Idiosyncratic Volatility Matters for the Cross-Section of Returns-- in More Ways than One!

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Idiosyncratic Volatility Matters for the Cross-Section of Returns-- in More Ways than One!

Abstract

This article re-examines the relationship between idiosyncratic volatility and the cross-section of stock returns. Previous studies, using total realized returns as proxies for expected returns, have found ambiguous and conflicting relationships between expected idiosyncratic volatility and expected returns. We decompose idiosyncratic volatility into expected and unexpected idiosyncratic volatility, and use unexpected idiosyncratic volatility to control for unexpected returns so that the relationship between expected returns and expected idiosyncratic volatility can be observed with more clarity. We find expected idiosyncratic volatility to be significantly and positively related to expected returns. In addition, we find evidence suggesting that unexpected idiosyncratic volatility is positively related to unexpected returns and that this relationship is consistent with the option effect proposed by Merton (1974).

JEL classification: G10, G11, G12

Idiosyncratic Volatility Matters for the Cross-Section of Returns-- in More Ways than One!

Standard asset pricing theory predicts that only systematic risks should be priced into the expected returns of assets. Empirically Fama and MacBeth (1973), concludes that "no measure of risk, in addition to portfolio risk, systematically affects average returns". Merton (1987) however argues that expected idiosyncratic volatility may explain expected stock returns if investors are under-diversified. Recently there have been renewed attempts to re-examine the relation between idiosyncratic risk and stock returns, which is in part due to Campbell et al (2001) documenting the phenomenon that idiosyncratic volatility has been on the rise in the past four decades and therefore warrants additional scrutiny. The evidence is mixed.

Lehmann (1990) and Malkiel and Xu (2002) find that idiosyncratic volatility is positively related to the variations in cross section of expected returns. In contrast, Ang et al (2004) find that portfolios with high idiosyncratic volatility subsequently have low returns. In addition, Jacobs and Wang (2004) present empirical evidence that idiosyncratic consumption risk is priced[1]. Zhang (2004) however argues that the evidence is not robust to different choices of conditioning variables, sample stocks, and sample periods.[2] Thus, to date, there is

[1] For theoretical models and evidence related to idiosyncratic risk, see e.g., Telmer (1993), Heaton and Lucas (1996), (2000), Constantinides and Duffie (1996), among others.

[2] At the aggregate level, Goyal and Santa-Clara (2003) find a significant positive relationship between equal-weighted average stock variance, which is largely idiosyncratic, and the average return. Bali, Cakici, Yan and Zhang (2004) find this relation is in part due to a liquidity premium. They find no evidence to support the claim that there is a statistically significant relationship between aggregate idiosyncratic risk and the market return. Meanwhile, Jiang and Lee (2004) find that idiosyncratic volatility is positively related to the market return after correcting for serial correlation in the idiosyncratic volatility. On the other hand, Guo and Savickas (2004) find that value-weighted idiosyncratic volatility, when evaluated jointly with market volatility, is negatively related to stock market return.

still an ensuing debate as to whether idiosyncratic risk matters, and if it does, whether the relationship with stock returns is positive or negative.

Theories suggest several tests on the relationship between the expected stock returns and the expected idiosyncratic volatility. However, the approach in many of these recent studies is to examine the relationship between expected (or total) idiosyncratic risk and the cross section of realized stock returns. This may be a cause for the inconclusive findings because total realized returns are poor proxies for expected returns. At the monthly horizon, the effects of unexpected returns can dominate that of expected returns, and can cloud the true relationship between expected returns and expected idiosyncratic volatility.

There is no easy way of decomposing total stock returns into expected and unexpected components. In this study, we propose a novel approach to address this issue. We decompose idiosyncratic volatility into expected idiosyncratic volatility (EIV) and unexpected idiosyncratic volatility (UIV). We then use UIV to control for the unexpected returns portion of total returns. Consequently, this enables us to more clearly observe the relationship between expected returns and expected idiosyncratic volatility.

There are several reasons, both theoretical and empirical, to support the use of UIV to control for unexpected returns:

1. If investors care about volatility, any unexpected change in the volatility of a stock could potentially have pricing implications and therefore affect unexpected stock returns contemporaneously.
2. Merton (1974) posits that we can view common stocks as call options on the assets of the firm. A positive shock to the idiosyncratic volatility of the stock implies that the volatility of the firm's assets has increased, and consequently means that the call

option (the common stock) should increase in value as well (henceforth termed the option effect).

3. Recent papers such as Dennis, Mayhew and Stivers (2004), Jiang and Lee (2004) and Ang, Hodrick, Xing and Zhang (2004) find that innovations in volatilities, albeit aggregated over all stocks, are useful in the study of stock returns. Given their findings, it is plausible that each individual stock's innovation in idiosyncratic volatility may also be important in studying the underlying stock's returns.

