Time-Varying International Stock Returns and Risk Sharing under Labor Income Risk

by

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I examine the importance of labor income risk for time-series variability of international stock returns and risk sharing. I find that interactions of stock returns with labor income growth within countries, but not across countries, are significant for explaining the time-varying risk premiums and volatilities of domestic and foreign stock markets. When each investor's investment opportunity set is expanded to include the human capital of the investor' own country, in addition to a subset of internationally-traded financial assets, the level of international risk sharing implied by the minimum-variance stochastic discount factors of domestic and foreign investors is drastically lower than the level implied by internationally-traded financial assets only. Time-varying international risk sharing associated with such discount factors, like risk sharing associated with marginal utility growth, is related to the comovement of labor income growth across countries in the last three decades.

Recent international finance literature finds that imperfect risk sharing across countries helps explain the risk premium and the volatility of the foreign exchange market (e.g., Brandt and Santa-Clara (2002), and Sarkissian (2003)). However, the literature also raises some controversy concerning the degree of international risk sharing and the importance of imperfect risk sharing across countries on international asset returns. For example, Brandt, *et al.* (2004) show domestic and foreign minimum-variance stochastic discount factors implied by asset returns differ only by exchange rate changes. By measuring the international risk sharing using such discount factors, they conclude that international risk sharing is nearly perfect or exchange rates are too smooth. The results are puzzling because they contradict the evidence on the lack of international consumption risk sharing and the effect of imperfect risk sharing on international asset returns.

To resolve the international risk sharing puzzle, I first study implications of Pareto optimal consumption allocations across countries under exchange rate volatility. Unlike the case of domestic risk sharing discussed by Constantinides (1982), Cochrane (1991) and Mace (1991) and the case for international risk sharing under constant exchange rates (Lewis (1996, 2000)), I demonstrate that unless exchange rates are constant, perfect international consumption risk sharing under Pareto optimal allocations of consumption is not equivalent to the equalization of intertemporal marginal rates of substitutions across countries. Instead, perfect international consumption risk sharing means that the intertemporal marginal rates of substitutions of domestic and foreign investors differ by changes in the exchange rate between the domestic and foreign countries. I then propose an exchange rate-adjusted timevarying international risk sharing index, which depends on the conditional second moments of domestic and foreign investors' stochastic discount factors and exchange rate changes. When the investment opportunity sets of domestic and foreign investors are restricted to a common subset of internationallytraded financial assets, including the currency exchange between the two countries, the index implied by the minimum-variance stochastic discount factors of domestic and foreign investors attains unity at all times, implying that risks associated with such stochastic discount factors, or more precisely, associated with the common subset of assets are perfectly shared. However, when the investment opportunity sets of domestic and foreign investors are allowed to include nonfinancial assets such as human capital, which are not traded across countries, the risk sharing index can vary with the conditional moments of asset returns, exchange rate changes and labor income growth of domestic and foreign countries because the difference between the minimum-variance domestic and foreign stochastic discount factors reflect not only exchange rate changes but also the difference between the domestic and foreign labor income growth.

To investigate the time-varying levels of international risk sharing, I focus on the role of countryspecific labor income risk in international markets. I implement a multivariate GARCH-in-means model to examine interactions among stock returns, labor income growth and exchange rate changes between the U.S. and the U.K. I document that the risk premiums for the U.S. and U.K. stock markets are more related to the conditional covariances of returns with the labor income growth within countries than the volatility of their own markets. I find significant interactions of volatilities between stock returns and labor income within countries, and between exchange rate changes and the U.K. labor income growth. The exchange rate-adjusted risk sharing index varies considerably over time, with average that is more consistent with the degree of international risk sharing implied from consumption data than the index implied by the subset of the financial assets alone.

For the investor of each country, labor income is both an important part of returns on wealth and a predominant source of consumption. This suggests that shocks to labor income should affect not only an investor's minimum-variance discount factor, formed from returns on her investment opportunity set including labor income, but also the investor's marginal utility growth. This further suggests that the comovement of domestic and foreign labor income growth should affect how much risks are shared between domestic and foreign investors where the risks are associated with either the minimum-variance or consumption-based discount factors. Estimation results are consistent with the hypothesis that timevarying international risk sharing measured from either asset-based or consumption-based discount factors is related to the comovement between labor income growth rates across countries. I perform a variety of robustness checks on the specifications of the conditional moments of stock returns and labor income. While the results on the levels of international risk sharing are qualitatively similar under alternative specifications, the analysis reveals some interesting facts about the importance of country-specific, idiosyncratic labor income risk for the risk premiums and volatilities of stock returns in international markets. I find that the interactions of conditional volatilities between stock returns and labor income growth and the relations between risk premiums and labor income risk are weaker across countries than within countries. The results suggest that prices of domestic stocks are determined to a greater extent by the stochastic discount factors of the domestic than foreign investor and vice versa. The evidence on the importance of idiosyncratic labor income risk is consistent with the insight of Constantinides and Duffie (1996), who argue that income shocks need to be uninsurable, persistent and heteroskedastic in order to explain the behavior of returns on financial assets. The result is also in accord with the model of Heaton and Lucas (1996), who examine an economy in which agents face aggregate dividend risk, aggregate labor income risk as well as idiosyncratic labor income risk.

The importance of human capital for asset pricing has been long recognized at least since Mayers (1972) and Fama and Schwert (1977). In the domestic asset pricing literature, Campbell (1996) and Jagannathan and Wang (1996) consider return on human capital as part of the returns on aggregate wealth and document that the risk premium associated with human capital, more particularly aggregate labor income growth, helps explain the cross section of expected stock returns. Lettau and Ludvigson (2001a,b) show that the ratio of consumption to wealth, that includes aggregate financial assets and labor income, has remarkable predictive power both in the time series and the cross-section. Lettau and Ludvigson (2005) find that changing forecasts of dividend growth, measured from consumption, dividends and labor income, covary with changing forecasts of excess stock returns in the post-war U.S. stock market. Santos and Veronesi (2006) show that the risk premium that investors require to hold stocks varies with the fluctuation of labor income relative to consumption. The empirical findings of this paper illustrate that human capital as part of investment opportunities is important not only to domestic financial markets but also to our understanding of the risk-return relation and risk sharing in international financial markets.

The rest of the article is organized as follows. In Section I, I present the exchange rate-adjusted risk sharing index. I describe data sources and summary statistics in Section II. In Section III I describe the econometric specification and estimation results of a multivariate model for stock returns, exchange rates and labor income growth of two countries. In Section IV I discuss a model for stock returns, exchange rates and consumption growth of two countries and estimation results. In Section V I compare the various risk sharing indices implied by asset-based and consumption-based discount factors. Results of robustness checks are presented in Section VI. In the last section I present conclusions.

I. International risk sharing under exchange rate volatility

A. The framework

I consider assets traded in the capital markets of two countries: a domestic and a foreign country. All of these markets are perfect, without transaction costs and taxes. To take into account the exchange rate volatility, I assume that there is a consumption good (standing for a basket of commodities) available in each country. I demonstrate that Pareto optimal allocations of consumption are not equivalent to the equalization of the intertemporal marginal rates of substitutions across countries, unless exchange rates are constant through time.

I consider a social planer's problem of maximizing expected utility over two countries with representative agents having utility functions $U(C^{j}(\mathbf{S}_{t}), X^{j}(\mathbf{S}_{t}))$, where the superscript indexes the domestic and foreign countries, j = d, f, the subscript t indexes time, \mathbf{S}_{t} is the value of a state vector at time t, $C^{j}(\mathbf{S}_{t})$ is the country j's consumption in the units of the country's consumption good at time t in state \mathbf{S}_{t} , and $X^{j}(\mathbf{S}_{t})$ can index arbitrary cross-sectional and intertemporal variation in preference.

Given these assumptions, the social planner has the objective:

$$\max \sum_{j=d,f} \omega^{j} \sum_{t=1}^{\infty} \sum_{\mathbf{S}_{t}} (\rho^{j})^{t} \Pi(\mathbf{S}_{t}) U(C^{j}(\mathbf{S}_{t}), X^{j}(\mathbf{S}_{t})),$$
(1)

where ω^{j} is the social planner's weight on country j's utility, ρ^{j} is the country j's time preference factor, and $\Pi(\mathbf{S}_{i})$ is the probability that the state \mathbf{S}_{i} occurs.

To consider the feasibility constraint faced by the social planner, let $E(\mathbf{S}_t)$ denote the real exchange rate for converting one unit of the foreign consumption good into the domestic consumption good. By multiplying foreign consumption denominated in the foreign consumption good by the real exchange rate, I obtain foreign consumption denominated in the domestic consumption good, $C^f(\mathbf{S}_t)E(\mathbf{S}_t)$. Assume that the aggregate endowment from both countries is $\Gamma(\mathbf{S}_t)$, expressed in the units of the domestic consumption good. Then the feasibility constraint is that aggregate consumption must be no greater than the aggregate endowment, at each time and in each state:

$$C^{d}(\mathbf{S}_{t}) + C^{f}(\mathbf{S}_{t})E(\mathbf{S}_{t}) \leq \Gamma(\mathbf{S}_{t}) \text{ for all } \mathbf{S}_{t}.$$
(2)

The first-order condition for the problem (1) subject to the constraint (2) with respect to the domestic consumption $C^{d}(\mathbf{S}_{t})$ is

$$\omega^{d}(\rho^{d})^{t}U_{c}(C^{d}(\mathbf{S}_{t}), X^{d}(\mathbf{S}_{t})) = \psi(\mathbf{S}_{t}), \qquad (3)$$

where U_c is a partial derivative with respect to consumption, and $\psi(\mathbf{S}_t)$ is the ratio of Lagrange multiplier associated with the feasibility constraint (2) to the probability, $\Pi(\mathbf{S}_t)$. Similarly, the first-order condition with respect to the foreign consumption $C^f(\mathbf{S}_t)$ is

$$\omega^{f}(\rho^{f})^{t}U_{c}(C^{f}(\mathbf{S}_{t}), X^{f}(\mathbf{S}_{t})) = \psi(\mathbf{S}_{t})E(\mathbf{S}_{t}).$$

$$\tag{4}$$

Taking the ratio of first-order conditions at times t+1 to those at time t with respect to domestic or foreign consumption, respectively, gives

$$M_{t+1}^{d} = \frac{(\rho^{d})U_{c}(C^{d}(\mathbf{S}_{t+1}), X^{d}(\mathbf{S}_{t+1}))}{U_{c}(C^{d}(\mathbf{S}_{t}), X^{d}(\mathbf{S}_{t}))} = \frac{\psi(\mathbf{S}_{t+1})}{\psi(\mathbf{S}_{t})},$$
(5)

$$M_{t+1}^{f} = \frac{(\rho^{f})U_{c}(C^{f}(\mathbf{S}_{t+1}), X^{f}(\mathbf{S}_{t+1}))}{U_{c}(C^{f}(\mathbf{S}_{t}), X^{f}(\mathbf{S}_{t}))} = \frac{\psi(\mathbf{S}_{t+1})}{\psi(\mathbf{S}_{t})} \frac{E(\mathbf{S}_{t+1})}{E(\mathbf{S}_{t})},$$
(6)

where M_{t+1}^d and M_{t+1}^f are respectively domestic and foreign intertemporal rates of substitution, or equivalently, marginal utility growth rates for the period *t*+1. To simply notation, I adopt the notation that $E_t^j \equiv E^j(\mathbf{S}_t), C_t^j \equiv C^j(\mathbf{S}_t)$, and so forth. Define

$$M_{t+1}^{a} = \frac{M_{t+1}^{f}}{E_{t+1}/E_{t}}$$
(7)

as an exchange rate-adjusted marginal utility growth Since ψ_t is constant across countries, equations (5)-(6) together imply the condition of perfect international risk sharing:

$$M_{t+1}^{d} = M_{t+1}^{a} \tag{8}$$

for every possible state S_t at time *t* and S_{t+1} at time *t*+1. Equation (8) implies that Pareto optimal allocations of consumption across countries do not imply the equalization of domestic and foreign marginal utility growth rates over a period unless exchange rates are constant over the period. However, perfect international consumption risk sharing does mean that the exchange rate-adjusted marginal utility growth rates are equalized across countries, state by state.

It has been noted in the literature that equation (8) holds for any domestic and foreign stochastic discount factors, including marginal utility growth rates, which price all assets, under the assumption of complete international markets.¹ For the sake of illustration, let $R_{i,t+1}^{j}$ denote real returns on an asset denominated in the units of the country *j*'s consumption good. No arbitrage implies that:

$$E_{t}\left[M_{t+1}^{j}R_{i,t+1}^{j}\right] = 1, \ j = d, f$$
(9)

where M_{t+1}^{j} is a stochastic discount factor of country *j*.

Since returns to foreign investors multiplied by exchange rate changes are returns to domestic investors,

$$R_{i,t+1}^{d} = R_{i,t+1}^{f} (E_{t+1} / E_{t}), \qquad (10)$$

the following holds

¹ See, e.g., Ahn and Gao (1999), Backus, Foresi, and Telmer (2001), Brandt and Santa-Clara (2002).

$$M_{t+1}^{a}R_{i,t+1}^{d} = M_{t+1}^{f}R_{i,t+1}^{f},$$
(11)

where M_{t+1}^{a} is the an exchange rate-adjusted discount factor given by equation (7). Substituting equation (11) into equation (9) implies

$$E_{t}\left[M_{t+1}^{a}R_{i,t+1}^{d}\right] = 1,$$
(12)

which says that M_{t+1}^a is a valid discount factor for returns denominated in units of the domestic consumption good. In complete international markets, the stochastic discount factor for returns on all assets in units of domestic consumption good is unique: M_{t+1}^a and M_{t+1}^d are equalized, state by state.²

The preceding discussions suggest that complete international markets imply that perfect international consumption risk sharing, namely, optimal Pareto consumption allocations, even under exchange rate volatility.

B. International risk sharing index

Unless noted otherwise throughout the rest of the paper, I use a lower-case letter to denote the natural logarithm of an upper-case letter. Equation (12) implies that the exchange rate-adjusted log discount factor,

$$m_{t+1}^a = m_{t+1}^f - \Delta e_{t+1}, \tag{13}$$

represents a valid log discount factor in units of the domestic consumption good.

