Asymmetric Co-movement Behaviors between Futures and Spot Positions and Dynamic Hedge Ratios under various Volatility Regime Combinations

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Abstract

This project analyzes dynamic co-movement behaviors between futures and spot positions via a state-varying framework. Specifically, we adopt Hamilton and Susmel (1994)'s Markov switching ARCH (hereafter SWARCH) model to identify high/low volatility regime at each time point by date itself and measuring co-movement sizes between spot and futures positions at various volatility states as well as establishing dynamic hedging ratio. Our empirical results are consistent with the following notions. First, the situation of both futures and spot positions during high volatility states will be associated with the maximum correlation measure between them. Second, by incorporating the character of state-varying correlation into the establishment of hedge ratio, we can create a more efficient futures hedge strategy with less risk.

Keywords: *futures index, futures and spot positions, hedge ratio, Markov-switching model*

I. Introduction

Risk Management has never been an easy task, but in recent years it has become even more difficult because of greater uncertainty in the economic environment, namely the prices of financial assets have become much more volatile. One of the key objectives of the risk manager is to reduce the risk via available financial derivative instruments in the markets. For stock market risk, specifically, the objective of hedging is to control or reduce the risk of adverse price change in physical assets.

To achieve this, investors can use stock index futures contracts. However, one of the key problems of hedging with stock index futures is to determine a hedge ratio, namely the ratio of futures contracts buying or selling for each unit of the underlying asset. As we know, the hedge ratio establishment heavily depends on co-movement sizes between the prices of futures and spot positions. To conclude, the question of how to accurately picture the correlation measure between the prices of futures contracts and underlying stock assets is one of the keys to futures hedging management.

Ederington (1979) and Figlewski (1984) model the hedger's problem and formulate the hedge ratio that minimizes the variance in the hedged spot position. However, their work has several limitations. Specifically, one of the most pressing limitations is that their work assumes that the variances of futures and spot positions as well as correlations between them are stable. One approach to addressing the above limitations of future hedging is to measure the variance and covariance via a time-varying framework. Specifically, Kroner and Sultan (1993), Park and Switzer (1995), Gagnon and Lypny (1995, 1997) and Kavussanos and Nomikos (2000) adopt GARCH (gemegalized autoregressive conditional heteroskedasticity[1]) to capture the time-varying variances and covariance as well as to establish dynamic hedge ratio approaches.

In contrast with those prior studies concentrating on analyzing non-constant variances via the time-varying approach, this paper focuses on examining asymmetric co-movement sizes between futures and spot positions via a state-varying framework. Specifically, we adopt Hamilton and Susmel (1994)'s Markov switching ARCH (hereafter SWARCH) model to identify high/low volatility regimes at each time point by date itself and measure the co-movement sizes between spot and futures positions at various volatility states as well as establishing a dynamic hedging ratio approach.

By examining the realized stock return data, one can easily find they are much more volatile during certain periods because the occurrences of financial crises or particular political events. Unfortunately, the simplified settings with constant parameters can not accurately picture the characters of various volatility regimes.

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[1] The most commonly used methods to characterize the volatility of financial assets' price returns are either the Engle (1982)'s ARCH (auto-regressive conditional heteroskedasticity) or Bollerslev (1986)'s GARCH (generalized ARCH).

Many prior studies including Hamilton and Susmel (1994), Ramchand and Susmel (1998a and b) and Li and Lin (2003) have demonstrated the existence of separate high/low volatility regimes in stock markets. Moreover, many prior studies (for example, King and Wadhwani (1990), Longin and Solnik (1995), and Jacquier and Marcus (2001) and Li (2005)) also note that the prices of stock assets appear to be much more closely correlated when markets are in a crisis period. Following the above line of observations and thought, in this paper, we investigate asymmetric co-movement behaviors between spot and futures positions under various volatility states and create a dynamic futures hedging approach via a state-varying correlation measure.

The analysis techniques in this paper are related to the framework of state variables adopted by Alizadeh and Nomikos (2004). They find a dynamic relationship between spot and futures index returns may be characterized by regime shift and suggest the hedge ratio to be dependent upon the state of the market.

Our analysis differs from theirs in the following respects. First, we adopt the ideas of the CAPM perspective to consider two *independent* elements: (1) *systematic* risk: the factor from the entire stock market, (2) *nonsystematic* risk: the individual stock asset itself. In contrast with Alizadeh and Nomikos (2004)'s analysis of the dual state specifications regarding high/low volatility states for the stock market, we consider that both the *market-wide* component from the futures position and the *idiosyncratic* component from the spot position should be subject to their own volatility state switching processes and discuss the quarterly correlations among the combinations of volatility states of futures and spot positions.