The UIV used in this study is the unexpected volatility at any month t, which is not in investors' information set at month t-1, hence by construction it should not be related to the expected stock returns (UIV refers to the realizations of innovations of idiosyncratic volatility, not to levels of idiosyncratic volatility or even to the volatility or uncertainty in time-varying idiosyncratic volatility, and therefore cannot be priced into expected returns). Consequently, this enables us to more clearly observe the relationship between expected returns and expected idiosyncratic volatility.

]In short, we use the results of our study to answer two questions: whether there is a relationship between expected returns and expected idiosyncratic volatility; and whether there is a relationship between unexpected returns and unexpected idiosyncratic relationship. The answers to both questions are two unequivocal affirmatives.

Our methodology is straightforward and extends upon the current literature. We first construct a measure of idiosyncratic volatility for each stock at each month along the lines of Ang et al (2004), Wei and Zhang (2003) and Zhang (2004). Then, using each stock's time series of idiosyncratic volatility, we model the evolution of the idiosyncratic volatility as an autoregressive process. The monthly predicted idiosyncratic volatility is labelled EIV, and the residual is labelled UIV. We then run cross-sectional regressions of returns on EIV alone as well as on both EIV and UIV. We find that consistent with the literature, EIV alone cannot explain stock returns. The point estimate is virtually zero, and statistically insignificant at 10% level. However, when used jointly with UIV, both EIV and UIV are positively and significantly related to contemporaneous stock returns. The average coefficients for EIV and UIV are 0.22 and 0.82 respectively, both statistically significant at 1% level.

These results suggest that EIV and UIV jointly matter for the cross-section of stock returns. As we discuss above, EIV and UIV serve different roles in explaining stock returns, hence a distinction between the two is necessary. By construction, UIV should not be related to the expected stock returns. We therefore deduce that the positive and significant coefficient for UIV is driven by its relationship with unexpected returns. This then makes the relationship between EIV and expected returns clearer-- a straightforward interpretation of the positive and significant coefficient for EIV is that EIV is positively related to the expected stock returns. Our results therefore show that using total return as proxy for expected returns clouds the true relation between expected return and expected idiosyncratic volatility. By separating the roles of EIV and UIV, our methodology instead provides an alternative approach to test this relation.

To ensure that our results are robust, we re-run the regression controlling for size, book-to-market, and past returns. The regression coefficients for EIV and UIV remain significant, indicating that our earlier results are not driven by common firm characteristics that have been widely shown to predict future returns. To control for potential market micro structure issues, we re-run the tests excluding the smallest size quintile stocks and NASDAQ stocks, the results remain robust. Our results are also robust in different sub-sample periods.

Since, to the best of our knowledge, this is the first study to investigate the relationship between UIV and stock returns, we perform a further robustness check. Using the standard asset pricing research methodology, we create ten EIV-stratified, UIV-sorted portfolios and measure the average return, CAPM alpha, Fama-French 3-factor alpha as well as Fama-French 3-factor plus momentum-factor alpha. For both equal-weighted and value-weighted portfolios, the risk-return relationship from a low UIV portfolio to a high UIV portfolio is unmistakably positive. In addition, the returns and alphas of the high-UIV portfolio are always significantly higher than the corresponding returns and alphas of the low-UIV portfolio. This produces additional empirical evidence that UIV can indeed serve as a good variable to control for unexpected returns[3].

Our results are consistent with Merton (1974)'s idea of viewing equity as a call option on the firm's assets. If the volatility of the firm's asset value unexpectedly increases, the option value (equity price) will increase and hence the contemporaneous return. The option effect also implies that the observed relationship be stronger for firms with higher financial leverage, since the equity of these firms are more option-like. To further test Merton's theory, we sort all firms into five quintiles based on their leverage. Our results show that the relationship between UIV and returns is positive for firms in all five quintiles and that the relationship gets monotonically stronger as leverage increases[4]. Thus we find evidence suggesting the idiosyncratic risk and stock return relationship is indeed driven by the option

[3] Popular theories in the asymmetric volatility literature such as the "leverage effect" (Black (1976) and Christie (1982)) and "positive feedback trading" (Campbell and Hentschel (1992)) theories predict a negative relation between current return and future volatility. These theories do not say anything about the relationship between volatility and contemporaneous returns, which is what we documented.

[4] In a different context, Duffee (1995) documents a similar relation between contemporaneous return and firm's total volatility. However he finds that this relation is negatively related to the leverage ratio. We contend that it is more appropriate to use unexpected volatility to test the option effect, since the expected volatility should have already been priced and should not affect the contemporaneous return.

effect.

It is worth noting the positive contemporaneous relation between UIV and unexpected return does not necessarily lead to a negative predictive relation. In Merton's option framework, if the risk premium of the underlying asset (total firm value) remains constant, an increase in volatility will lead to lower equity beta, and consequently to lower expected future returns for the equity of the firm (See e.g. Johnson (2004)). However, we find that there is a positive relation between expected volatility and future expected return, suggesting that the constant premium assumption is not valid. Instead, our evidence suggests that an increase in volatility leads to higher expected returns on the value of the underlying firm as well, and this effect dominates the negative effect of volatility on equity beta.