I define the mean squared dispersion between domestic and the exchange rate-adjusted log discount factors as

$$P_t^d(l) = \sum_{\mathbf{S}_{t+1}} M_{t+1}^k D_{t+1}^d \Pi_t(\mathbf{S}_{t+1}) = M^k(l) \Pi_t(l) \quad \text{for } k = d, a$$

² A quick proof: Let $P_t(l)$ denote the time *t* price of a state contingent claim for state $\mathbf{S}_{t+1} = l$, which pays one unit of domestic consumption good, $D_{t+1}^d = 1$, if this state occurs and none otherwise, then the pricing equations (9) and (12) imply

where $\Pi_t(\mathbf{S}_{t+1})$ is the conditional probability that state \mathbf{S}_{t+1} occurs at time t+1. Because the probability and the price of the contingent claim are the same for k = d, a, $M_{t+1}^d = M_{t+1}^a$ for every state at times t and t+1.

$$MSD_{t} \equiv E_{t}[m_{t+1}^{d} - m_{j,t+1}^{a}]^{2} = \sigma_{t}^{2}[m_{t+1}^{d} - m_{t+1}^{a}] + \left(E_{t}[m_{t+1}^{d} - m_{t+1}^{a}]\right)^{2},$$
(14)

In complete international markets, $m_{t+1}^d = m_{t+1}^a$; or equivalently, $MSD_t = 0$ at all times.

Assume that there exists a domestic real riskfree asset which pays one unit of the domestic consumption good. Then under conditional normality or a second-order Taylor expansion, the Euler equations (9) and (12) imply that the domestic log real riskfree rate satisfies

$$r_F^d = -E_t[m_{t+1}^d] - \frac{1}{2}\sigma_t^2[m_{t+1}^d] = -E_t[m_{t+1}^a] - \frac{1}{2}\sigma_t^2[m_{t+1}^a], \qquad (15)$$

Hence,

$$E_{t}[m_{t+1}^{d}] - E_{t}[m_{t+1}^{a}] = -\frac{1}{2} \Big(\sigma_{t}^{2}[m_{t+1}^{d}] - \sigma_{t}^{2}[m_{t+1}^{a}] \Big).$$
(16)

Note that $\sigma_t^2[m_{t+1}^d - m_{t+1}^a] = 0$ implies that $m_{t+1}^d - m_{t+1}^a = k_t$ which is known at time *t*, and thus $\sigma_t^2[m_{t+1}^d] = \sigma_t^2[m_{t+1}^a]$. Substituting equation (16) into equation (14) implies that $MSD_t = 0$, if and only if $\sigma_t^2[m_{N,t+1}^d - m_{j,t+1}^a] = 0$.

Given the preceding discussions, I define an exchange rate-adjusted risk sharing index:

$$ARSI_{t} = 1 - \frac{\sigma_{t}^{2}[m_{N,t+1}^{d} - m_{j,t+1}^{a}]}{\sigma_{t}^{2}[m_{t+1}^{d}] + \sigma_{t}^{2}[m_{t+1}^{a}]} = \frac{2\operatorname{cov}_{t}[m_{t+1}^{d}, m_{t+1}^{a}]}{\sigma_{t}^{2}[m_{t+1}^{d}] + \sigma_{t}^{2}[m_{t+1}^{a}]}$$
(17)

The index reaches a maximum of unity at time *t* if $m_{t+1}^d = m_{t+1}^a$; it is zero if the domestic and foreign exchange rate-adjusted log discount factors are uncorrelated. Note that the adjusted risk sharing index has the same sign as the correlation coefficient between the domestic and foreign exchange rate-adjusted log discount factors but is less than the correlation coefficient in magnitude unless the conditional variance of foreign exchange rate-adjusted discount factor is equalized to that of the domestic discount factor. Under exchange rate volatility, the adjusted index given by equation (17) can be greater than the conditional correlation between the domestic and foreign log discount factors.

When the domestic and foreign discount factors are marginal utility growth rates, the adjusted index given by equation (17) serves as a measure of consumption risk sharing between two countries.

However, if the domestic and foreign discount factors are minimum-variance discount factors for subsets of assets, which are projections of marginal utility growth rates onto subsets of assets held domestic and foreign investors, a unitary value of the index is not equivalent to the Pareto optimal consumption allocations across countries. Instead, the adjusted index is indicative of how much risks associated with such discount factors, or more precisely, the subsets of assets along with exchange rate risk are shared between domestic and foreign investors.

By substituting out the exchange rate-adjusted discount factor using equation (13), I express the adjusted risk sharing index as

$$ARSI_{t} = \frac{2\left(\operatorname{cov}_{t}[m_{t+1}^{d}, m_{t+1}^{f}] - \operatorname{cov}_{t}[m_{t+1}^{d}, \Delta e_{t+1}]\right)}{\sigma_{t}^{2}[m_{t+1}^{d}] + \sigma_{t}^{2}[m_{t+1}^{f}] + \sigma_{t}^{2}[\Delta e_{t+1}] - 2\operatorname{cov}_{t}[m_{t+1}^{f}, \Delta e_{t+1}]}.$$
(18)

Thus the adjusted index depends on the conditional second moments of domestic and foreign discount factors and exchange rate changes. The index in equation (17) is simplified to the following unadjusted risk sharing index under a constant real exchange rate:

$$RSI_{t} = 1 - \frac{\sigma_{t}^{2}[m_{t+1}^{d} - m_{t+1}^{f}]}{\sigma_{t}^{2}[m_{t+1}^{d}] + \sigma_{t}^{2}[m_{t+1}^{f}]} = \frac{2\operatorname{cov}_{t}[m_{t+1}^{d}, m_{t+1}^{f}]}{\sigma_{t}^{2}[m_{t+1}^{d}] + \sigma_{t}^{2}[m_{t+1}^{f}]}$$
(19)

which is similar to the risk sharing index introduced by Brandt, et al. (2004), except that I evaluate the index using conditional moments. When the real exchange rate is constant, both indices given by equations (18)-(19) attain unity for marginal utility growth rates under Pareto optimal consumption allocations or any discount factors under complete international market assumption. However, under a stochastic real exchange rate, only the adjusted risk sharing index reaches a maximum of unity. Therefore, the risk sharing index (18) serves as a more appropriate measure of the level of international risk sharing under the exchange rate volatility.

B.1 Minimum-variance stochastic discount factors for subsets of assets

The law of one price implies that every subset of assets can be priced by a unique minimumvariance discount factor formed by a payoff from portfolios of the subset of assets. The minimumvariance discount factor also represents a projection of marginal utility growth of investors, whose investment opportunity set entails the subset of assets, onto the subset of assets (see, e.g., Cochrane (2001)). In the international context, investors are faced with investment opportunities from domestic and foreign financial markets as well as nonfinancial assets such as human capital and real estates from their own countries. For both domestic and foreign investors, I consider a common subset of financial assets including a domestic stock, a foreign stock, and the currency exchange between the pair of countries. The entire subsets of assets faced by domestic and foreign investors are, respectively:

$$\Omega^{d} = \{ \text{domestic stock, foreign stock, currency, domestic human capital} \},$$

$$\Omega^{f} = \{ \text{domestic stock, foreign stock, currency, foreign human capital} \}.$$
(20)

Here I assume that labor markets exist within countries so the value of human capital can be determined by the law of one price, like financial assets. I also assume that domestic and foreign real riskfree rates are constant.

I denote continuously-compounded returns at time *t* in units of local consumption good on domestic or foreign stocks, riskfree assets, and human capital as r_t^j , r_F^j and $r_{hc,t}^j$ for j = d, *f*. For domestic investors, the vector of returns in units of domestic consumption good, in excess of the domestic riskfree rate, is represented by

$$\mathbf{r}_{t}^{d} = \begin{pmatrix} r_{t}^{d} - r_{F}^{d} \\ r_{t}^{f} + \Delta e_{t} - r_{F}^{d} \\ \Delta e_{t} + r_{F}^{f} - r_{F}^{d} \\ r_{hc,t}^{d} - r_{F}^{d} \end{pmatrix}.$$
(21)

Similarly, the vector of returns in units of foreign consumption good, in excess of the foreign riskfree asset, is given by

$$\mathbf{r}_{t}^{f} = \begin{pmatrix} r_{t}^{d} - \Delta e_{t} - r_{F}^{f} \\ r_{t}^{f} - r_{F}^{f} \\ -\Delta e_{t} + r_{F}^{d} - r_{F}^{f} \\ r_{hc,t}^{f} - r_{F}^{f} \end{pmatrix}$$
(22)

From Hansen and Jagannathan (1991), the domestic or foreign minimum-variance discount factor formed from portfolio returns on the subset of assets Ω^{j} given by equation (20) is

$$M_{t+1}^{j}(\Omega^{j}) = \frac{1}{R_{F}^{j}} \Big[1 - E_{t} [(\mathbf{R}_{t+1}^{j})'] \boldsymbol{\Sigma}_{t}^{-1}(\mathbf{R}_{t+1}^{j}) (\mathbf{R}_{t+1}^{j} - E_{t}(\mathbf{R}_{t+1}^{j})) \Big] \text{ for } j = d, f,$$
(23)

where \mathbf{R}_{t+1}^{j} is a vector of returns in simple compounding for period t+1 and $\Sigma_{t}(\mathbf{R}_{t+1}^{j})$ is the conditional covariance matrix of returns. I denote country *j*'s minimum-variance discount factor by $M_{t+1}^{j}(\Omega^{j})$ because it represents the projection of the domestic or foreign investor's marginal utility growth onto Ω^{j} . The corresponding log discount factor from a second-order Taylor expansion is then

$$m_{t+1}^{j}(\Omega^{j}) = -\left(r_{F}^{j} + \frac{1}{2}E_{t}[\mathbf{r}_{t+1}^{j}]\boldsymbol{\Sigma}_{t}^{-1}(\mathbf{r}_{t+1}^{j})E_{t}[\mathbf{r}_{t+1}^{j}]\right) - E_{t}[\mathbf{r}_{t+1}^{j}]\boldsymbol{\Sigma}_{t}^{-1}(\mathbf{r}_{t+1}^{j})\left(\mathbf{r}_{t+1}^{j} - E_{t}[\mathbf{r}_{t+1}^{j}]\right), \text{ for } j = d, f \qquad (24)$$

Note that equation (24) is exact in the continuous-time limit and similar to the expression of the log discount factor given by Brandt, et. al. (2004), except that the covariance matrix of returns here is country-specific. For the given log discount factors, I obtain following conditional moments:

$$\sigma_t^2[m_{t+1}^j(\Omega^j)] = E_t[\mathbf{r}_{t+1}^j] \mathbf{\Sigma}_t^{-1}(\mathbf{r}_{t+1}^j) E_t[\mathbf{r}_{t+1}^j], \text{ for } j = d, f$$
(25)

$$\operatorname{cov}_{t}[m_{t+1}^{j}(\Omega^{j}), \Delta e_{t+1}] = -E_{t}[\mathbf{r}_{t+1}^{j}] \mathbf{\Sigma}_{t}^{-1}(\mathbf{r}_{t+1}^{j}) \operatorname{cov}_{t}[\mathbf{r}_{t+1}^{j}, \Delta e_{t+1}], \text{ for } j = d, f$$
(26)

$$\operatorname{cov}_{t}[m_{t+1}^{d}(\Omega^{d}), m_{t+1}^{f}(\Omega^{f})] = E_{t}[\mathbf{r}_{t+1}^{d}] \boldsymbol{\Sigma}_{t}^{-1}(\mathbf{r}_{t+1}^{d}) \operatorname{cov}_{t}[\mathbf{r}_{t+1}^{d}, \mathbf{r}_{t+1}^{f}] \boldsymbol{\Sigma}_{t}^{-1}(\mathbf{r}_{t+1}^{f}) E_{t}[\mathbf{r}_{t+1}^{f}].$$
(27)

The adjusted risk sharing index, $ARSI_{t}$, for the subsets of assets can be calculated by substituting equations (25)-(27) into equation (18).

If human capital is excluded from the investment opportunity sets, then equations (21)-(22) suggest that domestic and foreign investors face shocks from the same subset of assets:

$$\Omega = \{ \text{domestic stock, foreign stock, and currency} \}$$
(28)

As discussed earlier (see equation (13)), the exchange rate-adjusted foreign log discount factor,

$$m_{t+1}^{a}(\Omega) = m_{t+1}^{f}(\Omega) - \Delta e_{t+1}, \qquad (29)$$

is a valid log discount factor for returns in units of domestic consumption good. Since shocks to Δe_{t+1} is part of shocks to Ω , $m_{t+1}^a(\Omega)$ and $m_{t+1}^d(\Omega)$ are linear combinations of shocks from the same subset of assets, Ω , so they must be equalized for every state of nature given the uniqueness of the minimumvariance discount factor that can be formed from linear combinations of a given set of shocks.³ The result implies that the adjusted risk sharing index, $ARSI_t$, must attain unity at any time, or equivalently, risks associated with domestic and foreign investors' investment opportunities are perfectly shared, if domestic and foreign investors are assumed to have the same investment opportunity set, including the currency of the two countries.

The preceding result breaks down when domestic and foreign investors are allowed to have asymmetric investment opportunity sets, $\Omega^d \neq \Omega^f$. Since the domestic discount factor, $m_{t+1}^d(\Omega^d)$, and the exchange rate-adjusted discount factor,

$$m_{t+1}^{a}(\Omega^{f}) = m_{t+1}^{f}(\Omega^{f}) - \Delta e_{t+1}, \qquad (30)$$

are no longer formed from linear combinations of the same set of shocks, in the presence of investor's own human capital in her portfolio, $m_{t+1}^{a}(\Omega^{f})$ and $m_{t+1}^{d}(\Omega^{d})$ are not necessary equalized unless shocks to human capital are equalized across countries, state by state.

The presence of human capital in the investment opportunity sets can affect international risk sharing in two ways. First, the volatility of a country's minimum-variance discount factor, $M_{t+1}^{j}(\Omega^{j})$, formed from the investment opportunity set including human capital should be greater than the volatility of the minimum-variance discount factor, $M_{t+1}^{j}(\Omega)$, formed from a subset including only financial assets because the former discount factor, $M_{t+1}^{j}(\Omega^{j})$, can price the subset of assets Ω but not vice versa. Second, if returns from human capital are weakly correlated across countries unlike returns from financial

³ Brandt, etl al. (2004) derives the result in a continuous time model.

assets, then the domestic and foreign minimum-variance discount factors, $M_{t+1}^{j}(\Omega^{j})$, formed from the investment opportunity set including human capital should be less correlated than $M_{t+1}^{j}(\Omega)$, formed from the subset including only financial assets.

B.2 Consumption-based stochastic discount factors

Under the assumption that investors maximize lifetime utility of consumption, the domestic and foreign marginal utility growth rates serve as discount factors for all assets held by domestic and foreign investors, respectively. These assets include, but are not limited to, the subset of assets, Ω^d , for the domestic investor or Ω^f for the foreign investor.