Our ideas are presented as the following notations. As we know, one reason stock index futures are so popular is that they substitute for holdings in the underlying stock themselves. Specifically, index futures allow investors to participate in broad market movement because futures can represent *synthetic* holdings of the market position. Moreover, the transaction cost involved in establishing futures positions are much lower that what would be required to take actual spot positions. On the other hand, it is diffcult for investors to establish a broad market portfolio in their spot position.

Alizadeh and Nomikos (2004) employ the market index as the proxy variable of underlying stock asset's price. The assumption behind it is that investors hold a *well-diversified* market portfolio. One will expect that the market index and futures index will almost move together. In other words, the co-movement behaviors between them are quite stable and the hedging ratio will be closed to unity. Nevertheless, from the practical point of view, investors could only invest in a sub-set of market portfolio or might hold an individual security only in their spot positions. Following the above line of thought, we propose that index futures could serve as a *systematic* element in contrast with individual security in spot positions being *idiosyncratic* ones. Moreover, when we measure co-movement processes between them, we have to consider both of their dynamic volatility processes.

Additionally, because the establishment of the futures hedging ratio relies heavily on the measurement of correlation between futures and spot positions, we want to further address and examine the following problems. First, are the co-movement sizes between them consistent among various volatility regimes? If not, what are the relationships between the correlations and volatility regimes? Last but not the least, could the framework of state-varying correlation help investors to design a futures hedging strategy with less risk?

II. Identification of Volatility States for Futures and Spot Positions

1. Data

In this paper, we adopt the U.K. FTSE-100 and the U.S. S&P 500 future indices. Besides, for keeping away from the ad hoc problems of sample security selection, we use the sub stock index of various industry categories to be the proxy variable of the price of individual security in spot positions. Specifically, for the U.S. stock markets, there are ten sub stock indices including (1) Consumer Discretionary, (2) Consumer Staples, (3) Energy, (4) Financials, (5) Health Care, (6) Info Technology, (7) Industrials, (8) Materials, (9) Telecom Service and (10) Utility. Moreover, ten sub stock indices in the U.K. market include (1) Basic Industries, (2) Cycle Consumer GDS, (3) Cycle Services, (4) Financial, (5) General Industrials, (6) Info Technology, (7) Non-cycle Consumer GDS, (8) Non-cycle Services, (9) Resources and (10) Utilities. To conclude, we create 20 (2x10) hedged individual securities with index futures for the following discussions. The data are obtained daily from January 1st, 1995 to December 31st, 2004 for 2610 observations. All market price indices are valued in U.S. dollars and were obtained from the Data Stream database. Tables 1 and 2 summarize the descriptive statistics and correlation matrices of various daily stock index returns for the U.K. and U.S. markets, respectively.

2. Methodology

In this paper, we adopt Hamilton and Susmel's (1994) SWARCH model to analyze the price returns of spot and futures positions and identify the volatility regimes of each of them at each time point. Denoting R_t as the change rates of the prices of futures and spot positions, Hamilton and Susmel's (1994) SWARCH models are presented as follows:

$$R_t = \phi_0 + \phi_1 R_{t-1} + \cdots + \phi_p R_{t-p} + e_t$$

$$e_t = \sqrt{g_{s_t}}\, u_t$$

$$u_t = \sqrt{h_t} v_t$$

$$h_t = a_0 + a_1 u_{t-1}^2 + a_2 u_{t-2}^2 + ... + a_q u_{t-q}^2 ,$$

where v_t is a Gaussian distribution with a unit standard error, s_t, that is an unobservable state variable with the possible values of 1, 2,…, k. For the two volatility states, s_t=1 (s_t =2) represents the stock market in the low (high) volatility state. The transition probabilities for state variables are presented as follows:

$$p(s_t = 1 | s_{t-1} = 1) = p_{11} , \quad p(s_t = 2 | s_{t-1} = 1) = p_{12}$$
$$p(s_t = 2 | s_{t-1} = 2) = p_{22} , \quad p(s_t = 1 | s_{t-1} = 2) = p_{21}$$

where p_{11}+p_{12}=p_{21}+p_{22}=1[2]. It is worth noting that u_t is a standard ARCH (q) setting. Moreover, when s_t=1 (s_t=2), e_t equals u_t multiplied by $\sqrt{g_1}$ ($\sqrt{g_2}$). Without losing the generalization principle, we set g_1 = 1. This means that the volatility of state 2 is g_2 times state 1. If the estimates of g_2 are significantly greater than the estimates of g_1, then we can conclude that state 1 (2) is the low (high) volatility state.