Our analysis is organized as follows. Section I introduces the data used in this paper and defines idiosyncratic volatility, expected idiosyncratic volatility, and unexpected idiosyncratic volatility. Section II analyzes the relationship between EIV, UIV and returns. Section III further investigates the role of UIV and also tests the validity of the option effect. Section IV concludes the paper.

I. Data and definitions

A. Data

We obtain daily and monthly stock returns from CRSP. The sample period is from January 1963 to December 2003. All common stocks (share code 10 and 11) traded at NYSE, AMEX and NASDAQ are included. In later sections we also use data on firm characteristics such as size, book-to-market ratio and financial leverage. These characteristics are constructed from the Compustat annual file and we merge them with the return data. Specifically, we define the book-to-market ratio every month as the ratio between previous year's ending book value of common equity (item 60) and the current month's market capitalization. Financial leverage is defined as the book value of total debt (item 181) divided by the sum of the book value of debt, other equity and the market value of common equity (last year end total assets (item 6) – last year end common equity + current month market capitalization). We exclude firms with negative book-to-market ratio. Following the literature, we exclude financial firms (SIC code 4900 to 4999) and utility firms (SIC code 6000 to 6999) whenever we need leverage information. We also exclude firms whose leverage ratio is outside the range of [0, 1]. We obtain the daily and monthly risk free rate and Fama and French three factor returns from 1963 to 2003 from Kenneth French's website[5].

B. Volatility definitions

For each month, we first run the Fama-French (1993) regression using daily returns:

$$r_{i,d,m} = \alpha_{i,m} + \beta_{i,m}^{MKT} MKT_{i,m} + \beta_{i,m}^{SMB} SMB_{i,m} + \beta_{i,m}^{HML} HML_{i,m} + \varepsilon_{i,d,m} \tag{1}$$

where $\alpha_{i,m}$ and $\beta_{i,m}$ are constant for each month *m*, but may vary across different months. Similar to Ang et al (2003) and Zhang (2004), we then define the idiosyncratic volatility measure for stock *i* in month *m* as:

$$IV_{i,m} = \sum_{d=1}^{N_m} \varepsilon_{i,d,m}^2 \tag{2}$$

[5] We thank Kenneth French for making the data public.

For each stock i, we impose an AR(2)[6] model on the time series evolution of the idiosyncratic volatility. We run the following time-series regression:

$$IV_{i,m} = \phi_0 + \phi_1 IV_{i,m-1} + \phi_2 IV_{i,m-2} + \xi_{i,m} \tag{3}$$

We define expected idiosyncratic volatility for stock i in month m as the fitted value:

$$EIV_{i,m} = \phi_0 + \phi_1 IV_{i,m-1} + \phi_2 IV_{i,m-2} \tag{4}$$

We define unexpected idiosyncratic volatility for stock i in month m as the residual:

$$UIV_{i,m} = \xi_{i,m} \tag{5}$$

[Table I here]

Table I displays the time-series averages of the cross-sectional mean, standard deviation, skewness, and kurtosis for $IV_{i,m}$, $EIV_{i,m}$ and $UIV_{i,m}$. The monthly mean and standard deviation of IV is 0.031 and 0.130 respectively; these numbers are comparable to those reported in Zhang (2004). IV is positively skewed and fat-tailed. The mean of EIV is close to that of IV, but less volatile; its standard deviation is 0.068. UIV has a close-to-zero mean[7] but is as volatile as the IV; the standard deviation is 0.114. Similar to IV, both the EIV and UIV are positively skewed and fat-tailed.

[6] We examine the use of an AR(1) model on the time series evolution of idiosyncratic volatility. However, the residuals from the AR(1) have high serial autocorrelation. The AR(2) model eliminates most of the serial autocorrelation. Additionally, we also examine the use of a varying-order AR(p) model, where we use AIC to choose the order. This results in similar estimates, but with the drawback that a large amount of data is wasted in determining and fitting the optimal model. In any case, the main results of this paper are qualitatively unaffected by the choice of the time series model.

[7] The mean of UIV is not exactly zero because it is calculated from the differences between the forecasted IV and realized IV, not from the residual of any regression.

II. Main Results

A. Regressions of returns on EIV and UIV

We first explore the relationship between returns and EIV (the usual analysis in previous studies). For each month, we run the following cross-sectional regression:

$$R_{i,m} = \varphi_0 + \varphi_1 EIV_{i,m} + \eta_{i,m} \tag{6}$$

We test the time-series of the parameter estimates using the Newey-West (1987) t-statistics. The means and significance of the parameter estimates are reported as Model 1 of Table II. The coefficient for EIV is not significantly different from zero as the Newey-West t-statistics is only -0.19. This means that when used singly, EIV cannot explain the cross-section of stock returns.

We then explore the relationship between returns, EIV and UIV via the following cross-sectional regression every month:

$$R_{i,m} = \varphi_0 + \varphi_1 EIV_{i,m} + \varphi_2 UIV_{i,m} + \eta_{i,m} \tag{7}$$

Again, we compute the time-series average and the Newey-West t-statistics. The results are reported as Model 2 of Table II. With the inclusion of UIV, both EIV and UIV are now significant, with t-stats of 3.18 and 8.51 respectively. This result suggests that the decomposition of idiosyncratic volatility is essential in explaining the cross-sectional returns.