Assuming that the preferences of domestic and foreign representative agents can be represented by power utility:

$$U(C_t^j) = \left[(C_t^j)^{1-A^j} - 1 \right] / (1 - A^j),$$
(31)

with constant relative risk aversion, A^{j} , the log marginal utility growth rates as discount factors are

$$m_{i+1}^{j} = \ln(\rho^{j}) - A^{j} \Delta c_{i+1}^{j}, \quad j = d, f.$$
(32)

Substituting equation (32) into equations (18) yields

$$ARSI_{t} = \frac{2A^{d} \left(A^{f} \operatorname{cov}_{t}[\Delta c_{t+1}^{d}, \Delta c_{t+1}^{f}] - \operatorname{cov}_{t}[\Delta c_{t+1}^{d}, \Delta e_{t+1}]\right)}{(A^{d})^{2} \sigma^{2}[\Delta c_{t+1}^{d}] + (A^{f})^{2} \sigma^{2}[\Delta c_{t+1}^{f}] + \sigma_{t}^{2}[\Delta e_{t+1}] - 2A^{f} \operatorname{cov}_{t}[\Delta c_{t+1}^{f}, \Delta e_{t+1}]}.$$
(33)

The adjusted risk sharing index varies with the conditional volatility of the exchange rate change between the two countries, in addition to the conditional volatility of consumption growth of both countries. If exchange rate changes are uncorrelated with consumption growth of either country, ignoring the exchange rate volatility will overestimate the level of international consumption risk sharing, according to equation (33).

Alternately, assume that preferences of the representative agents are given by external habit utility,

$$U(C_t^j, S_t^j) = [(C_t^j S_t^j)^{1-\gamma} - 1]/(1-\gamma), \text{ for } j = d, f$$
(34)

with the same level of utility curvature, γ . Then the log marginal utility growth rates are

$$m_{t+1}^{j} = \ln(\rho^{j}) - \gamma(\Delta c_{t+1}^{j} + \Delta s_{t+1}^{j}), \text{ for } j = d, f$$
(35)

where S_t^{j} is country *j*'s surplus consumption of. For the sake of tractability, I adopt the habit specification of Campbell and Cochrane (1999) for each country by assuming that each country's log surplus consumption is given by an AR(1) process:

$$s_{t+1}^{j} = (1-\varphi)\overline{s}^{j} + \varphi s_{t}^{j} + \lambda_{t}^{j} \left(\Delta c_{t+1}^{j} - E_{t}[\Delta c_{t+1}^{j}]\right), \text{ for } j = d, f$$
(36)

where for simplicity the habit persistence parameter $0 < \phi < 1$ is assumed to be identical across countries and the sensitivity functions are

$$\lambda_t^j = \max\left\{0, \ \frac{1}{\overline{S}^j}\sqrt{1 - 2(s_t^j - \overline{s}^j)} - 1\right\}, \text{ for } j = d, f$$
(37)

with the steady-state surplus consumption, $\overline{S}^{j} = \sigma_{c}^{j} \sqrt{\gamma(1-\varphi)}$. Unlike Campbell and Cochrane (1999), I do not assume that the conditional moments of each country's consumption growth and the real risk free rates are constant. The log marginal utility growth rates are

$$m_{t+1}^{j} = \ln(\rho^{j}) - \gamma E_{t}[\Delta c_{t+1}^{j}] - \gamma(\varphi - 1)(s_{t}^{j} - \overline{s}^{j}) - \gamma(1 + \lambda_{t}^{j}) \Big(\Delta c_{t+1}^{j} - E_{t}[\Delta c_{t+1}^{j}]\Big), \text{ for } j = d, f$$
(38)

where $A_t^j \equiv \gamma(1 + \lambda_t^j)$ can be considered as a conditional measure of risk aversion for the country *j*'s investor. The power utility is a special case: $A_t^j \equiv A^j$. Then the adjusted risk sharing index is similar to equation (33) where A^j is replaced with A_t^j . As each country's time-varying sensitivity function reflects the country's changing investor risk aversion, the adjusted risk sharing index takes into account not only time-varying conditional volatilities of consumption growth and exchange rates but also time-varying investor risk aversion across business cycles. Similar to the case of power utility, overlooking the exchange rate volatility may also overstate the level of international consumption risk sharing.

For either the domestic or foreign investor, the conditional variance of the minimum-variance discount factor given by equation (25) serves as a lower bound for the conditional variance of any consumption-based discount factor:

$$\sigma_t^2[m_{t+1}^j] = (A_t^j)^2 \sigma^2[\Delta c_{t+1}^j] \ge E_t[\mathbf{r}_{t+1}^j] \mathbf{\Sigma}_t^{-1}(\mathbf{r}_{t+1}^j) E_t[\mathbf{r}_{t+1}^j], \text{ for } j = d, f.$$
(39)

For both the power and external habit utilities, the Hansen-Jagannathan (1991) relation given by equation (39) is unlikely to be satisfied at all times. For this reason, I define the following preference specification:

$$U(C_{t+1}^{j}, A_{t}^{j}) = \left[(C_{t+1}^{j})^{1-A_{t}^{j}} - 1 \right] / (1 - A_{t}^{j}), \qquad (40)$$

where the level of relative risk aversion at time t is given by

$$A_{t}^{j} \equiv \sqrt{E_{t}[\mathbf{r}_{t+1}^{j}]\mathbf{\Sigma}_{t}^{-1}(\mathbf{r}_{t+1}^{j})E_{t}[\mathbf{r}_{t+1}^{j}]} / \sigma_{t}[\Delta c_{t+1}^{j}], \text{ for } j = d, f.$$
(41)

Equation (41) defines the minimum level of investor risk aversion at each time needed to satisfy equation (39). Hence, in the rest of paper, this type of investor preference is called minimum risk aversion utility. The log marginal utility growth and the adjusted risk sharing index under this type of utility are similar to those given by equations (32)-(33), where A^{j} is replaced with A_{t}^{j} given by equation (41).

II. The data and summary statistics

Before presenting the method for estimating the adjusted risk sharing indices, I first discuss the sample properties of the data to be used for the study in order to gain some intuition. Without loss of generality, I assume that the U.S. is the domestic country. I study quarterly returns on U.S. and U.K. national stock market indices from the Morgan Stanley Capital International (MSCI). Quarterly data for three-month Treasury bill rates, seasonally adjusted aggregate consumption, exchange rates, and Consumer Price Indices (CPI) for both countries are obtained from the International Financial Statistics (IFS). Per-capita seasonally adjusted real consumption for each country is obtained by interpolating the annual population data from IFS into quarterly observations. Per capita national labor income in a country is the country's seasonally adjusted compensation of employees obtained from the OECD national accounts deflated by

each country's CPI and population.⁴ Real stock returns and real riskfree rates are local log stock returns minus local log CPI growth rates. The real exchange rate between the U.S. and U.K. in units of the U.S. consumption good is the dollar price of pound multiplied by the ratio of the U.K. CPI index to the U.S. CPI index. The sample period spans from the first quarter of 1970 to the fourth quarter of 2003.

In Table 1 I present summary statistics for excess stock and currency returns and labor income growth from the U.S. and U.K. where returns are annualized in percent, converted into units of the U.S. consumption good, and in excess of the U.S. real riskfree rate. Real riskfree rate in a country is taken as the sample average of the real Treasury bill rate in the country. Over the sample period, the U.K. stock returns exceed the corresponding U.S. figures in terms of the sample means and standard deviations. Similarly, the mean (1.9%) and standard deviation (2.9%) of the U.K. annual labor income growth rate are higher than the mean (1.3%) and standard deviation of (1.8%) of the annual U.S. labor income growth rate. The mean consumption growth rate in the U.S. (1.8%) or the U.K. (2.4%) exceeds the mean consumption growth in the country. While the standard deviation of the U.S. consumption growth (1.7%) is similar to that of the U.S. labor income growth. The standard deviations of labor income and consumption growth from both countries are, however, much lower than that of the real exchange rate (9.6%) and those of stock returns.

I now examine the sample correlations. The stock returns between the U.S. and U.K. are known to be highly correlated with a sample correlation of 65%. While U.S. stock returns are weakly correlated with real exchange rate between the two countries, U.K. stock returns and the exchange rate have a correlation coefficient of 26%. We also note high correlations between labor income and consumption growth within countries. The correlations are 71% for the U.S. and 40% for the U.K. Other correlations are lower. For example, the correlation between U.S. and U.K. labor income growth is 14%, while the correlation between U.S. and U.K. consumption growth is 22%.

⁴ Japan's labor income data are not available from the same data source. Germany's consumption and labor income data are for West Germany before 1991 and for whole Germany since 1991 while population estimates are for East and West Germany for the whole period.

III. Estimating the moments of the minimum-variance discount factors

A. Econometric specifications

In this section I discuss the econometric specifications for the conditional moments of the minimumvariance discount factors and results of the estimating the moments and the adjusted risk sharing index. Expected stock returns are given by conditional asset pricing models. The conditional volatilities of stock returns, the exchange rate, and labor income are modeled by a multivariate GARCH model.

Unlike stock returns and riskfree rates, returns on human capital are unobservable. Following Fama and Schwert (1977) and Jagannathan and Wang (1996), I assume that returns on human capital can be proxied by the growth rate of labor income. This assumption is justified if the value of human capital, $V_{hc,t}^{j}$, is proportional to labor income, L_{t}^{j} , in each country: $V_{hc,t}^{j} = L_{t}^{j}/k$ at any time where k is constant, and the return on human capital is defined as the growth rate of human capital, so $r_{hc,t}^{j} \equiv \Delta v_{hc,t}^{j} = \Delta l_{t}^{j}$ for j= d, f.

To investigate the time variation in international risk sharing, it is necessary to model the timevarying expectations and volatilities of domestic and foreign discount factors. For the domestic and foreign investment opportunity sets Ω^{j} described earlier, returns on financial and human capital are determined by shocks from the following five sources: domestic and foreign stock returns, the exchange rate, and domestic and foreign labor income growth. Therefore, I stack returns on financial assets to domestic investors and labor income growth of domestic and foreign investors into the following 5×1 vector:

$$\mathbf{y}_{t} = \begin{pmatrix} \mathbf{r}_{t}^{d} - \mathbf{r}_{F}^{d} \\ \mathbf{r}_{t}^{f} + \Delta \mathbf{e}_{t} - \mathbf{r}_{F}^{d} \\ \Delta \mathbf{e}_{t} + \mathbf{r}_{F}^{f} - \mathbf{r}_{F}^{d} \\ \Delta \mathbf{l}_{t}^{d} \\ \Delta \mathbf{l}_{t}^{f} \end{pmatrix}.$$
(42)

The vectors of excess returns on financial and human capital Ω^{j} for domestic and foreign investors are, respectively,

$$\mathbf{r}_{t}^{d} = \begin{pmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} - \mathbf{r}_{F}^{d} \end{pmatrix}, \quad \mathbf{r}_{t}^{f} = \begin{pmatrix} y_{1t} - y_{3t} \\ y_{2t} - y_{3t} \\ -y_{3t} \\ y_{5t} - \mathbf{r}_{F}^{f} \end{pmatrix}.$$
(43)

By defining the following vector and matrices,

$$\mathbf{M}^{d} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \qquad \mathbf{M}^{f} = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{k}^{d} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -r_{F}^{d} \end{pmatrix}, \qquad \mathbf{k}^{f} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -r_{F}^{f} \end{pmatrix}, \quad (44)$$

the vectors of excess returns can be written as $\mathbf{r}_t^d = \mathbf{M}^d \mathbf{y}_t + \mathbf{k}^d$, and $\mathbf{r}_t^f = \mathbf{M}^f \mathbf{y}_t + \mathbf{k}^f$. To model the conditional means, variances, and the covariance of \mathbf{r}_{t+1}^d and \mathbf{r}_{t+1}^f , therefore, it is only necessary to model the first two conditional moments of \mathbf{y}_{t+1} . To this end, Let

$$\mathbf{y}_{t+1} = E_t[\mathbf{y}_{t+1}] + \boldsymbol{\varepsilon}_{t+1}, \quad \boldsymbol{\varepsilon}_{t+1} \sim N(\mathbf{0}, \mathbf{H}_{t+1})$$
(45)

where $\mathbf{\varepsilon}_{t+1}$ is a 5×1 vector of shocks with *i*th element $\varepsilon_{i,t+1}$, and \mathbf{H}_{t+1} is a 5×5 conditional covariance matrix with (*i*,*j*) element $h_{ij,t+1}$. Then

$$\boldsymbol{\Sigma}_{t}(\mathbf{r}_{t+1}^{j}) = \mathbf{M}^{j} \mathbf{H}_{t+1} \mathbf{M}^{j}, \quad j = d, f, \qquad \operatorname{cov}_{t}(\mathbf{r}_{t+1}^{d}, \mathbf{r}_{t+1}^{f}) = \mathbf{M}^{d} \mathbf{H}_{t+1} \mathbf{M}^{f}, \quad (46)$$

and so forth.

Asset pricing theory suggests that expected returns on any asset should be related to its conditional covariance with any discount factor that prices the asset. In the current setting, the domestic or foreign log discount factor is a linear combination of four shocks. Because I work with low frequency, quarterly data in this article, it is beyond the scope of the paper to study a model of expected returns which depend explicitly on the conditional covariancess associated with each of the shocks. As an approximation, I assume that expected domestic or foreign equity market return is linearly related its own

volatility, its conditional covariance with labor income growth within the country, and an instrument which proxies for unspecified covariances or time-varying prices of risks from a linear approximation:⁵

$$y_{1,t+1} \equiv r_{t+1}^d - r_F^d = \alpha_{10} + \alpha_1 \ cay_t + \alpha_{11}h_{11,t+1} + \alpha_{14}h_{14,t+1} + \varepsilon_{1,t+1}$$
(47)

$$y_{2,t+1} \equiv r_{t+1}^f + \Delta e_{t+1} - r_F^d = \alpha_{20} + \alpha_2 \ cay_t + \alpha_{22}h_{22,t+1} + \alpha_{25}h_{25,t+1} + \varepsilon_{2,t+1}$$
(48)

Here I use the lagged log consumption-wealth ratio (*cay*) of Lettau and Ludvigson (2001a,b) as an instrument because of its strong predictive power for U.S. stock returns and the high correlations between U.S. and U.K. stock returns. I assume that each of the remaining elements of \mathbf{y}_t follows an AR(1) given the weak interdependence among them:

$$y_{i,t+1} = \alpha_{i0} + \alpha_i y_{it} + \varepsilon_{i,t+1}, \quad i = 3, 4, 5,$$
 (49)

Given the specifications of conditional means, I assume that the conditional covariance matrix follows the multivariate GARCH(1,1) of Engle and Kroner (1995):

$$\mathbf{H}_{t+1} = \mathbf{C}'\mathbf{C} + \mathbf{B}'\mathbf{H}_{t}\mathbf{B} + \mathbf{A}'\boldsymbol{\varepsilon}_{t}\boldsymbol{\varepsilon}_{t}'\mathbf{A},$$
(50)

where A, B and C are 5×5 matrices of constant coefficients and C is restricted to be lower triangular.