Although the regime variable s_t is unobservable, we can estimate the probability of a specific state at any time point using the data itself. Specifically, when the information set for estimation includes signals dated up to time t, the regime

[2] For satisfying $1 > p_{ij} > 0$, i (j)=1 or 2, we use the following probability settings :

$$p_{11} = \theta_{11}^2 /(1 + \theta_{11}^2), \qquad p_{12} = 1 - p_{11} = 1/(1 + \theta_{11}^2)$$
$$p_{22} = \theta_{22}^2 /(1 + \theta_{22}^2), \qquad p_{21} = 1 - p_{22} = 1/(1 + \theta_{22}^2)$$

probability is $p(s_t|R_t, R_{t-1},\ldots)$ or a *filtering probability*. On the other hand, one could also use the overall sample period information set to estimate the state at time t: $p(s_t|R_T, R_{T-1},\ldots)$, or a *smoothing probability*. In contrast, a *predicting probability* denotes the regime probability for an ex ante estimation, with the information set including signals dated up to the period t-1: $p(s_t|R_{t-1},R_{t-2},\ldots)$. In this paper, we use the smoothing probability and the criteria of 0.5 which follows Hamilton (1989) to identify the volatility regime at each time point. Specifically, we conclude that the market will be in state 1 (or 2) if the associated smoothing probability is greater than or equal to 0.5.

It should be noted that our paper extends the framework of Alizadeh and Nomikos (2004) to consider the volatility states of both futures and spot positions. Nevertheless, SWARCH models become complicated when extended to estimate a multivariate system. From a theoretical point of view, one can extend the number of SWARCH processes to infinity. However, from a practical perspective, the number of states is limited. Reviewing prior empirical studies, the maximum number of states is only 3. This paper sets two outcomes of the discrete state variable s_t to represent high and low volatility regimes and two orders of prior-period error squares. Thus one needs to consider 2^3 (8) possible states for any univariate index return at each date. Therefore, in a hedged security with futures index, one would need to consider 8^2 (64)

possible state combinations.

Besides, stock index futures can serve as a *synthetic* holding of market position and we know that the *idiosyncratic* risk of an individual security will be reduced to an arbitrarily low level. Following the above line of thought, in this paper, we set up futures and spot positions as being subject to their own switching processes of the volatility state.

Accordingly, we adopt a computationally simpler design. First, we use smoothing probability $p(s_t \mid y_T, y_{T-1}, ...)$ to identify the volatility regimes of each of the futures and spot positions. Specifically, if $p(s_t = 1 \mid y_T, y_{T-1}, ...)$ > 0.5 (<0.5), then we conclude that the element is at the low (high) volatility state.

With our setting of two distinct volatility regimes for the two components including futures and spot positions, we create four possible combinations of volatility regimes: (1) both futures and spot positions in a high volatility state (Futures=HV and Spot=HV), (2) futures position in a high volatility state with spot position in a low volatility state (Futures=HV and Spot=LV), (3) futures position in a low volatility state with spot position in a high volatility state (Futures=LV and Spot=HV), and (4) both futures and spot positions in a low volatility state (Futures=LV and Spot=LV).

3. Results

In the SWARCH model estimation processes, we set the order of auto-regression setting for the stock returns to be unity, namely, p=1, and the number of orders in ARCH to be two, namely, q=2[3], as well as the number of states to be two. We used OPTIMUM, a package from GAUSS, and the built-in BFGS algebra functions[4] to derive the negative minimum likelihood function.[5]

Table 3 presents the g_2 estimates of the SWARCH models for various daily sub stock index returns and futures index returns. The g_2 estimates were significantly greater than one in all cases. For example, consider the U.K. FTSE-100 future index as an example, the g_2 estimate is 3.8711 with a standard deviation of 0.2991. This means that the volatility of state 2 is 3.8711 times that of state 1 in the U.K. futures market. We conclude that state 2 (state 1) is the high (low) volatility state in the following discussion.

Next, we take a hedged security of Basic Industries in a spot position with the U.K. FTSE 100 index futures as an example. Figure 1 presents the daily index returns and smoothing probabilities of the high volatility state (or state 2) for futures and spot

[3] In the SWARCH model, the third-order ARCH parameter estimate appears to have a non-significant variation from zero in all cases. Therefore, for the specifications on the SWARCH model, we only take into account the second order ARCH setting.
[4] Boyden, Fletcher, Goldfarb, and Shanno (BFGS) algebra is effective for deriving the maximum value of the non-linear likelihood functions. See Luenberger (1984).
[5] We randomly generated 50 sets of initial values and derived the ML function value for each of the 50 sets of initial values, respectively. The mapped converged measure of the greatest ML function value then serves to estimate the parameter.

positions. Moreover, by using the criteria of 0.5, we create four possible volatility state combinations for this hedged security with index futures in Figure 2.

III. Asymmetric Co-movement Behaviors between Futures and Spot Positions at various Volatility Regime Combinations

Because of the occurrence of political and economic shocks and crises, the prices of financial assets appear much more volatile in particular periods. The next question we address is whether the co-movement sizes between futures and spot positions are consistent with various volatility state combinations.