It is important at this point to examine the implications of Model 2 in greater detail. The significance of the coefficients of both EIV and UIV implies that both these variables are strongly associated with returns. As mentioned earlier, UIV should have little relation with expected returns. This implies that the positive and significant coefficient for UIV must be largely driven by its relationship with unexpected returns. Therefore, UIV serves as a control variable by controlling for the effects of unexpected returns. Consequently, the coefficient for

EIV now reflects the marginal effect of EIV on expected returns. As we discuss in the introduction, our methodology hence provides an alternative approach for testing the relation between the expected return and expected idiosyncratic volatility. The key result here is that we need UIV to bring out the significance of EIV's relationship with expected returns— a fact that many previous studies overlooked.

The significant coefficient for the UIV variable is neither a statistical artifact nor a direct product of the construction of the study. A high UIV implies that the realized IV for the month is unexpectedly high. This does not automatically mean that the return for the month should be unexpectedly high as well. In contrast, an unexpectedly low (negative) return will also generate a high UIV statistic. Therefore, a priori, there is no reason to believe that high UIVs should be contemporaneously associated with high returns[8].

[Table II here]

For completeness, we repeat (8), but with an additional regressor, lagged returns. It is a well documented result that at the firm-level, returns in month m-1 are negatively related to returns in month m. By adding in the lagged returns as an explanatory variable, we can verify whether our earlier results are due to this phenomenon. The results are reported as Model 3 in Table II. The parameter estimates and t-stats for both EIV and UIV are virtually identical for Model 2 and Model 3, which implies that our results for EIV and UIV are not related to lagged returns.

[8] As a further test of the robustness of the significance of UIV, we re-compute the time-series average (and the corresponding t-stats) of the UIV variable in Model 2 of Table II by excluding all months where the CRSP index return (NYSE/AMEX/NASDAQ) in excess of 1-month Tbill rate is above zero . The average coefficient is 0.2085 for EIV and 0.8924 for UIV, both significant at 5% level. This indicates that the positive contemporaneous relationship between UIV and returns is not mechanically driven by the construction of our variables (contemporaneous high returns and UIV) because the relationship holds in months where the equity market as a whole went down as well.

B. Control Variables and size quintiles

It is also well documented that size and the book-to-market ratio have explanatory power for the cross-sectional returns (see Fama and French (1992)). To check for the robustness of our results, we use these two characteristics as additional explanatory variables. We also add past one year return (returns from previous 12 to 2 months ago.) to control for the momentum effect. (See e.g. Jagadish and Titman (1993)). It is possible that market micro-structure issues for small and illiquid stocks may drive the results. Thus we conduct two additional robustness tests. In one test, every month we sort all stocks in our sample into 5 size quintiles based on previous month's market capitalization, and we exclude the smallest quintile from the sample. In another test, we exclude NASDAQ stocks entirely from our sample. We then repeat the cross-sectional regression analysis. The results for both tests together with the full sample results are reported in Table III. Across all sub-sample, the coefficients for both EIV and UIV remain statistically significant, after controlling for firm characteristics. Moreover, the point estimates are stable across different samples. Interestingly, the size effect reverses after controlling for the volatility effects, suggesting that the size effect may be partially explained by the IVs. We leave further investigation on this issue to future studies.

[Table III here]

III. Unexpected Idiosyncratic Volatility

This study is among the first in examining the role of firm-level (un-aggregated) UIV in explaining cross-sectional stock returns. Since the previous section documents an empirical positive and highly significant relationship between UIV and stock returns, these results merit closer examination. We do this through two separate investigations. First, by constructing UIV sorted portfolios and observing how returns and alphas vary across the different portfolios. Second, we test the option effect by observing how the relationship between UIV and returns change as the leverage of the firm changes.

A. Returns and Alphas of Portfolios Stratified by EIV and Sorted by UIV

We construct equal-weighted and value-weighted portfolios stratified by EIV, and then sorted by UIV. In other words, every month we first sort the stocks into ten deciles based on their EIV. Next, within each EIV decile, we sort the stocks into ten more deciles based on their UIV. Then, we aggregate all the stocks in the same UIV deciles across all the ten EIV deciles. The ten resulting portfolios will thus have different UIVs but have a similar spread of EIVs. For each portfolio and each month, we construct four measures: raw returns, alphas based on CAPM, alphas based on Fama-French (1993) three factor model (FF-3), and alphas based on Fama-French three factor model plus a momentum factor (FF-3 + MOM).

Table IV reports the means and significance of these measures for ten equal-weighted portfolios. As we move from the low UIV portfolio to the high UIV portfolio, all measures (both returns and alphas) increase almost monotonically. Specifically, raw returns increase from -0.96% to 8.08% per month, a difference of more than 9% per month. CAPM alphas, FF-3 alphas and FF-3+MOM alphas also register significant increases from approximately -1.80% per month for the low UIV portfolio to approximately 6.50% per month for the high

UIV portfolio. The t-stats for testing the differences of means between the low UIV portfolios and the high UIV portfolios range from 22.14 for raw returns to 27.53 for the FF-3 alpha.