Equation (50) is very appealing because it directly imposes positive definiteness on the covariance matrix and allows interactions among the variances and covariances. The specification, however, is very difficult to estimate due to the large number of unknown parameters. Some authors impose the restriction that both **A** and **B** are diagonal (e.g., De Santis and Gerald (1997)), while others assume that conditional volatility follows alternative univariate models (e.g., Scruggs (1998), Duffee (2005)). While allowing interactions of volatility between domestic and foreign stock returns, between stock returns and labor income within countries, and between foreign stock returns or labor income and exchange rates, I impose the following restrictions on elements of **A** and **B**:

⁵ For evidence on the time-varying price of covariance risk associated with a world market portfolio, see Bekaert and Harvey (1995) and De Santis and Gerard (1997).

$$a_{13} = a_{31} = b_{13} = b_{31} = 0,$$

$$a_{15} = a_{51} = b_{15} = b_{51} = 0,$$

$$a_{24} = a_{42} = b_{24} = b_{42} = 0,$$

$$a_{34} = a_{43} = b_{34} = b_{43} = 0,$$

$$a_{45} = a_{54} = b_{45} = b_{54} = 0.$$

(51)

For example, to see how the restrictions $b_{31} = b_{51} = 0$ affect $h_{11,t+1}$, let us examine the (1,1)th element of the second term in equation (50), which is given by

$$b_{11}(h_{11,t}b_{11} + h_{12,t}b_{21} + h_{14,t}b_{41}) + b_{21}(h_{21,t}b_{11} + h_{22,t}b_{21} + h_{24,t}b_{41}) + b_{41}(h_{41,t}b_{11} + h_{42,t}b_{21} + h_{44,t}b_{41})$$

$$= b_{11}^{2}h_{11,t} + b_{21}^{2}h_{22,t} + b_{41}^{2}h_{44,t} + 2(b_{11}b_{21}h_{12,t} + b_{11}b_{41}h_{14,t} + b_{21}b_{41}h_{24,t}).$$
(52)

Thus the domestic stock variance can depend on the lagged variances of domestic and foreign stock returns and domestic labor income growth as well as the lagged covariances between any pairs of these three terms. The specification, however, rules out the dependence of the domestic stock variance on the lagged variances of the exchange rate change and foreign labor income growth and lagged covariances of these variables with any other variables. In general the *j*th column of **B** indicates how the conditional variance of *j*th variable relates to lagged conditional moments and the *j*th column of **A** indicates how the conditional variance of *j*th variable relates to lagged disturbance terms.

Similarly, the *i*th and *j*th columns of **B** deliver information about how $h_{ij,t+1}$ is related the lagged conditional moments and correspondingly, *i*th and *j*th columns of **A** deliver information about how $h_{ij,t+1}$ is related the lagged disturbance terms. For example, $h_{12,t+1}$ is related to the (1,2)th element of the second term in equation (50):

$$b_{11}(h_{11,t}b_{12} + h_{12,t}b_{22} + h_{13,t}b_{32} + h_{15,t}b_{52}) + b_{21}(h_{21,t}b_{12} + h_{22,t}b_{22} + h_{23,t}b_{32} + h_{25,t}b_{52}) + b_{41}(h_{41,t}b_{12} + h_{42,t}b_{22} + h_{43,t}b_{32} + h_{45,t}b_{52}) = b_{11}b_{12}h_{11,t} + b_{21}b_{22}h_{22,t} + (b_{11}b_{22} + b_{21}b_{12})h_{12,t} + b_{11}(b_{32}h_{13,t} + b_{52}h_{15,t}) + b_{21}(b_{32}h_{23,t} + b_{52}h_{25,t}) + b_{41}(b_{12}h_{41,t} + b_{22}h_{42,t} + b_{32}h_{43,t} + b_{52}h_{45,t})$$
(53)

Thus the conditional covariance between domestic and foreign stock returns, $h_{12,t+1}$, can be related to the lagged conditional variances of domestic and foreign stock returns and lagged conditional covariance between them as well as other lagged conditional covariances.

Likewise, $h_{45,t+1}$ is related to the (4,5)th element of the second term in equation (50):

$$b_{14}(h_{12,t}b_{25} + h_{13,t}b_{35} + h_{15,t}b_{55}) + b_{44}(h_{42,t}b_{25} + h_{43,t}b_{35} + h_{45,t}b_{55}) = b_{44}b_{55}h_{45,t} + b_{14}b_{25}h_{12,t} + b_{14}(h_{13,t}b_{35} + h_{15,t}b_{55}) + b_{44}(h_{42,t}b_{25} + h_{43,t}b_{35}).$$
(54)

Thus the conditional covariance between domestic and foreign labor income growth can vary with its own lag and lagged covariance between domestic and foreign stock returns, as well as other lagged covariances.

I estimate the model using the method of maximum likelihood. Given the complexity of the model, a good set of initial parameter estimates is necessary. First, for the U.S. and U.K. stock returns, I estimate univariate GARCH(1,1) models by zeroing out the GARCH-in-mean parameters λ_{11} and λ_{22} to get initial estimates of conditional variances of returns. Second, I run univariate OLS regressions of stock returns on estimated conditional stock variances and lagged *cay* to obtain disturbance terms for stock returns of both countries. For the rest of variables, I estimate univariate AR(1) model to obtain initial estimate of associated disturbance terms. Second, by setting H_0 and C'C to be the covariance matrix of the initial disturbance terms and holding the parameters of conditional means fixed, I search the GARCH parameters given by equation (50) under restrictions (51) by maximizing the log likelihood function. Finally, all parameters of conditional means and unrestricted parameters of GARCH terms are estimated jointly by the maximum log likelihood method. I compute the *t*-statistics using standard errors that are robust to nonmormality of the disturbance terms.

B. The estimation results

Table 2 presents the results of estimating the multivariate model of stock returns, currency returns, and labor income growth for U.S. and U.K. The top section of table displays parameter estimates

of conditional means, the middle section displays parameter estimates of conditional covariance matrices, and bottom section reports the maximized value of the log likelihood function and the adjusted risk sharing index implied by the estimated parameters.

Consider first the coefficients of the mean equations. The estimated coefficients on lagged *cay* are 1.22 for U.S. stocks and 1.30 for U.K. stocks and significant at the 5% level for both countries, implying that the lagged consumption-wealth ratio can predict both countries' stock returns. For the expected U.S. stock return, its own conditional variance, $h_{11,t+1}$, enters the mean equation with a coefficient of -8.69 and a *t*-statistic of -1.56, while its conditional covariance with the U.S. labor income, $h_{14,t+1}$, enters the mean equation with a coefficient of 83.37 and a *t*-statistic of 1.85. Note that magnitude of the covariance is much smaller than the variance of stock returns. Thus U.S. expected stock return is unrelated to its own variance but related to its covariance with U.S. labor income growth at the 10% level.

For the expected U.K. stock return, the coefficient on its own variance, $h_{22,t+1}$, is 2.50 with a *t*-statistic of 1.79 and the coefficient on its covariance with U.K. labor income, $h_{25,t+1}$, is 31.82 with a *t*-statistic of 2.03. The results imply that the expected U.K. stock return is positively related to its variance at the 10% level and significantly related to its covariance with the U.K. labor income at the 5% level. The adjusted R^2 s indicate that approximately 8.20% of the variability of U.S. stock returns is explained by the variability of its time-varying expected returns while 3.85% of the variability of U.K. stock returns is explained R^2 s is mainly caused by the difference between the explanatory powers of the consumption-wealth ratio for stock returns of the two countries.

I now discuss the results of estimating AR(1) coefficients for currency returns and labor income growth of the two countries. The coefficients are 0.18 for currency returns, 0.53 for U.S. labor income and 0.10 for U.K. labor income. Except for U.K. labor income, the AR(1) coefficients are statistically significant at the 5% level. The adjusted R^2 s of the three mean equations are 2.13%, 26.30% and -1.55% respectively.

Next, I examine the results of estimated coefficients of the multivariate GARCH models for the conditional covariance matrix. To converse space, the estimated coefficients associated with the matrix \mathbf{C} are not reported. The estimated values of unrestricted elements of the matrix \mathbf{B} are significant at the 5% level, except for b_{21} and b_{23} , suggesting that the conditional variances and covariances are not only related to their own lagged values but also lagged values of other second moments. For instance, from the first column of \mathbf{B} , the variance of U.S. stock returns is related to its own lagged variance, the lagged variance of U.S. labor income, and the lagged covariance of U.S. and U.K. stock returns do not have any significant effects on the variance of U.S. stock returns. The estimates reported in the second column of \mathbf{B} imply that the variance of U.K. stock returns varies with its own lag, the lagged variances of U.S. stock returns, the exchange rate, and U.K. labor income, as well as lagged covariance is related to its own lag, the lagged variance of U.K. labor income, and the lagged covariance of the exchange rate and U.K. labor income.

Similarly, the results reported in the last two columns of B indicate that the conditional variances of U.S. and U.K. labor income growth are related to their own lags and lagged variances of stock returns in their own countries as well as the lagged covariances of stock returns and labor income in their own countries. The variance of U.K. labor income growth is also related to the lagged variance of the exchange rate and lagged covariance between the exchange rate and U.K. stock returns and labor income.

The conditional covariances vary with lagged variances and covariances in a more complicated way. For example, the conditional covariance between U.S. and U.K stock returns varies with the lagged variance of U.S. stock returns, lagged covariance between U.S. and U.K stock returns and other lagged covariances. The conditional covariance between U.S. and U.K. labor income growth is related to the lagged covariance between stock returns of the two countries and lagged covariance between labor income growth of the two countries.

The estimated elements of matrix A reveal that the diagonal elements, a_{11} , a_{33} and a_{44} , are significant at the 5% level, indicating that conditional variances of domestic stock returns, exchange rate changes and domestic labor income growth are sensitive to the magnitudes of their own lagged shocks. Many off-diagonal elements such as a_{14} , a_{41} , a_{25} and a_{52} are also significant, implying that the conditional variances of stock returns and labor income growth in either country are also sensitive to the lagged cross-products of shocks between stock returns and labor income growth in the country and the magnitudes of the lagged shocks of each other. The significance of these off-diagonal elements also imply that the conditional covariance between U.S. and U.K. stock returns is sensitive to the lagged cross-product of shocks to labor income growth of the two countries and the conditional covariance between U.S. and U.K. labor income growth is sensitive to the lagged cross-product of shocks to stocks returns of the two countries.

To examine the degree of international risk sharing, I first compute the conditional moments of U.S. and U.K. log discount factors using equations (25)-(27) and then the exchange rate-adjusted risk sharing index given by equation (18). As noted earlier, when domestic and foreign investors are assumed to have symmetric investment opportunity without human capital, the adjusted risk sharing index implied by the minimum-variance discount factors are 100% at any time. However, when domestic and foreign investors are assumed to have asymmetric investment opportunity by admitting human capital, the average adjusted risk sharing index falls drastically to 58.9%, with a standard deviation of 32.2%. The sample distribution of the index is skewed to the left, so the median of 70% is higher than the mean but is still far below 100%. The high volatility of the index is consistent with the wide range of index, between - 43.4% and 99.4%. Although negative values of the index are not ruled out, 90% of the index falls within the range from 5% to 97%.

If the exchange rate is constant and the conditional variances of domestic and foreign log discount factors are always the same, then the index is simply the conditional correlation between the log discount factors of two countries. As seen from Figure 1, the movement of the index is less extreme than

the conditional correlation between the log discount factors in the mid-1970s and early 2000s. Figure 2 shows that the volatility of the U.S. log discount factor is more than twice that of the U.K., and the volatility of the real exchange rate can be high, e.g., in the mid-1970s, so the index is a more accurate measure of international risk sharing than the conditional correlation between the log discount factors.

As discussed earlier, the domestic minimum-variance discount factor is related to shocks to domestic labor income growth while the foreign minimum-variance discount factor is related to shocks to foreign labor income growth, in addition to shocks to internationally-traded financial assets. The estimation of the multivariate model indicates that conditional variances and covariances between U.S. and U.K. labor income growth are time varying. Because the multivariate GARCH model does not impose the restriction of constant conditional correlation between the U.S. and U.K. labor income growth, the conditional correlation between the U.S. and U.K. labor income growth, the conditional correlation between the U.S. and U.K. labor income growth is conspicuously evident on Figure 3. Hence, the time series variability of the risk sharing index is explained, to some extent, by the time-varying conditional correlation between the U.S. and U.K. labor income growth. I discuss more evidence on this in the next section.

IV. Estimating the moments of consumption-based discount factors

A. Econometric specifications

According to Campbell (1996), consumption shocks today are related to shocks to returns on wealth today and revisions in expectations about future returns on wealth and future consumption growth. This implies that the volatility of consumption growth can depend on shocks to stock returns. I use a GARCH-in-mean model to estimate the conditional moments of stock returns, the exchange rate and the consumption growth together. Consider the following 5×1 vector:

$$\mathbf{y}_{t} = \begin{pmatrix} r_{t}^{d} - r_{F}^{d} \\ r_{t}^{f} + \Delta e_{t} - r_{F}^{d} \\ \Delta e_{t} + r_{F}^{f} - r_{F}^{d} \\ \Delta c_{t}^{d} \\ \Delta c_{t}^{f} \end{pmatrix}.$$
(55)

Let \mathbf{H}_{t+1} denote the 5×5 conditional covariance matrix of \mathbf{y}_t , with (i,j) element $h_{ij,t+1}$. Then the moments in the consumption-based adjusted risk sharing indices given by equations (33) can be calculated as follows:

$$\sigma_t^2[\Delta e_{t+1}] = h_{33,t}, \sigma_t^2[\Delta c_{t+1}^d] = h_{44,t}, \sigma_t^2[\Delta c_{t+1}^f] = h_{55,t}, \operatorname{cov}_t[\Delta c_{t+1}^d, \Delta e_{t+1}] = h_{43,t}, \operatorname{cov}_t[\Delta c_{t+1}^f, \Delta e_{t+1}] = h_{53,t}.$$
 (56)

I estimate a conditional consumption-based CAPM for each country as follows:

$$y_{1,t+1} \equiv r_{t+1}^d - r_F^d = \alpha_{10} + \alpha_1 \ cay_t + \alpha_{14}h_{14,t+1} + \varepsilon_{1,t+1}$$
(57)

$$y_{2,t+1} \equiv r_{t+1}^f + \Delta e_{t+1} - r_F^d = \alpha_{20} + \alpha_2 \ cay_t + \alpha_{25}h_{25,t+1} + \varepsilon_{2,t+1}$$
(58)

The specifications of conditional expected returns are similar to the labor-income based model given by equations (47)-(48), except that $h_{14,t+1}$ now refers to the conditional covariance between domestic stock returns and domestic consumption growth and $h_{25,t+1}$ now refers to the conditional covariance between foreign stock returns and foreign consumption growth. The log consumption-wealth ratio, cay_t , is included in both equations, to capture the time-varying prices of consumption risk from a linear approximation, or the unspecified time-varying conditional covariance such as the conditional covariance of stock returns from one country with consumption growth of the other country.

