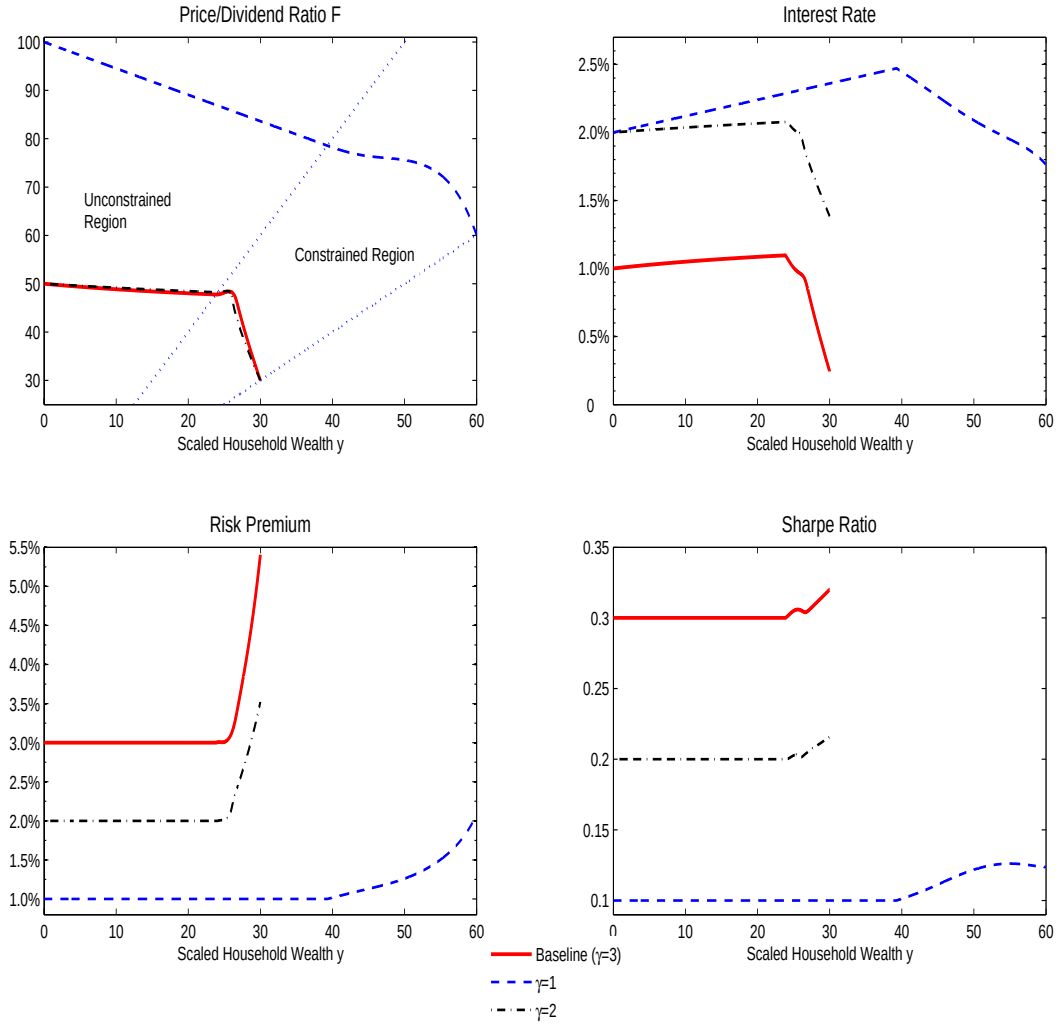
In Figure 13 we illustrate the behavior of the economy for different values of α , which determines $F(y^b)$. We note that changing α mainly affects the speed (in terms of y) at which the economy hits the bankruptcy boundary. For example, in the risk premium panel, we see that the risk premium take on similar values across the constrained region; however, in the case where α is smallest, the economy hits the highest risk premium more quickly.

Figure 11: Effect of varying f .