[Table IV here]

Table V reports the corresponding numbers for ten value-weighted portfolios. Again, all four measures are virtually monotonically increasing as UIV increases. The t-stats for the differences in means between the highest and lowest UIV sorted portfolio for the four measures ranges from 4.40 to 5.13. Even though the t-stats are lower than that of the equal-weighted portfolios, the level of statistical significance is still high. Results from tables IV and V therefore provide strong evidence that innovations in idiosyncratic volatilities are related to stock returns.

[Table V here]

It is worth noting that although we document significant alpha differentials among UIV sorted portfolios, the portfolio sorting is based on contemporaneous information (UIV) and therefore the observed pattern does not imply a profitable investment strategy. Nevertheless, our results highlight the important role of UIV in explaining the contemporaneous cross-sectional returns, implying that UIV is indeed a suitable control variable.

B. Linkage between UIV and stock returns: Testing the option effect

The option effect of Merton (1974) predicts that since equity is a call option on the firm's assets, an (unexpected) increase in the volatility should result in a higher value for the call option. The more highly leveraged the firm is, the more option-like the equity will be. Therefore, the option effect further predicts that the positive relationship between UIV and returns should be stronger for high-leverage vis-à-vis low-leverage firms. Our results in Section II and Section III.A corroborate with the prediction that higher UIV is associated with higher returns.

To test the proposition that high leverage firms will see a stronger relationship between UIV and returns than low leverage firms, we first divide the firms into five quintiles sorted by leverage and assign five dummy variables (D1,…,D5 corresponding to each of the five leverage quintiles) to each firm. To equitably compare the strength of the relationship between UIV and returns (in essence comparing the magnitude of the coefficients of the UIV variable) across firms in different leverage quintiles, we first standardize the UIV variable so that the variance of the UIV variable is equal across all quintiles. We do this by dividing the monthly demeaned UIV by the time-series standard deviation of the UIV for that firm. This not only results in equal variances across the five quintiles, but also results in equal unconditional variances for the UIV variable for all firms in all months. The standardized UIV (UIVS) for firm i in month m is defined as follows:

$$UIVS_{i,m} = \frac{UIV_{i,m}}{SD(UIV_{i,.})} \tag{8}$$

Similarly, we also standardize the returns (RS) and EIV (EIVS) for easy comparison. We then perform the following regression:

$$RS_{i,m} = \varphi_0 + \varphi_1 EIVS_{i,m} + \varphi_2 Size + \varphi_3 btm + \varphi_4 D1 * UIVS_{i,m} + \varphi_5 D2 * UIVS_{i,m}$$

$$+\varphi_6 D3 * UIVS_{i,m} + \varphi_7 D4 * UIVS_{i,m} + \varphi_8 D5 * UIVS_{i,m} + \eta_{i,m} \qquad (9)$$

We interpret the coefficient φ_j (j = 4…8) as the effect on returns, of a 1-sigma realization in the UIV. In other words, if in month m, firm i has a UIV that is one standard deviation above zero, then this should, on average, add φ_j to the month's standardized returns.

Panel A of Table VI shows the results of this regression. The means of the coefficients φ_j increase monotonically from 0.161 to 0.242 as j increases from 4 to 8. This finding is consistent with the prediction of the option effect, that is, as leverage increases from low to high, the relationship between UIV and returns get stronger. Furthermore, the differences between the means of φ_4 and φ_8 is statistically significant (with a t-stat of 8.90), implying that the relationship for firms in the high-leverage quintile is significantly stronger than the relationship for firms in the low-leverage quintile. This result, coupled with those reported in Tables II through V lends additional support for the option effect.

Campbell et al (2001) document a sharp rise in individual stock volatilities from 1962 to 1997. To be sure that our results are not driven by any particular period (whether during a low volatility period or high volatility period), we divide the sample into two sub-periods: from 07/1963 to 12/1983 and from 01/1984 to 12/2003. We then repeat regression (9) for both sub-periods. The results for these regressions are reported in Panel B and Panel C of Table VI. For both sub-periods, the effect of UIV increases monotonically as leverage increases, implying that the result for the full sample is robust across all sub-samples as well.

It is worth noting that the option effect of UIV on contemporaneous stock price does not lead to a negative relation between expected volatility and future expected stock return. In Merton (1974) framework, the expected return on firm's equity depends on both the risk premium on the underlying asset (total firm value), and the sensitivity of the equity value to the change in value of the underlying total asset (equity beta). If the premium on the total assets of the firm remains constant, it can be shown that an increase in current volatility of the firm will lead to lower equity beta, and in turn lower expected future return on firm equity (See e.g. Johnson (2004)). Our empirical evidence earlier however suggests that the assumption of constant premium on firm's total assets is not valid. The significant positive coefficient for EIV suggests that idiosyncratic volatility has a positive effect on the premium of the firm's total assets, and this effect dominates the negative effect of volatility on equity beta. As a result, we empirically observe a positive relation between the expected stock (equity) return and the expected volatility.