I assume that each of the rest of variables, including the exchange rate change, the domestic and foreign consumption growth, follows an AR(1). I also assume that the conditional covariance matrix, \mathbf{H}_{t+1} , follows the restricted multivariate GARCH(1,1), given by equations (50)-(51). In this way, I allow interactions of volatilities between stock returns and consumption growth within countries.

B. Estimation results

The results of estimating the GARCH-in-mean model for U.S. and U.K. stock returns, the exchange rate and consumption growth are presented in Table 3. The log consumption-wealth ratio, *cay*, is still precisely estimated. The coefficients associated with the estimated conditional covariances of stock returns with consumption growth for the U.S. and U.K. are negative and less precisely estimated. For the U.S. case, a negative price of consumption risk has been reported by Duffee (2005). For the U.K. case, the estimated price of risk is -14.41 with a *t*-statistics of -1.82, so the coefficient is significant at the 10% level. The negative prices of risks are contradictory to the assumption that investors are risk averse. The adjusted R^2 s associated with the mean equations for stock returns are 11.2% and 6.7%, which are higher than those in Table 2, mostly due to the reduction in the explanatory variables for stock returns. The AR(1) model captures significantly positive autocorrelation of U.S. consumption growth and negative autocorrelation of U.K. consumption growth.

The estimated GARCH parameters, b_{41} , indicate that the conditional variance of U.S. stock returns is related to the lagged variance of U.S. consumption and the lagged covariance between U.S. stock returns and U.S. consumption growth. There is also evidence of time-varying conditional variance of U.K. consumption growth, as b_{35} , b_{55} and c_{55} are statistically significant. The model, however, reveals no evidence of time-varying volatility of U.S. consumption growth and time-varying covariance of U.S. stock returns with U.S. consumption growth, because none of the coefficients in column 4 of **B** and **A** are estimated precisely. The result on the lack of time-varying volatility of U.S. consumption growth is consistent with findings of Li (2001).

To estimate the level of the international consumption risk sharing, it is necessary to make assumptions about the risk aversion coefficients A^{j} in the power utility model and utility curvature γ and habit persistence parameter φ in the external habit model. Following the external habit literature (Campbell and Cochrane (1999), Li (2001, 2005)), I set $\gamma = 2$ and $\varphi = 0.95$. According to Li and Zhong (2004), the risk aversion coefficient, A^{j} , in the power utility model for each country needed to explain the magnitude of the global equity premium is the same as the mean risk aversion, $\overline{A^{j}} = \gamma^{j} / \overline{S}^{j}$, in the external habit model. For this reason, I set $A^{j} = \gamma / \overline{S}^{j}$. The implied risk aversion coefficients are 39 for the U.S. investor and 17 for the U.K. investor.

The summary statistics for the adjusted international consumption risk sharing index is reported at the bottom section of Table 3. The index is 0.19 under power utility and 0.18 under external habit utility, which are slightly lower than the correlation between U.S. and U.K. consumption growth. The standard deviations and other statistics of the estimated index values are also similar for both utility specifications. Like the risk sharing index for the minimum-variance discount factors, the consumptionbased indices are negatively skewed, with wide ranges, (-0.4, 0.6), and positive autocorrelations (0.33).

IV. Common component of minimum-variance and consumption-based risk sharing indices

A. Econometric specifications

The results of estimating the adjusted risk sharing indices for the minimum-variance and consumptionbased discount factors show some similarities of empirical distributions. I examine further the common component of minimum-variance and consumption-based risk sharing indices.

To this end, I estimate a multivariate model of domestic and foreign stock returns, labor income growth, consumption growth, and the exchange rate change. By stacking all of these variables into the following 7×1 vector:

$$\mathbf{y}_{t} = \begin{pmatrix} r_{t}^{d} - r_{F}^{d} \\ r_{t}^{f} + \Delta e_{t} - r_{F}^{d} \\ \Delta e_{t} + r_{F}^{f} - r_{F}^{d} \\ \Delta e_{t}^{d} \\ \Delta l_{t}^{d} \\ \Delta l_{t}^{f} \\ \Delta c_{t}^{d} \\ \Delta c_{t}^{f} \end{pmatrix},$$
(59)

I can estimate the joint conditional moments of these variables. The conditional covariance matrix of \mathbf{y}_t is given by a multivariate GARCH(1,1) model, equation (50). The parameters of the model, however,

become even more difficult to estimate precisely, due to the increased dimension of parameter space and limited sample size. As the estimated levels of international risk sharing are robust to the specifications of expected stock returns, to be discussed later, I simplify the model of expected stock returns as follows,

$$r_{t+1}^{d} - r_{F}^{d} = \alpha_{10} + \alpha_{1} \ cay_{t} + \varepsilon_{1,t+1}, \tag{60}$$

$$r_{t+1}^{f} + \Delta e_{t+1} - r_{F}^{d} = \alpha_{20} + \alpha_{2} \ cay_{t} + \varepsilon_{2,t+1}$$
(61)

Each of the other variables is given by an AR(1). I impose restrictions on the GARCH parameters to lower the dimension of the parameter space. These restrictions allow interactions of volatilities among stock returns, labor income growth, and consumption growth within countries, but not across countries.

B. Estimation results

The results of estimating the multivariate system is presented in Table 4. The estimated coefficients on the log consumption-wealth ratio, *cay*, remain significant at the 5% level for both U.S. and U.K. stock returns. The adjusted R^2 s are 7.20% and 3.39%, respectively, which are somewhat lower than those reported earlier in Tables 2 and 3. The results of estimating mean equations for labor income growth are similar to those in Table 2, but the estimates for expected consumption growth are imprecise here, unlike those reported in Table 3.

Among the estimated GARCH parameters, all of the diagonal elements of the matrix **B**, except b_{11} , and four of the diagonal elements of the matrix **A**, are statistically significant at the 5% level, indicating that the conditional variances of most variables are related to their own lags and shocks. Several off-diagonal elements of matrix **B** are also precisely estimated. For instance, coefficients b_{21} , b_{41} and b_{61} are significant at the 5% level, implying that the conditional variance of the U.S. stock returns is related to the lagged variances of U.K. stock returns, U.S. labor income and consumption growth as well as lagged covariances between pairs of these variables. In addition, b_{57} is significant at the 5% level, and

 b_{75} is significant at the 10% level, suggesting interactions of volatilities between U.S. and U.K. consumption growth.

Among the off-diagonal elements of matrix **A**, coefficients $a_{52}a_{46}, a_{27}$ and a_{57} are precisely estimated. Thus the conditional variance of U.K. stock returns is related to shocks to U.K. labor income, the conditional variance of U.S. consumption growth is related to shocks to U.S. labor income growth, and the conditional variance of U.K. consumption growth is related to shocks to U.K. stock returns and labor income growth.

The evidence on the time-varying conditional volatility of U.S. consumption growth presented here contrasts directly with the lack of evidence in Table 3, where labor income shocks are excluded in the GARCH model. Because coefficients, b_{41} , b_{61} and b_{66} are significantly, the conditional covariance between U.S. stock returns and consumption growth varies with the lagged covariance between U.S. labor income and consumption growth and lagged consumption volatility. The evidence on the time-varying conditional covariance of U.S. stock returns and consumption growth here is in accord with the finding of Duffee (2005), who assumes conditional moments are functions of market capitalization to consumption ratio and consumption to wealth ratio.

C. Implied statistics of the adjusted risk sharing indices and discount factor loadings

In Table 5 I present implied statistics of the adjusted risk sharing index for the asset-based minimum-variance discount factors (ARSIMV), the indices for consumption-based discount factors under power utility (ARSIPW), external habit utility (ARSIHB), and the minimum risk aversion utility (ARSI). For the purpose of comparison, I also present implied statistics for the conditional correlations between consumption growth (CORCG) and labor income growth (CORLB). On again, the estimated risk sharing indices under power and external habit utilities are qualitatively similar, with the means of ARISPW and ARIHB being 0.20 and 0.21, respectively, which are slightly higher than the mean of CORCG (0.19). The index under minimum risk aversion utility is also similar to those under power or habit utility, with a

mean of 0.19. While these indices average lower than the asset-based index, ARIMV (0.62), the later is more volatile with a higher standard deviation of 0.33, compared with standard deviations of 0.25 or lower for consumption-based indices.

It is noteworthy that the estimated average conditional correlation between domestic and foreign labor income growth is 0.45, which lie between the averages of risk sharing indices for minimumvariance discount and the consumption-based discount factors. The median of each index or correlation is higher than the mean. All of indices, like the conditional correlation between U.S. and U.K. labor income growth, are negatively skewed and positively autocorrelated, with wide ranges. The consumption-based indices are also highly correlated with one another and with the conditional correlation between U.S. and U.K. consumption growth, with sample correlations of 0.96-0.99. More interestingly, the asset-based index and the conditional correlation between U.S. and U.K. labor income growth have a correlation of 0.19. The consumption-based indices are also correlated with the conditional correlation between U.S. and U.K. labor income growth, with correlations of 0.43-0.45.

To examine the relations between the adjusted risk sharing indices and the conditional labor income correlation, I run regressions of each index on the conditional labor income correlation. The results are also reported in Table 5. The *t*-statistics in parentheses are adjusted for heteroskedasticity and autocorrelations of residuals up to lag 40, given the high level of autocorrelations of these variables. When the dependent variable is ARSIMV, the coefficient on CORLB is 0.35 with a *t*-statistic of 1.79, so the labor income correlation is significant at the 10% level for explaining the variability of the asset-based index, with an adjusted R^2 of 3.9%. With ARSIPW, ARSIHB, or ARSI as the dependent variable, the regression coefficient is significant with a *t*-statistic of 4.6 and an adjusted R^2 of 18% or higher, implying that the labor income correlation is highly significant for explaining the variability of the consumption-based risk sharing indices. Similar results are obtained when the dependent variable is the conditional consumption correlation, CORCG. Overall, the results are consistent with the hypothesis that time-varying international risk sharing measured from either asset-based or consumption-based discount

factors is related to the comovement between labor income growth rates across countries. I plot the assetbased index, ARSIMV, the consumption-based index, ARSIPW, and labor income correlation, CORLB on Figure 3. The figure confirms the common variations of the risk sharing indices and the comovement between labor income growth rates.

To get a better sense of the discount factors, I show in Table 6 the loadings on the log discount factors, given by equation (24) for the minimum-variance discount factors and equations (32) for the consumption-based discount factors. For the U.S. log minimum-variance stochastic discount factors, the loadings, $E_t[\mathbf{r}_{t+1}^d] \mathbf{\Sigma}_t^{-1}(\mathbf{r}_{t+1}^d)$, are on shocks to U.S. stocks returns, U.K. stock returns, currency returns, and U.S. labor income growth. For the U.K. log minimum-variance stochastic discount factors, the loadings, $E_t[\mathbf{r}_{t+1}^f] \mathbf{\Sigma}_t^{-1}(\mathbf{r}_{t+1}^f)$, are on shocks to U.S. stocks returns, U.K. stock returns, currency returns, and U.S. labor income growth. For the consumption-based discount factors the loadings are the risk aversion coefficient A_t^d for the U.S. and A_t^f for the U.K. The standardized loadings are loadings multiplied by the conditional standard deviations of shocks. For the consumption-based models, the standardized loadings are the conditional standard deviations of the log marginal utility growth.

First I discuss the loadings of the minimum-variance discount factors reported in panel A. Excluding human capital from domestic and foreign investor's investment opportunity sets, domestic and foreign log discount factors differ only by the exchange rate change, so the loadings on the domestic and foreign stock returns should be the same for domestic and foreign discount factors. This result no longer holds when domestic or foreign investor's investment opportunity set entails her own human capital. Although the means and standard deviations of the loadings on the U.S. or U.K. stock returns are still similar for both U.S. and U.K. discount factors, the mean loadings on the exchange rate change for U.S. and U.K. discount factors differ by more than one, as labor income shocks from a country only affect the discount factor of the country. The loading on the labor income growth is -2.8 on average for the U.S. with a standard deviation of 57 but is 9.6 on average for the U.K. with a lower standard deviation of 15.

To compare the magnitude of the loadings across assets, I examine the standardized loadings, which are loadings on standardized shocks with conditional standard deviations of unity. For U.S. and U.K. discount factors, the means of the standardized loadings on U.S. stock returns are larger than those on the labor income growth. However, in terms of the standard deviations, the loading on the U.S. labor income growth is more volatile than those on financial assets for the U.S. discount factor and the loading on the U.K. labor income growth is almost as volatile as the loading on the U.K. stock returns. Thus the effects of labor income shocks on the discount factor vary substantially over time, which helps explain the high volatility of risk sharing associated with the minimum-variance discount factors.

Next I discuss the loadings on the consumption-based discount factors, reported in panel B. Although the loading on U.S. consumption growth, which is the investors risk aversion coefficient under power utility, is more than twice as large as that on U.K. consumption growth, the standardized loadings of the two countries are similar, due to higher volatility of U.K. consumption growth. The mean loading for either country under external habit utility is similar to the risk aversion coefficient under power utility, because the latter is assumed to be the same as the steady-state value of investor risk aversion under external habit utility growth under power or habit utility turns out to be lower than the conditional volatility of the asset-based factor for either country, which contradicts the notion that the asset-based discount factor is a minimum-variance discount factor. However, the conditional volatility of the U.S. or U.K. marginal utility growth under minimum risk aversion utility matches that of the asset-based discount factor is a minimum-variance discount factor. However, the conditional volatility of the U.S. or U.K. marginal utility growth under minimum risk aversion utility matches that of the asset-based discount factor exactly at any time, although the implied risk aversion coefficient for either country is much higher under minimum risk aversion utility than that under power or external habit utility.