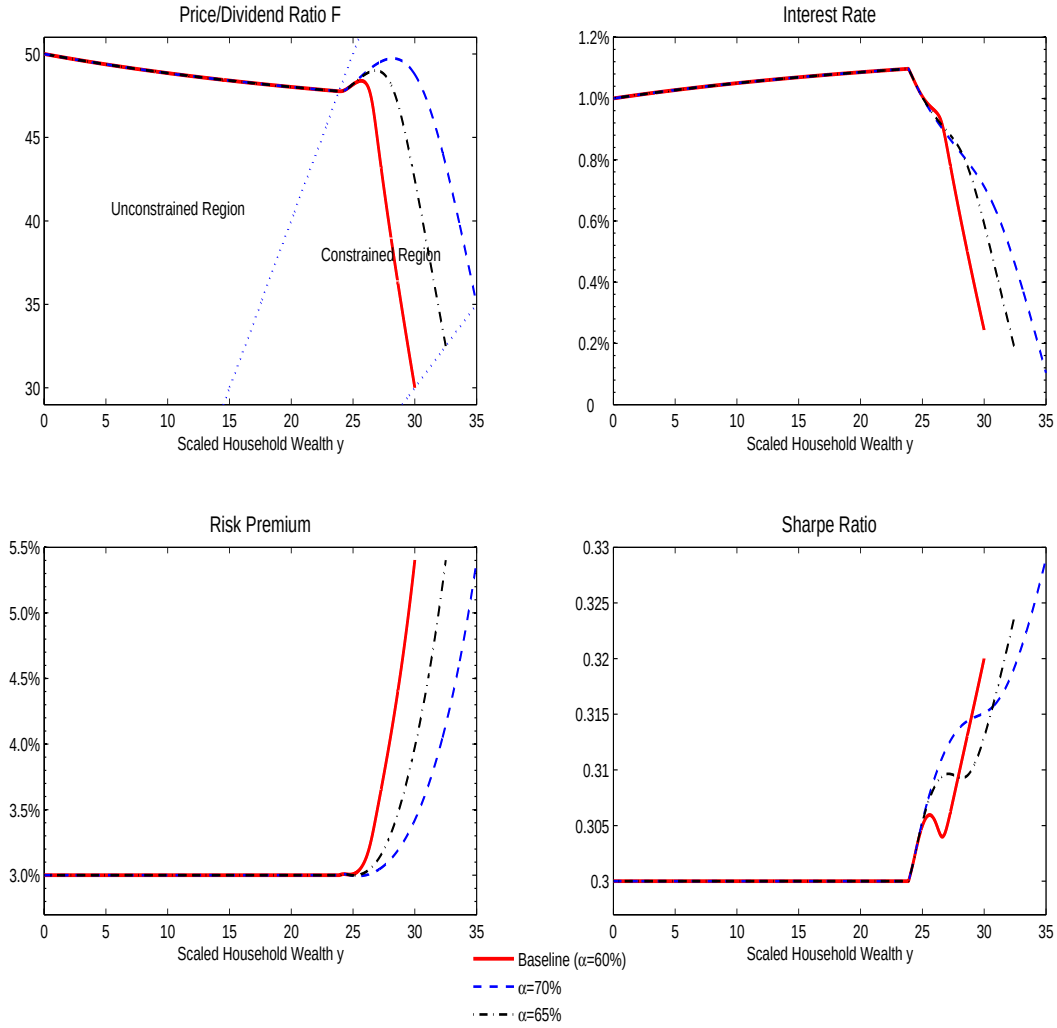
The price/dividend ratio F , interest rate, risk premium, and Sharpe ratio for the case of $f = 1.1\%$ and $f = 1.5\%$. Baseline case result is plotted in solid line for comparison.

Figure 12: Effect of varying γ .



The price/dividend ratio F , interest rate, risk premium, and Sharpe ratio for the case of $\gamma = 1$ and $\gamma = 2$. Baseline case result is plotted in solid line for comparison.

Figure 13: Effect of varying α .



The price/dividend ratio F , interest rate, risk premium, and Sharpe ratio for the case of $\alpha = 65\%$ and $\alpha = 70\%$. Baseline case result is plotted in solid line for comparison.