[Table VI here]

IV. Conclusion

This paper re-examines whether idiosyncratic volatility matters to the cross-section of stock returns. We find evidence suggesting that the conflicting results in the current literature can be attributed to an errors in variables problem because realized returns are often used as proxies for expected returns. We correct for this problem by introducing unexpected idiosyncratic volatility to control for the noise caused by unexpected returns thereby allowing the true relationship between expected idiosyncratic volatility and expected returns to surface.

We show this effect via 2 regressions: (1) using only expected idiosyncratic volatility to explain stock returns; and (2) using both expected and unexpected idiosyncratic volatility.

The former gives a statistically insignificant coefficient for the slope of expected idiosyncratic volatility, while the latter results in highly statistically significant coefficients for both expected and unexpected idiosyncratic volatilities. The interpretation is that both expected and unexpected idiosyncratic volatilities are important in explaining stock returns. Specifically, expected idiosyncratic volatility is positively related to expected stock returns while unexpected idiosyncratic volatility is positively related to unexpected stock returns. Excluding unexpected idiosyncratic volatility from the analysis leads to incorrect conclusions about the role of expected idiosyncratic volatilities. We include several robustness checks to ensure that our results are not due to characteristics such as size and book-to-market factors, or our choice of sample periods.

Also, to further investigate the importance and role of unexpected idiosyncratic volatility in explaining cross-sectional stock returns we perform two additional tests. For the first test, we construct both equal- and value-weighted portfolios that are EIV-stratified and UIV-sorted. We show that for both types of portfolios, the average returns as well as the Jensen's alphas (over a wide range of models) increases monotonically as we move from a low UIV portfolio to a high UIV portfolio. For the second test, we investigate the validity of the option effect. We group firms into five leverage quintiles. We then standardize the unexpected idiosyncratic volatility measure and show that the magnitude of the regression coefficients for the standardized unexpected idiosyncratic volatility increases monotonically as leverage increases. This leads us to conclude that the unexpected idiosyncratic volatility is indeed important and this importance stem from the consequences of the option effect.

To summarize, this paper addresses a keenly contested debate on whether idiosyncratic volatility helps to explain the cross-sectional returns. The answer lies in decomposing idiosyncratic volatility into the expected and unexpected component. With this decomposition, we unambiguously show that both components are significantly related to returns, albeit in different ways.

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Table I

Summary Statistics for Idiosyncratic Volatility, Expected Idiosyncratic Volatility (EIV) and Unexpected Idiosyncratic Volatility (UIV)

The sample is from 1963:07 to 2003:12. At each month m, for every common stock traded in NYSE/AMEX/NASDAQ, we regress the daily returns on Fama-French three factors and obtain the daily residuals ε_{is}. We then define $IV_{i,m} \equiv \sum_{s=1}^{n} \varepsilon_{is}^2,\ s \in m$, as stock i's idiosyncratic volatility (IV) for month m. We fit an AR(2) on the monthly IV series for every stock. We define the fitted values as the expected IV (EIV) and the residuals as the unexpected IV (UIV). We report the time-series averages of the cross-sectional mean, standard deviation, skewness, and kurtosis for these three volatility measures.

	IV	EIV	UIV
Mean	0.0306	0.0309	-0.0003
Stdev.	0.1304	0.0677	0.1144
Skewness	19.7859	12.7923	14.2754
Kurtosis	820.5728	440.5348	834.1148

Table II

Cross-sectional Regressions of Returns on Contemporaneous Expected and Unexpected Idiosyncratic Volatilities

The sample is from 1963:07 to 2003:12. The volatility measures EIV and UIV are constructed in Table I. Each month, for all common stocks traded in NYSE/AMEX/NASDAQ, we run cross-sectional regressions of monthly stock returns on the contemporaneous unexpected volatility (UIV), as well as the expected volatility (EIV), one month lagged EIV, and one month lagged returns, ret_{-1}. We report the time-series averages of parameter estimates and Newey-West (1987) adjusted t-statistics. We also report the time-series averages of the cross-sectional adjusted R^2s.

	Model 1		Model 2		Model 3	
	parameter	t-stat	parameter	t-stat	parameter	t-stat
C	0.0131	5.15	0.0102	4.18	0.0104	4.06
EIV	-0.0083	-0.19	0.2196	3.18	0.2283	3.13
UIV			0.8209	8.51	0.8107	8.33
ret_{-1}					-0.0509	-12.26
Adjusted R^2	0.017		0.082		0.089	

Table III

Cross-sectional Regressions of Returns on EIV, UIV: controlling for firm characteristics