V. Robustness

A. The effects of cross-country labor income risk

In our basic model, expected stock returns from a country are related to the labor income risk of the same country. If international markets are integrated in that domestic investors hold foreign stocks and foreign investors hold domestic stocks, then the domestic minimum-variance discount factors including shocks to domestic labor income growth can be used to determine expected foreign stock returns and vice versa. This implies the one country's stock returns should be related to the labor income risk of another country. To test this hypothesis, I replace the within-country conditional covariance of returns with labor income growth with the cross-country conditional covariance in the mean equations for expected stock returns as follows:

$$y_{1,t+1} \equiv r_{t+1}^d - r_F^d = \alpha_{10} + \alpha_1 \ cay_t + \alpha_{11}h_{11,t+1} + \alpha_{15}h_{15,t+1} + \varepsilon_{1,t+1}$$
(62)

$$y_{2,t+1} \equiv r_{t+1}^f + \Delta e_{t+1} - r_F^d = \alpha_{20} + \alpha_2 \ cay_t + \alpha_{22}h_{22,t+1} + \alpha_{24}h_{24,t+1} + \varepsilon_{2,t+1}$$
(63)

I also replace the following restrictions on the GARCH parameters:

$$a_{15} = a_{51} = b_{15} = b_{51} = 0,$$

$$a_{24} = a_{42} = b_{24} = b_{42} = 0,$$
(64)

with

$$a_{14} = a_{41} = b_{14} = b_{41} = 0,$$

$$a_{25} = a_{52} = b_{52} = b_{52} = 0,$$
(65)

to allow cross-country instead of within-country interactions of volatilities between stock returns and labor income growth.

The results of estimating the modified model are reported in Table 7. While lagged *cay* is still a significant predictor of U.S. and U.K. stock returns, none of other variables including conditional variances and covariances are statistically significant. Unlike the results on within-country interactions of GARCH effects between stock returns and labor income growth, some of the cross-country interactions of GARCH effects such as b_{15} , b_{51} , and a_{24} are insignificant. The adjusted R^2 for U.S. stocks (6.82%) and the maximized value of the log likelihood function (2036.24) from the modified model are noticeably lower than those from the basic model. The results suggest that within-country labor income risk is more important than cross-country labor income for determining expected stock returns and volatility of

returns. Nonetheless, the mean and median of the adjusted risk sharing index implied by the model with cross-country interactions of stock returns and labor income growth are 52.4% and 55.5%, respectively, which are even lower than the estimates implied by the basic model. Overall, the estimation results show that the conclusion about the effects of asymmetric investment opportunities on the inferences about international risk sharing is robust to the specifications of conditional means and volatilities of returns.

B. The effects of excluding labor income risk for expected stock returns

I conduct a variety of robustness checks of the model specifications. I first examine what happens if the conditional covariances of returns with labor income are omitted from the mean equations for expected stock returns of both countries. The specification of the expected stock returns is then

$$y_{1,t+1} \equiv r_{t+1}^d - r_F^d = \alpha_{10} + \alpha_1 \, cay_t + \alpha_{11} h_{11,t+1} + \varepsilon_{1,t+1} \tag{66}$$

$$y_{2,t+1} \equiv r_{t+1}^f + \Delta e_{t+1} - r_F^d = \alpha_{20} + \alpha_2 \ cay_t + \alpha_{22}h_{22,t+1} + \varepsilon_{2,t+1}$$
(67)

which is similar to the model for the U.S. market studied by Guo and Whitelaw (2005). The results of estimating the reduced model along with summary statistics for the adjusted risk-sharing index are summarized in Table 8. In the absence of labor income risk in the model, expected stock returns from both countries are still positively and significantly related to lagged *cay*. The relation between expected stock returns from either country and the conditional variance of returns is positive and the relation is significant at the 5% level for U.K. More specifically, the coefficient on the conditional variance is 0.40 with a *t*-statistic of 0.07 for the U.S. and 3.24 with a *t*-statistic of 2.07 for the U.K. Thus omitting the risk of labor income makes the price of local market risk more precisely estimated for U.K. For the U.S. case, the result is largely consistent with the finding of Guo and Whitelaw (2005), who report that the significance of the conditional market volatility depends on the specification of the volatility. However, AR(1) coefficients for currency returns and labor income growth of both countries are now imprecisely estimated. Excluding the labor income risk for expected stock returns also lowers the precision of estimated coefficients of the GARCH model for volatilities, e.g., coefficients b_{14} , b_{25} and a_{14} . The

reduction of precision can be attributed to the omitted-variable problem in the mean equations for stock returns, as the multivariate model is estimated jointly.

The adjusted R^2 s reported near the bottom of table confirm the importance of the effects of the labor income risk on stock returns. The adjusted R^2 s here for both U.S. and U.K. stock returns are 7.97% and 3.66%, which are lower than the corresponding values reported in Table 2. The maximized value of the log likelihood function also drops from 2040.94 to 2038.36. In spite of these, the reduced model delivers an estimate of the average adjusted risk sharing index of 58.7% with a standard deviation of 31.6% and a median of 66.2%, which are slightly lower than the previous estimates.

C. The effects of cross-market risk

In a second robustness check, I replace the conditional covariance of stock returns with labor income growth for either country's expected stock returns with the conditional covariance of stock returns between countries:

$$y_{1,t+1} \equiv r_{t+1}^d - r_F^d = \alpha_{10} + \alpha_1 \ cay_t + \alpha_{11}h_{11,t+1} + \alpha_{12}h_{12,t+1} + \mathcal{E}_{1,t+1}, \tag{68}$$

$$y_{2,t+1} \equiv r_{t+1}^f + \Delta e_{t+1} - r_F^d = \alpha_{20} + \alpha_2 \ cay_t + \alpha_{21}h_{21,t+1} + \alpha_{22}h_{22,t+1} + \varepsilon_{2,t+1}.$$
(69)

The model is similar to the bivariate model of daily U.S. and Japan equity markets studied by Chan, Karolyi, and Stulz (1992), except that I also include an instrument *cay*, which are still significant at the 5% level for both countries. Similar to them, the results reported in Table 9 indicate that expected U.S. stock market is unrelated to its own conditional variance, with a coefficient of -0.59 and a *t*-statistic of - 0.40, but positively related to its conditional covariance with foreign stock returns, with a coefficient of 7.13 and a *t*-statistic of 5.99. Interestingly, I also find that the U.K. stock market remains positively and significantly related to its covariance with the U.S. market, with a coefficient of -7.65 and a *t*-statistic of -8.33. Hence, for stock returns of both countries, the price of the U.S. stock market risk is negative or insignificant but the price of the U.K. stock market risk is positive and significant, which implies that

investors of both countries require lower or the same expected returns on domestic or foreign stocks if these stocks are expected to covary more with the U.S. stock market but investors require higher expected returns on stocks if they are expected to covary more with the U.K. stock market.

Despite of the modification of the specification for expected stock returns, the estimation results for conditional means of other variables and the dynamics of conditional covariance matrix are qualitatively similar. The modified model produces a higher adjusted R^2 of 10.52% for U.S. stock returns but lower adjusted R^2 of 3.59% for U.K. stock returns. The maximized value of the log likelihood function is almost unchanged. Finally, the time-series distribution of the adjusted risk sharing index shifts only slightly to the right, compared with the basic model for expected stock returns. The mean and median of the index are 63.4% and 71.7%, respectively.

VI. Conclusions

In this paper, I study the importance of labor income risk on time-varying risk premiums and volatilities of U.S. and foreign stock markets and the impacts of asymmetric investment opportunities as a result of admitting human capital on the time series variability of international risk sharing. By extending the domestic finance literature on the roles of human capital, the results of the paper suggest that the interactions of stock returns with returns on human capital, more specifically, labor income growth, go a long way toward our understanding of the time variations of the risk premiums and volatilities of the U.S. and foreign stock markets. The results highlight the roles of country-specific, idiosyncratic labor income risks in international equity markets.

I also report that the comovement of labor income growth, like the comovement of the consumption growth across countries, varies considerably through time. Accordingly, the level of the risks shared between U.S. and foreign investors fluctuates greatly over the past three decades, if human capital that is not traded across countries is recognized as one of the investment vehicles, along with

internationally-traded financial capital. Uninsurable and heteroskedastic shocks to labor income growth are therefore one of the main reasons for the time-varying stock market volatility and predictability as well as for incomplete consumption risk sharing in international markets.

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Table 1. Summary statistics

Returns on U.S. stocks, U.K. stocks and the currency are real returns in units of the U.S. consumption good, in excess of the U.S. real riskfree rate. The currency returns are for buying pounds and lending at the U.K. real riskfree rate. USLB and UKLB are per capita real labor income growth of U.S. and U.K., respectively. USCG and UKCG are per capita real consumption growth of U.S. and U.K., respectively. The sample period is from the 1970:Q1 to 2003:Q4. All series are annualized in percent and logs.

			Correlations									
Returns	Mean	Std dev.	US stocks	UK stocks	Currency	USLB	UKLB	USCG				
US stocks	4.20	17.28										
UK stocks	4.71	22.74	0.65									
Currency	1.16	9.55	-0.01	0.26								
USLB	1.28	1.75	0.11	0.11	-0.04							
UKLB	1.93	2.89	-0.17	-0.05	-0.09	0.14						
USCG	1.76	1.71	0.16	0.13	-0.08	0.71	0.16					
UKCG	2.38	3.87	0.03	0.04	-0.07	0.08	0.40	0.22				

Table 2. Estimates of a multivariate model of domestic and foreign stock returns, currency returns and domestic and foreign labor income growth

 $r_t^j r_F^j$ and Δl_t^j are real stock returns, riskfree rate, and labor income growth at time *t* for U.S. (*j=d*) and U.K. (*j=f*), respectively. Δe_t is the change in the log real exchange rate between the two countries, in units of U.S. consumption good. cay_t is the U.S. log consumption-wealth ratio. The sample period is from the 1970:Q1 to 2003:Q4. The following system of equation is estimated by the method of maximum likelihood:

$$\begin{aligned} r_{t+1}^{d} - r_{F}^{d} &= \alpha_{10} + \alpha_{1} \ cay_{t} + \alpha_{11}h_{11,t+1} + \alpha_{14}h_{14,t+1} + \varepsilon_{1,t+1} \\ r_{t+1}^{,f} + \Delta e_{t+1} - r_{F}^{,d} &= \alpha_{20} + \alpha_{2} \ cay_{t} + \alpha_{21}h_{22,t+1} + \alpha_{25}h_{25,t+1} + \varepsilon_{2,t+1} \\ \Delta e_{t+1} + r_{F}^{,f} - r_{F}^{,d} &= \alpha_{30} + \alpha_{3}(\Delta e_{t} + r_{F}^{,f} - r_{F}^{,d}) + \varepsilon_{3,t+1} \\ \Delta l_{t+1}^{,d} &= \alpha_{40} + \alpha_{4}\Delta l_{t}^{,d} + \varepsilon_{4,t+1} \\ \Delta l_{t+1}^{,f} &= \alpha_{50} + \alpha_{5}\Delta l_{t}^{,f} + \varepsilon_{5,t+1} \end{aligned}$$

The conditional covariance matrix of the disturbance terms is given by $\mathbf{H}_{t+1} = \mathbf{C}'\mathbf{C} + \mathbf{B}'\mathbf{H}_{t}\mathbf{B} + \mathbf{A}'\boldsymbol{\varepsilon}_{t}\boldsymbol{\varepsilon}_{t}'\mathbf{A}.$

		Conditional means											
-	_		τ	J.S. stock	CS				U	J.K. stocl	ks		
		Const.	cay_t	$h_{11,t+1}$	$h_{14,t+1}$	\overline{R}^2 , %		Const.	cay_t	$h_{22,t+1}$	$h_{25,t+1}$	\overline{R}^2 , %	
Coeff		6.36	1.22	-8.69	83.37	8.20		-0.31	1.30	2.50	31.82	3.85	
T-Stat		1.83	2.23	-1.56	1.85			-0.22	2.52	1.79	2.03		
	_	(Currency	7		<u> </u>	J.S. labo	or		1	U.K. lat	or	
		Const.	AR(1)	\overline{R}^2 , %		Const.	AR(1)	\overline{R}^{2} , %		Const.	AR(1)	\overline{R}^2 , %	
Coeff		0.50	0.18	2.13		0.14	0.53	26.30		0.47	0.10	-1.55	
T-Stat		2.27	2.44			2.50	8.14			6.90	1.04		
_					Conditional covariance matrices								
				В						А			
		(1,1)	(1,2)	(1,3)	(1,4)	(1,5)		(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	
Coeff		1.01	0.31		0.01			-0.20	0.11		-0.02		
T-Stat		15.77	4.33		2.84			-2.24	1.01		-2.96		
		(2,1)	(2,2)	(2,3)	(2,4)	(2,5)		(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	
Coeff		-0.09	0.72	0.00		0.01		0.07	-0.10	0.20		-0.08	
T-Stat		-1.55	11.78	-0.17		2.51		0.89	-1.28	3.71		-6.37	
		(3,1)	(3,2)	(3,3)	(3,4)	(3,5)		(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	
Coeff			0.37	0.60		0.05			-0.07	-0.48		0.03	
T-Stat			2.20	3.22		3.06			-0.43	-2.89		1.95	
		(4,1)	(4,2)	(4,3)	(4,4)	(4,5)		(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	
Coeff		-2.44			0.85			3.12			0.26		
T-Stat		-7.11			20.14			4.19			2.99		
		(5,1)	(5,2)	(5,3)	(5,4)	(5,5)		(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	
oeff			-1.18	0.91		0.74			1.16	1.78		-0.11	
T-Stat			-2.56	2.72		9.54			2.00	8.25		-0.80	
Log likelihood	2	2040.94											
_		Adjus	sted inter	mational	risk shar	ing index	k for mir	nimum-va	riance di	scount fa	actors		
	Mean	Std.	Dev. N	Median	Skewne	ess Mini	mum M	laximum	5 %til	e 95 9	%tile 4	Auto Cor.	
	0.589	0.3	22	0.700	-0.862	2 -0.4	434	0.994	0.050	0.9	970	0.439	

Table 3. Estimates of a multivariate model of domestic and foreign stock returns, currency returns and domestic and foreign consumption growth

 $r_t^j r_F^j$, and Δc_t^j are real stock returns, riskfree rate, consumption growth at time *t* for U.S. (*j=d*) and U.K. (*j=f*), respectively. Δe_t is the change in the log real exchange rate between the two countries, in units of U.S. consumption good. *cay_t* is the U.S. log consumption-wealth ratio. The sample period is from the 1970:Q1 to 2003:Q4. The following system of equation is estimated by the method of maximum likelihood:

$$\begin{aligned} r_{t+1}^{d} - r_{F}^{d} &= \alpha_{10} + \alpha_{1} \ cay_{t} + \alpha_{14}h_{14,t+1} + \varepsilon_{1,t+1} \\ r_{t+1}^{f} + \Delta e_{t+1} - r_{F}^{d} &= \alpha_{20} + \alpha_{2} \ cay_{t} + \alpha_{25}h_{25,t+1} + \varepsilon_{2,t+1} \\ \Delta e_{t+1} + r_{F}^{f} - r_{F}^{d} &= \alpha_{30} + \alpha_{3}(\Delta e_{t} + r_{F}^{f} - r_{F}^{d}) + \varepsilon_{3,t+1} \\ \Delta c_{t+1}^{d} &= \alpha_{40} + \alpha_{4}\Delta c_{t}^{d} + \varepsilon_{4,t+1} \\ \Delta c_{t+1}^{f} &= \alpha_{50} + \alpha_{5}\Delta c_{t}^{f} + \varepsilon_{5,t+1} \end{aligned}$$