The sample period is from 1963:07 to 2003:12. The expected volatility EIV and the unexpected volatility UIV are constructed in Table I. We define the book-to-market ratio every month as the ratio between last year end book value of common equity (item 60) and the current month's market capitalization, where the book value is from Compustat annual file and merged with CRSP data. Each month, for all common stocks (share code 10 and 11) traded in NYSE/AMEX/NASDAQ that have positive book and market value, we run cross-sectional regressions of monthly stock returns on EIV, UIV, the one-month lagged log of market capitalization (size), the book-to-market ratio (b-t-m), last month's return, and the return from 12 to 2 months ago. The first column reports the results for the full sample. Besides using the full sample, we also sort stocks into five quintiles based on the previous month's market capitalization. We then delete the smallest quintile and repeat the cross-sectional regressions In the last column we exclude NASDAQ stocks and repeat the cross-sectional regressions. For the full sample and each sub-sample, we report the time-series averages of parameter estimates and Newey-West (1987) adjusted t-statistics. We also report the time-series averages of the cross-sectional adjusted R^2s.

		Full Sample	Excluding smallest size quintile	Excluding NASDAQ stocks
Const.	parameter	-0.0140	-0.0129	-0.0144
	t-stat	-3.64	-3.26	-3.76
EIV	parameter	0.4248	0.4017	0.3870
	t-stat	5.52	3.97	5.11
UIV	parameter	1.0774	1.2118	1.1286
	t-stat	9.72	8.44	10.21
Size	parameter	0.0007	0.0007	0.0007
	t-stat	2.14	2.11	3.17
b-t-m	parameter	0.0103	0.0095	0.0068
	t-stat	13.31	11.61	11.72
ret_{-1}	parameter	-0.0551	-0.0493	-0.0584
	t-stat	-12.61	-11.31	-12.54
Ret_{-2-12}	Parameter	0.0101	0.0109	0.0115
	t-stat	7.70	8.39	7.53
Adj. R^2		0.116	0.102	0.116

Table IV
Equal-weighted monthly returns and alphas on portfolios sorted UIV (EIV stratified)

The sample period is from 1963:07 to 2003:12. The unexpected volatility measure UIV is constructed in Table I. Each month, we first sort common stocks (share code 10 and 11) traded in NYSE/AMEX/NASDAQ into ten deciles based on their EIV rankings. Within each EIV decile, we sort stocks into ten deciles based on their UIV rankings. We then aggregate all stocks with the same UIV ranking across EIV sorted portfolios and construct ten EIV stratified UIV sorted portfolios. Decile 1 contains stocks with the lowest UIVs, while Decile10 with the highest. The portfolios are rebalanced every month. The first two rows report the average equal-weighted monthly returns and *t*-stats for each portfolio. The portfolio H-L is the return difference between decile 10 and 1. For each portfolio, we then regresses monthly equal-weighted returns in excess of the one month T-bill rate on the market factor (CAPM), Fama-French (1993) three factors (FF-3) and Fama-French three factor plus the momentum factor. The factors realizations are downloaded from Kenneth French's website. For each model, we report the regression intercepts (alpha), as well as the *t*-stats. It must be emphasized that the portfolios are constructed after the realizations for UIV (and hence returns) are known. Therefore, one cannot construct a profitable trading strategy to exploit the results in this table.

		Low UIV	2	3	4	5	6	7	8	9	High UIV	H-L
Raw returns	r	-0.0096	-0.0096	-0.0069	-0.0031	0.0014	0.0068	0.0134	0.0221	0.0365	0.0808	0.0904
	t-stat	-7.05	-5.27	-3.27	-1.35	0.57	2.48	4.49	6.79	9.89	15.94	22.14
CAPM	alpha	-0.0169	-0.0179	-0.0159	-0.0125	-0.0083	-0.0033	0.0029	0.0113	0.0251	0.0684	0.0853
	t-stat	-20.03	-18.31	-14.99	-10.89	-6.66	-2.31	1.81	6.10	11.18	18.13	25.25
FF-3	alpha	-0.0182	-0.0196	-0.0176	-0.0144	-0.0104	-0.0055	0.0004	0.0086	0.0218	0.0639	0.0821
	t-stat	-29.73	-30.71	-27.59	-20.82	-14.16	-6.14	0.35	7.12	13.76	20.93	27.53
FF-3+MOM	alpha	-0.0178	-0.0190	-0.0170	-0.0136	-0.0095	-0.0043	0.0017	0.0099	0.0233	0.0648	0.0826
	t-stat	-28.48	-29.41	-26.33	-19.59	-12.95	-4.83	1.66	8.22	14.49	20.63	26.90

Table V
Value-weighted monthly returns and Alphas on portfolios sorted UIV (EIV stratified)

The sample period is from 1963:07 to 2003:12. The unexpected volatility measure UIV is constructed in Table 1. Each month, we first sort common stocks (share code 10 and 11) traded in NYSE/AMEX/NASDAQ into ten deciles based on their EIV rankings. Within each EIV decile, we sort stocks into ten deciles based on their UIV rankings. We then aggregate all stocks with the same UIV ranking across EIV sorted portfolios and construct ten EIV stratified UIV sorted portfolios. Decile 1 contains stocks with the lowest UIVs, while Decile10 with the highest. The portfolios are rebalanced every month. The first two rows report the average value-weighted monthly returns and *t*-stats for each portfolio. The portfolio H-L is the return difference between decile 10 and 1. For each portfolio, we then regresses monthly value-weighted returns in excess of the one month T-bill rate on the market factor (CAPM), Fama-French (1993) three factors (FF-3) and Fama-French three factor plus the momentum factor. The factors realizations are downloaded from Kenneth French's website. For each model, we report the regression intercepts (alpha), as well as the *t*-stats.

		Low UIV	2	3	4	5	6	7	8	9	High UIV	H-L
Raw returns	r	0.0027	0.0059	0.0069	0.0081	0.0098	0.0093	0.0107	0.0130	0.0139	0.0160	0.0134
	t-stat	1.65	3.18	3.52	4.09	4.73	4.53	4.9	5.53	5.51	4.77	5.13
CAPM	alpha	-0.0054	-0.0028	-0.0021	-0.0009	0.0005	0.0000	0.0011	0.0031	0.0038	0.0051	0.0132
	t-stat	-7.22	-4.29	-3.49	-1.48	0.84	0.08	1.98	4.79	4.38	2.64	4.63
FF-3	alpha	-0.0058	-0.0029	-0.0024	-0.0007	0.0006	-0.0001	0.0009	0.0027	0.0035	0.0047	0.0105
	t-stat	-7.69	-4.25	-3.76	-1.15	0.88	-0.11	1.45	4.03	3.97	2.47	4.60
FF-3 +MOM	alpha	-0.0058	-0.0028	-0.0026	-0.0004	0.0006	-0.0002	0.0011	0.0027	0.0034	0.0046	0.0103
	t-stat	-7.38	-4.02	-4.10	-0.54	0.95	-0.43	1.90	3.96	3.71	2.36	4.40

Table VI
Stock Returns, UIV and Leverage

The sample is from 1963:07 to 2003:12. The expected volatility EIV and the unexpected volatility UIV are constructed in Table I. The information on book value of equity and total liability is from Compustat annual industrial file. We define the book-to-market ratio every month as the ratio between previous year-end book value of common equity (item 60) and the current month's market capitalization. Financial leverage is defined as the book value of total debt (item 181) divided by the sum of the book value of debt, other equity and the market value of common equity (last year end total assets (item6) – last year-end common equity + current month market capitalization).We exclude financial firms (SIC code 4900 to 4999) and utility firms (SIC code 6000 to 6999). We also exclude firms with negative book value and those with leverage ratio outside [0,1]. Each month, we sort common stocks (share code 10 and 11) traded in NYSE/AMEX/NASDAQ into quintiles based on their financial leverage rankings from the previous month. For every firm, D1=1 if the firm is in the quintile with the lowest leverage ratio ranking, and 0 otherwise. Similarly, D5=1 if the firms is in the quintile with the highest leverage ratio ranking and 0 otherwise. The values of D2 to D4 are assigned in the same fashion. We then define five variables $UIVS_i \equiv UIVS*Di$ (i=1 to 5). Every month, we run cross-sectional regressions of normalized returns on the EIVS, the log of previous month's market capitalization and book-to-market ratio, and $UIVS_i$s. We report the time-series averages of parameter estimates and Newey-West (1987) adjusted t-statistics. We also report the time-series averages of the cross-sectional adjusted R^2s. The column for DIV reports the difference between the coefficients of $UIVS_5$ and $UIVS_1$ and the result for the null hypothesis that both coefficients are equal. Panel A reports the results for the full sample, Panel B reports those for the first half of the sample period, and Panel C for the second half.

Panel A: 1963:07-2003:12

	Const.	EIVS	Size	btm	$UIVS_1$	$UIVS_2$	$UIVS_3$	$UIVS_4$	$UIVS_5$	DIV	Adj. R^2
parameter	-0.0102	0.0089	-0.0045	0.0525	0.1605	0.1871	0.2042	0.2202	0.2421	0.0817	0.094
t-stat	-0.45	2.97	-1.65	11.01	9.40	11.58	14.48	15.55	18.41	8.90	

Panel B: 1963:07-1983:12

	Const.	EIVS	Size	btm	$UIVS_1$	$UIVS_2$	$UIVS_3$	$UIVS_4$	$UIVS_5$	DIV	Adj. R^2
parameter	-0.0416	0.0057	0.0008	0.0498	0.2003	0.2352	0.2427	0.2638	0.2867	0.0865	0.126
t-stat	-1.10	1.49	0.18	5.94	9.38	11.87	14.12	15.34	16.82	6.83	

Panel C: 1984:01 to 2003:12

	Const.	EIVS	Size	btm	$UIVS_1$	$UIVS_2$	$UIVS_3$	$UIVS_4$	$UIVS_5$	DIV	Adj. R^2
parameter	0.0073	0.0277	-0.0075	0.0584	0.0944	0.1119	0.1413	0.1569	0.1871	0.0927	0.064
t-stat	0.32	10.61	-2.15	11.87	5.04	7.18	9.69	11.43	14.49	7.80	