The conditional variance-covariance matrix of the disturbance terms is given by $\mathbf{H}_{t+1} = \mathbf{C}'\mathbf{C} + \mathbf{B}'\mathbf{H}_t\mathbf{B} + \mathbf{A}'\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\mathbf{A}.$

						Conditi	onal me	ans				
	_		U.S. s	stocks					U.K. s	stocks		_
		Const.	cay_t	$h_{14,t+1}$	\overline{R}^2 , %			Const.	cay_t	$h_{25,t+1}$	\overline{R}^2 , %	
Coeff		6.26	1.47	-265.91	11.22			1.86	1.03	-14.41	6.70	
T-Stat		0.73	3.45	-0.81				2.96	2.31	-1.82		
	_	(Currency	7		U.S	. consu	mption		U.K	. consur	nption
		Const.	AR(1)	\overline{R}^2 , %		Const.	AR (1) \overline{R}^2 , %		Const.	AR(1)	\overline{R}^{2} , %
Coeff		0.74	0.17	2.14		0.34	0.26	9.01		0.65	-0.19	1.59
T-Stat		2.40	2.60			4.88	2.92			5.62	-1.96	
_				Co	onditiona	l varian	ce-cova	riance matri	ices			
				В			_			А		
		(1,1)	(1,2)	(1,3)	(1,4)	(1,5)		(1,1)	(1,2)	(1,3)	(1,4)	(1,5)
Coeff		-0.97	-1.19		0.04			0.27	0.53		-0.03	
T-Stat		-9.03	-7.17		1.76			2.98	4.65		-1.75	
		(2,1)	(2,2)	(2,3)	(2,4)	(2,5)		(2,1)	(2,2)	(2,3)	(2,4)	(2,5)
Coeff		0.53	0.92	0.40		-0.01		-0.31	-0.77	0.05		0.01
T-Stat		5.75	11.72	7.83		-0.25		-1.95	-5.46	1.16		0.66
		(3,1)	(3,2)	(3,3)	(3,4)	(3,5)		(3,1)	(3,2)	(3,3)	(3,4)	(3,5)
Coeff			-1.47	-0.20		-0.09			-0.16	-0.46		0.05
T-Stat			-10.40	-1.87		-2.22			-1.64	-5.35		1.93
		(4,1)	(4,2)	(4,3)	(4,4)	(4,5)		(4,1)	(4,2)	(4,3)	(4,4)	(4,5)
Coeff		-2.88			-0.36			-0.42			-0.14	
T-Stat		-2.90			-1.70			-1.10			-0.85	
		(5,1)	(5,2)	(5,3)	(5,4)	(5,5)		(5,1)	(5,2)	(5,3)	(5,4)	(5,5)
Coeff			-0.40	-0.17		-0.85			-0.10	-0.34		-0.46
T-Stat			-1.08	-1.01		-18.51			-0.45	-1.72		-6.58
Log likelihood		1976.56	ì									
				Adjusted	internat	ional co	nsumpti	on risk sha	ring inde	х		
Utility	Mean	Std.]	Dev. 1	Median	Skewn	ess Mi	nimum	Maximum	5 %ti	le 95	%tile	Auto Cor.
Power	0.191	0.1	42	0.203	-0.68	2 -0	0.403	0.601	-0.04	8 0	.379	0.327
Habit	0.184	0.1	33	0.191	-0.58	9 -(0.383	0.604	-0.04	0 0	.363	0.326

Table 4. Estimates of a multivariate model of domestic and foreign stock returns, currency returns, and domestic and foreign labor income, and domestic and foreign consumption growth

 $r_t^j r_F^j$, Δl_t^j and Δc_t^j are real stock returns, riskfree rate, labor income growth, consumption growth at time *t* for U.S. (*j=d*) and U.K. (*j=f*), respectively. *cay*_t is the U.S. log consumption-wealth ratio. The sample period is from the 1970:Q1 to 2003:Q4. The following system of equation is estimated by the method of maximum likelihood:

$$r_{t+1}^{a} - r_{F}^{a} = \alpha_{10} + \alpha_{1} \ cay_{t} + \varepsilon_{1,t+1},$$

$$r_{t+1}^{f} + \Delta e_{t+1} - r_{F}^{d} = \alpha_{20} + \alpha_{2} \ cay_{t} + \varepsilon_{2,t+1}$$

$$\Delta e_{t+1} + r_{F}^{f} - r_{F}^{d} = \alpha_{30} + \alpha_{3} (\Delta e_{t} + r_{F}^{f} - r_{F}^{d}) + \varepsilon_{3,t+1}$$

$$\Delta l_{t+1}^{d} = \alpha_{40} + \alpha_{4} \Delta l_{t}^{d} + \varepsilon_{4,t+1}, \ \Delta l_{t+1}^{f} = \alpha_{50} + \alpha_{5} \Delta l_{t}^{f} + \varepsilon_{5,t+1}$$

$$\Delta c_{t+1}^{a} = \alpha_{60} + \alpha_7 \Delta c_t^{a} + \varepsilon_{6,t+1}, \ \Delta c_{t+1}^{c} = \alpha_{70} + \alpha_7 \Delta c_t^{c} + \varepsilon_{7,t+1}$$

The conditional variance-covariance matrix of the disturbance terms is given by

$\mathbf{H}_{t+1} = \mathbf{C}'\mathbf{C} + \mathbf{B}'\mathbf{H}_{t}\mathbf{B} + \mathbf{A}'\boldsymbol{\varepsilon}_{t}\boldsymbol{\varepsilon}_{t}'\mathbf{A}.$

							Condition	nal means						
	U	J.S. stock	KS		U	.K. stoc	ks			(Currency	y		
	Const.	cay_t	\overline{R}^2 , %		Const.	cay_t	\overline{R}^2 , %			Const.	AR(1)	\overline{R}^2 , %		
Coeff	1.95	1.05	7.20		1.92	0.74	3.39			0.27	0.22	1.91		
T-Stat	3.40	2.78			2.50	1.52				0.98	3.10			
	I	J.S. labo	or		U	J.K. labo	or	U.S.	consum	ption		U.K.	consum	ption
	Const.	AR(1)	\overline{R}^2 , %		Const.	AR(1)	\overline{R}^{2} , %	Const.	AR(1)	\overline{R}^{2} , %		Const.	AR(1)	\overline{R}^{2} , %
Coeff	0.28	0.34	22.95		0.61	-0.15	-3.23	0.50	-0.01	-1.69		0.69	-0.10	1.37
T-Stat	3.50	4.59			5.24	-2.66		7.28	-0.20			5.11	-1.58	
						Condi	tional cov	variance m	atrices					
				В							Α			
	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)
Coeff	-0.17	-0.84		0.01		-0.02		-0.04	0.09		0.00		0.01	
T-Stat	-1.10	-5.57		0.41		-1.49		-0.40	0.84		-0.09		1.08	
	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)
Coeff	0.38	0.95	0.18		-0.06		-0.02	0.22	0.21	0.00		0.01		-0.04
T-Stat	4.37	5.03	4.08		-1.46		-1.32	1.76	1.22	-0.14		0.50		-2.92
	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)	(3,7)	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)	(3,7)
Coeff		-0.42	-0.96						0.54	0.29				
T-Stat		-2.57	-42.31						3.33	4.14				
	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)	(4,7)	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)	(4,7)
Coeff	5.41			-0.85		-0.14		0.27			-0.10		-0.27	
T-Stat	2.24			-9.45		-0.61		0.25			-1.72		-3.68	
	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)	(5,7)	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)	(5,7)
Coeff		-0.15			-0.95		-0.26		3.72			-0.26		-0.40
T-Stat		-0.12			-19.74		-3.76		5.19			-2.45		-4.77
	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)	(6,7)	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)	(6,7)
Coeff	-7.75			-0.08		-0.68		1.70			-0.14		-0.36	
T-Stat	-6.56			-0.48		-5.45		1.43			-1.04		-2.55	
	(7,1)	(7,2)	(7,3)	(7,4)	(7,5)	(7,6)	(7,7)	(7,1)	(7,2)	(7,3)	(7,4)	(7,5)	(7,6)	(7,7)
Coeff		-0.75			0.06		-0.72		-0.85			0.07		0.63
T-Stat		-1.58			1.79		-8.79		-1.28			1.56		4.60
Log likel	ihood		3158.08											

Table 5. Summary statistics of adjusted international risk sharing indices

ARSIMV is the adjusted international risk sharing index for U.S. and U.K. minimum-variance discount factors. ARSIPW and ARSIHB are the adjusted international risk sharing indices for U.S. and U.K. marginal utility growth under power utility and external habit utility, respectively. ARSI is the adjusted international risk sharing index for U.S. and U.K. marginal utility growth under minimum-risk aversion utility. CORCG is the conditional correlation between U.S. and U.K. consumption growth. CORLB is the conditional correlation between U.S. and U.K. labor income growth. The *t*-statistics in parentheses are adjusted for heteroskedasticity and autocorrelations of residuals up to lag 40.

Series	Mean	Std Dev	Median	Skewness	Minimum	Maximum	5 %ile	95 %ile	Auto Cor
ARSIMV	0.617	0.325	0.717	-0.843	-0.266	0.992	-0.238	0.969	0.361
ARSIPW	0.204	0.249	0.238	-0.646	-0.661	0.783	-0.198	0.559	0.597
ARSIHB	0.207	0.239	0.233	-0.520	-0.486	0.746	-0.277	0.527	0.619
ARSI	0.194	0.231	0.214	-0.697	-0.546	0.672	-0.164	0.512	0.538
CORCG	0.192	0.264	0.234	-0.671	-0.727	0.773	0.035	0.543	0.602
CORLB	0.446	0.204	0.518	-1.242	-0.187	0.692	0.001	0.649	0.900

Correlations												
ARSIMV ARSIPW ARSIHB ARSI COR												
ARSIPW	0.011											
ARSIHB	0.006	0.987										
ARSI	0.061	0.964	0.955									
CORCG	-0.063	0.986	0.973	0.963								
CORLB	0.188	0.435	0.445	0.431	0.423							

Regressions										
	Constant	CORLB	\overline{R}^2 , %							
ARSIMV	0.444	0.352	3.93							
	(5.993)	(1.785)								
ARSIPW	-0.027	0.524	18.29							
	(-0.657)	(4.591)								
ARSIHB	-0.020	0.515	19.22							
	(-0.505)	(4.749)								
ARSI	-0.026	0.491	18.70							
	(-0.873	(5.421)								
CORCG	-0.035	0.523	16.49							
	(-0.788)	(4.811)								

Table 6. Discount factor loadings

For the U.S. log minimum-variance discount factor, the loadings, $E_t[\mathbf{r}_{t+1}^d] \Sigma_t^{-1}(\mathbf{r}_{t+1}^d)$, are on shocks to U.S. stocks returns, U.K. stock returns, currency returns, and U.S. labor income growth. For the U.K. log minimum-variance stochastic discount factors, the loadings, $E_t[\mathbf{r}_{t+1}^f] \Sigma_t^{-1}(\mathbf{r}_{t+1}^f)$, are on shocks to U.S. stocks returns, U.K. stock returns, currency returns, and U.K. labor income growth. The conditional standard deviation of the log minimum-variance discount factor is $\sqrt{E_t[\mathbf{r}_{t+1}^j] \Sigma_t^{-1}(\mathbf{r}_{t+1}^j]}$ for the U.S. investor (j = d) or the foreign investor (j = f). For consumption-based discount factors, the loadings are the risk aversion coefficients, A_t^j for the U.S. (j = d) and the U.K. (j = f). Under minimum-risk aversion utility, the time-varying risk aversion coefficient A_t^j of an investor is chosen so the conditional standard deviation of the log marginal utility growth of the investor is equal to the conditional standard deviation of the log minimum-variance discount factor for each point in time. The standardized loadings are loadings multiplied by the conditional standard deviations of shocks.

	Loa	dings	Standardize	ed loadings
	Mean	Std Dev	Mean	Std Dev
Panel A. Minimum-variance disco	unt factors			
Shocks to the U.S. discount factor				
U.S. stock returns	3.565	4.460	0.281	0.357
U.K. stock returns	0.442	2.219	0.035	0.273
Currency returns	1.953	5.975	0.089	0.314
U.S. labor income growth	-2.779	57.437	-0.038	0.445
Shocks to the U.K. discount factor				
U.S. stock returns	3.532	4.305	0.336	0.411
U.K. stock returns	0.536	2.198	0.061	0.253
Currency returns	-6.477	7.473	-0.309	0.372
U.K. labor income growth	9.620	15.015	0.118	0.211
Standard deviations of log discourt	t factors			
U.S. investor			0.554	0.227
U.K. investor			0.443	0.198
Panel B. Consumption-based disce	ount factors			
Shocks to discount factors under p	ower utility			
U.S. consumption growth	39.048		0.335	0.129
U.K. consumption growth	16.507		0.309	0.132
Shocks to discount factors under e	xternal habit utilit	y (utility curvature =	2, habit persistence $= 0$.95)
U.S. consumption growth	34.944	14.179	0.316	0.249
U.K. consumption growth	15.128	4.952	0.272	0.124
Shocks to discount factors under n	ninimum risk aver	sion utility		
U.S. consumption growth	72.934	36.970	0.554	0.227
U.K. consumption growth	27.776	17.265	0.443	0.198

Table 7. Estimates of a multivariate model of domestic and foreign stock returns, currency returns and domestic and foreign labor income growth: Cross-country labor income risk

 $r_t^j r_F^j$ and Δl_t^j are real stock returns, riskfree rate, and labor income growth at time *t* for U.S. (*j=d*) and U.K. (*j=f*), respectively. Δe_t is the change in the log real exchange rate between the two countries, in units of U.S. consumption good. cay_t is the U.S. log consumption-wealth ratio. The sample period is from the 1970:Q1 to 2003:Q4. The following system of equation is estimated by the method of maximum likelihood:

$$\begin{aligned} r_{t+1}^{d} - r_{F}^{d} &= \alpha_{10} + \alpha_{1} \ cay_{t} + \alpha_{11}h_{11,t+1} + \alpha_{14}h_{15,t+1} + \varepsilon_{1,t+1} \\ r_{t+1}^{f} + \Delta e_{t+1} - r_{F}^{d} &= \alpha_{20} + \alpha_{2} \ cay_{t} + \alpha_{21}h_{22,t+1} + \alpha_{25}h_{24,t+1} + \varepsilon_{2,t+1} \\ \Delta e_{t+1} + r_{F}^{f} - r_{F}^{d} &= \alpha_{30} + \alpha_{3}(\Delta e_{t} + r_{F}^{f} - r_{F}^{d}) + \varepsilon_{3,t+1} \\ \Delta l_{t+1}^{d} &= \alpha_{40} + \alpha_{4}\Delta l_{t}^{d} + \varepsilon_{4,t+1} \\ \Delta l_{t+1}^{f} &= \alpha_{50} + \alpha_{5}\Delta l_{t}^{f} + \varepsilon_{5,t+1} \end{aligned}$$

The conditional covariance matrix of the disturbance terms is given by

 $\mathbf{H}_{t+1} = \mathbf{C}'\mathbf{C} + \mathbf{B}'\mathbf{H}_{t}\mathbf{B} + \mathbf{A}'\boldsymbol{\varepsilon}_{t}\boldsymbol{\varepsilon}_{t}'\mathbf{A}.$

		Conditional means											
-	-		ι	J.S. stock	KS .				U	.K. stocl	KS		
		Const.	cay_t	$h_{11,t+1}$	$h_{15,t+1}$	\overline{R}^2 , %		Const.	cay_t	$h_{22,t+1}$	$h_{24,t+1}$	\overline{R}^2 , %	
Coeff		-10.95	1.17	21.38	83.57	6.82		-0.69	1.52	2.42	-57.69	9.39	
T-Stat		-0.58	3.60	0.62	0.60			-0.29	3.16	0.96	-0.72		
		1	Currency	/		ι	U.S. labo	or		ι	J.K. lab	or	
		Const.	AR(1)	\overline{R}^2 , %		Const.	AR(1)	\overline{R}^{2} , %		Const.	AR(1)	\overline{R}^2 , %	
Coeff		0.28	0.19	2.10		0.16	0.51	26.31		0.49	0.16	-3.13	
T-Stat		0.94	1.94			3.06	7.19			4.90	1.79		
					Conditi	onal cov	ariance	matrices					
-				В						А			
		(1,1)	(1,2)	(1,3)	(1,4)	(1,5)		(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	
Coeff		-0.27	-0.86			0.02		-0.36	-0.46			0.07	
T-Stat		-1.06	-5.06			0.79		-2.43	-3.45			2.15	
		(2,1)	(2,2)	(2,3)	(2,4)	(2,5)		(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	
Coeff		0.16	1.00	0.13	-0.02			0.01	-0.05	-0.14	-0.01		
T-Stat		1.40	13.71	2.46	-3.70			0.12	-0.52	-1.87	-1.20		
		(3,1)	(3,2)	(3,3)	(3,4)	(3,5)		(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	
Coeff			-0.33	-0.14		0.15			-0.14	0.13		0.01	
T-Stat			-1.50	-0.29		3.11			-0.90	0.93		0.64	
		(4,1)	(4,2)	(4,3)	(4,4)	(4,5)		(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	
Coeff			2.49		0.96				-1.86		0.20		
T-Stat			8.06		27.47				-4.00		3.38		
		(5,1)	(5,2)	(5,3)	(5,4)	(5,5)		(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	
Coeff		0.47		2.52		0.41		-1.62		0.55		0.45	
T-Stat		0.40		2.28		1.03		-3.71		0.91		5.19	
Log likelihood		2036.24											
_		Adju	sted inter	rnational	risk shari	ing index	x for mir	nimum-va	riance di	scount fa	actors		
	Mean	Std.	Dev. 1	Median	Skewne	ss Mini	mum M	laximum	5 %tile	e 95 %	%tile A	uto Cor.	
	0.524	0.2	280	0.555	-0.260	-0.	148	0.995	0.075	0.9	940	0.173	

Table 8. Estimates of a multivariate model of domestic and foreign stock returns, currency returns and domestic and foreign labor income growth: Excluding labor income risk for expected stock returns

 $r_t^j r_F^j$ and Δl_t^j are real stock returns, riskfree rate, and labor income growth at time *t* for U.S. (*j=d*) and U.K. (*j=f*), respectively. Δe_t is the change in the log real exchange rate between the two countries, in units of U.S. consumption good. cay_t is the U.S. log consumption-wealth ratio. The sample period is from the 1970:Q1 to 2003:Q4. The following system of equation is estimated by the method of maximum likelihood:

$$\begin{aligned} r_{t+1}^{d} - r_{F}^{d} &= \alpha_{10} + \alpha_{1} \ cay_{t} + \alpha_{11}h_{11,t+1} + \varepsilon_{1,t+1} \\ r_{t+1}^{f} + \Delta e_{t+1} - r_{F}^{d} &= \alpha_{20} + \alpha_{2} \ cay_{t} + \alpha_{22}h_{22,t+1} + \varepsilon_{2,t+1} \\ \Delta e_{t+1} + r_{F}^{f} - r_{F}^{d} &= \alpha_{30} + \alpha_{3}(\Delta e_{t} + r_{F}^{f} - r_{F}^{d}) + \varepsilon_{3,t+1} \\ \Delta l_{t+1}^{d} &= \alpha_{40} + \alpha_{4}\Delta l_{t}^{d} + \varepsilon_{4,t+1} \\ \Delta l_{t+1}^{f} &= \alpha_{50} + \alpha_{5}\Delta l_{t}^{f} + \varepsilon_{5,t+1} \end{aligned}$$

The conditional covariance matrix of the disturbance terms is given by

$$\mathbf{H}_{t+1} = \mathbf{C}'\mathbf{C} + \mathbf{B}'\mathbf{H}_{t}\mathbf{B} + \mathbf{A}'\boldsymbol{\varepsilon}_{t}\boldsymbol{\varepsilon}_{t}'\mathbf{A}$$

						Conditio	nal mea	ans						
_			U.S. s	stocks		_				U.K	. stock	s		_
		Const.	cay_t	$h_{11,t+1}$	\overline{R}^2 , %			Cor	ıst.	cay_t	h_{22}	, <i>t</i> +1	\overline{R}^2 , %)
Coeff		0.50	1.78	0.40	7.97			-1.	77	1.47	3.2	24	3.66	
T-Stat		0.12	2.48	0.07				-1.	13	2.95	2.0	07		
	_	(Currency	,			U.S. lał	oor				U	.K. lab	or
		Const.	AR(1)	\overline{R}^2 , %		Const.	AR (1) \overline{R}^2	, %		Co	nst.	AR(1)	\overline{R}^2 , %
Coeff		0.42	0.16	2.14		0.16	0.50	26.	43		0.	51	0.05	-1.00
T-Stat		1.05	2.42			3.28	9.22				5.	75	0.76	
_					Condi	tional cov	variance	e matrio	ces					
_				В							А			
		(1,1)	(1,2)	(1,3)	(1,4)	(1,5)		(1,1)	(1,	2)	(1,3)	(1	1,4)	(1,5)
Coeff		0.95	0.27		0.02			-0.30	0.0)5		-0	0.01	
T-Stat		10.24	3.48		1.80			-2.15	0.3	30		-0).84	
		(2,1)	(2,2)	(2,3)	(2,4)	(2,5)		(2,1)	(2,	2)	(2,3)	(2	2,4)	(2,5)
Coeff		-0.02	0.71	-0.04		0.02		0.05	-0.	12	0.22			-0.07
T-Stat		-0.44	11.42	-0.89		1.29		0.89	-1.	56	4.13			-5.10
		(3,1)	(3,2)	(3,3)	(3,4)	(3,5)		(3,1)	(3,	2)	(3,3)	(3	3,4)	(3,5)
Coeff			0.50	0.56		0.04			-0.	07	-0.54			0.03
T-Stat			3.17	5.17		1.37			-0.4	43	-3.36			1.73
		(4,1)	(4,2)	(4,3)	(4,4)	(4,5)		(4,1)	(4,	2)	(4,3)	(4	1,4)	(4,5)
Coeff		-3.03			0.84			3.72				0	.22	
T-Stat		-4.69			19.97			3.56				2	.22	
		(5,1)	(5,2)	(5,3)	(5,4)	(5,5)		(5,1)	(5,	2)	(5,3)	(5	5,4)	(5,5)
Coeff			-1.33	0.95		0.76			1.3	34	1.91			-0.11
T-Stat			-4.52	5.16		15.62			2.6	51	8.28			-1.71
Log likelihood	/	2038.36												
_		Adjus	sted inter	national	risk sha	ring inde	x for m	inimur	n-var	iance	discou	nt fac	ctors	
	Mean	Std.	Dev. N	Aedian	Skewn	ess Min	imum	Maxim	um	5 %t	ile	95 %	tile /	Auto Cor.
	0.587	0.3	16	0.662	-0.66	7 -0.	242	0.99	4	0.04	40	0.97	72	0.417

Table 9. Estimates of a multivariate model of domestic and foreign stock returns, currency returns and domestic and foreign labor income growth: Cross-market risk

 $r_t^j r_F^j$ and Δl_t^j are real stock returns, riskfree rate and labor income growth at time *t* for U.S. (*j=d*) and U.K. (*j=f*), respectively. Δe_t is the change in the log exchange rate between the two countries, in units of U.S. consumption good. cay_t is the U.S. log consumption-wealth ratio.. The sample period is from the 1970:Q1 to 2003:Q4. The following system of equation is estimated by the method of maximum likelihood:

$$\begin{aligned} r_{t+1}^{d} - r_{F}^{d} &= \alpha_{10} + \alpha_{1} \ cay_{t} + \alpha_{11}h_{11,t+1} + \alpha_{12}h_{12,t+1} + \varepsilon_{1,t+1} \\ r_{t+1}^{,f} + \Delta e_{t+1} - r_{F}^{,d} &= \alpha_{20} + \alpha_{2} \ cay_{t} + \alpha_{21}h_{21,t+1} + \alpha_{22}h_{22,t+1} + \varepsilon_{2,t+1} \\ \Delta e_{t+1} + r_{F}^{,f} - r_{F}^{,d} &= \alpha_{30} + \alpha_{3}(\Delta e_{t} + r_{F}^{,f} - r_{F}^{,d}) + \varepsilon_{3,t+1} \\ \Delta l_{t+1}^{,d} &= \alpha_{40} + \alpha_{4}\Delta l_{t}^{,d} + \varepsilon_{4,t+1} \\ \Delta l_{t+1}^{,f} &= \alpha_{50} + \alpha_{5}\Delta l_{t}^{,f} + \varepsilon_{5,t+1} \end{aligned}$$

The conditional covariance matrix of the disturbance terms is given by

 $\mathbf{H}_{t+1} = \mathbf{C}'\mathbf{C} + \mathbf{B}'\mathbf{H}_t\mathbf{B} + \mathbf{A}'\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\mathbf{A}.$

	Conditional means											
-			ι	J.S. stock	CS				ι	J.K. stocl	ks	
		Const.	cay_t	$h_{11,t+1}$	$h_{12,t+1}$	\overline{R}^2 , %		Const.	cay_t	$h_{21,t+1}$	$h_{22,t+1}$	\overline{R}^2 , %
Coeff		-2.14	1.85	-0.59	7.13	10.52		0.83	1.26	-7.65	4.44	3.59
T-Stat		-16.28	5.93	-0.40	5.99			2.27	3.78	-8.33	6.64	
	_		Currency	7		U.S. labor			U.K. labo			or
		Const.	AR(1)	\overline{R}^2 , %		Const.	AR(1)	\overline{R}^{2} , %		Const.	AR(1)	\overline{R}^{2} , %
Coeff		0.44	0.15	2.11		0.15	0.52	26.31		0.48	0.08	-1.24
T-Stat		1.89	2.89			2.82	9.65			7.40	1.32	
					Condit	ional cov	ariance 1	matrices				
-				В						А		
	-	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)		(1,1)	(1,2)	(1,3)	(1,4)	(1,5)
Coeff		0.88	0.19		0.02			-0.38	-0.04		-0.01	
T-Stat		20.72	6.49		6.25			-5.22	-0.68		-2.15	
		(2,1)	(2,2)	(2,3)	(2,4)	(2,5)		(2,1)	(2,2)	(2,3)	(2,4)	(2,5)
Coeff		-0.01	0.73	-0.01		0.02		0.14	-0.11	0.20		-0.07
T-Stat		-0.35	27.06	-0.44		4.63		2.73	-1.95	7.41		-11.15
		(3,1)	(3,2)	(3,3)	(3,4)	(3,5)		(3,1)	(3,2)	(3,3)	(3,4)	(3,5)
Coeff			0.43	0.48		0.04			-0.13	-0.58		0.03
T-Stat			11.13	14.40		7.51			-1.26	-7.49		2.18
		(4,1)	(4,2)	(4,3)	(4,4)	(4,5)		(4,1)	(4,2)	(4,3)	(4,4)	(4,5)
Coeff		-2.88			0.84			3.63			0.18	
T-Stat		-11.07			34.65			5.92			2.67	
		(5,1)	(5,2)	(5,3)	(5,4)	(5,5)		(5,1)	(5,2)	(5,3)	(5,4)	(5,5)
Coeff			-1.46	0.97		0.75			1.71	1.92		-0.13
T-Stat			-5.79	6.42		18.96			5.45	10.92		-1.59
Log likelihood		2041.10)									
_		Adju	sted inter	mational	risk sha	ring index	k for mir	imum-va	riance d	iscount fa	actors	
	Mean	Std.	Dev. I	Median	Skewne	ess Mini	mum M	laximum	5 %til	e 95 %	%tile A	Auto Cor.
	0.634	0.2	290	0.717	-0.632	2 -0.0	030	0.995	0.070	0.9	980	0.371



Figure 1. Exchange rate-adjusted international risk sharing index for U.S. and U.K. minimum-variance stochastic discount factors

ARSI is the adjusted international risk sharing index for U.S. and U.K. minimum-variance stochastic discount factors. CORM is the conditional correlation between U.S. and U.K. minimum-variance log stochastic discount factors. CORLBR is the conditional correlation between U.S. and U.K. per capita real labor income growth. The results are implied from a multivariate model for U.S. and U.K. stock returns, exchange rate, and labor income growth.





SDMUS is the conditional standard deviation of the U.S. minimum-variance log stochastic discount factor. SDMUK is the conditional standard deviation of the U.K. minimum-variance log stochastic discount factor. SDEXCH is the conditional standard deviation of the real exchange rate between U.K. and the U.S. in units of the U.S. consumption good. The results are implied from a multivariate model for U.S. and U.K. stock returns, exchange rate, and labor income growth.



Figure 3. Comparisons of exchange rate-adjusted international risk sharing indices

ARSIMV is the adjusted international risk sharing index for U.S. and U.K. minimum-variance stochastic discount factors. ARSIPW is the adjusted international risk sharing index for U.S. and U.K. marginal utility growth under power utility. CORLB is the conditional correlation between U.S. and U.K. labor income growth. The results are implied from a multivariate model for U.S. and U.K. stock returns, exchange rate, labor income growth, and consumption growth.