Table 4 presents the correlation coefficient estimates between futures and spot positions in various market volatility state combinations. Quite interestingly, the maximum values appear in the situation of both futures and spot positions in the high volatility state (namely, Futures=HV and Spot=HV) for most cases, specifically, 9 in 10 cases for the U.K. market and 7 in 10 cases for the U.S. market.[6] Nevertheless, the values of the three alternative state combinations are inconsistent for most cases.[7]

Additionally, the values of the correlation coefficient for the whole sample

[6] The finding of a maximum of correlation measure in the situation of Futures=HV and Spot=HV is robust for most cases except the following four cases: (1) FTSE 100 index futures and Technology for the U.K. stock market and (2) S&P 500 index futures and Consumer Staples, (3) S&P 500 index futures and Energy as well as (4) S&P 500 index futures and Utility for the U.S. stock market.

[7] For example, for the U.K. stock market, the second maximum correlation estimate for the case of FTSE 100 index futures and Basic Industries shows in the situation of Future=LV and Indl=HV. In contrast, the second maximum correlation estimate is in the situation of Future=HV and Indl=LV for the case of FTSE 100 index futures and Cycle Consumer GDS.

period are among the values in the four state combinations. In other words, this finding indicates that using the setting with one correlation measure will underestimate true co-movement size between futures and spot positions in the situation of Futures=HV and Spot=HV and overestimate it in other, alternative situations.

According to the empirical results of this paper, we find that the maximum correlations are associated with futures and spot positions that are simultaneously in a high volatility state. Here we provide one appealing explanation for our findings. First, the high volatility regime of futures position consistently coincide with a *market-wide* (namely, *systematic*) financial or economic crisis. Moreover, a volatile individual security in our spot position will be associated with *individual-security-specific* (namely, *idiosyncratic*) shocks and generally follows a recession status for this specific individual security. (Please refer to Chen, Roll and Ross (1986), Schwert (1990), Chen (1991), and Hamilton and Lin (1996)) During both *market-wide* crises and *idiosyncratic* shocks, the common factors from market will be more influential than those from individual security. Because all of the individual securities share similar common elements from the market, the co-movement behaviors between our futures and spot positions will be much more remarkable in this situation.

IV. Establishment of Dynamic Hedge Ratio via a State-varying Correlation Framework

The establishment of hedge ratio with index futures relies heavily on one key variable: correlation between our futures and spot positions. Specifically, the hedge ratio that minimizes the variance of spot position is a function of the correlation between futures and spot positions and standard errors of futures and spot positions. This can be expressed as:

$$HR = \frac{Cov(\Delta S_t, \Delta F_t)}{Var(\Delta F_t)} = \rho_{S,F} \times \frac{SD(\Delta S_t)}{SD(\Delta F_t)}$$

where ΔS_t and ΔF_t denote change rates of the prices of futures and spot positions, respectively. $COV(\Delta S_t, \Delta F_t)$ and $Var(\Delta F_t)$ are the covariance of ΔS_t and ΔF_t and the variance of ΔF_t,, respectively. Moreover, $SD(\Delta S_t)$ and $SD(\Delta F_t)$ are the standard deviations of ΔS_t and ΔF_t, respectively. $\rho_{S,F}$ is the correlation coefficient between ΔS_t and ΔF_t.

Apparently, the correlation between futures and spot positions (namely, $\rho_{S,F}$) is one of key variables in the above formulation. Furthermore, derivation of an accurate correlation measure should serve as a key point for an efficient hedge ratio establishment.

Here we want to examine whether investors can use the framework of the state-varying correlation measure to establish a more efficient hedge ratio. We focus on discussing change in volatility regime conditions in order to make the dynamics spot-futures relationship and hedge ratios. In this paper, we use the absolute value of the realized return rate to serve as the proxy of standard deviation of futures and spot positions. Moreover, in the setting with one measure of correlation, we use the entire sample to estimate the one constant correlation. In contrast, in the setting with the four measures of correlation, we estimate the state-varying correlations for various volatility state combinations.

Table 5 presents the risk of hedged spot positions with index futures. Specifically, the standard deviations (namely, *SD*) of hedged spot position with index futures can be presented as:

$$SD(\Delta S_t - HR \cdot \Delta F_t)$$

where ΔS_t and ΔF_t denote change rates of the prices of futures and spot positions, respectively, and *HR* is the hedged ratio. Remarkably, our empirical results indicate that the risk of the hedged spot position being established via the state-varying correlation framework is smaller than that via a constant correlation for all cases. Our conclusion is clear. By incorporating the framework of state-varying correlations into the establishment of hedge ratio formulation, we can create a more efficient hedging strategy with less risk.

In Table 6, we use the risk of spot positions with no hedge as a benchmark to calculate risk reduction performance of various alternatives. Specifically, the risk reduction percentage can be expressed as:

Risk reduction %=-100(risk of hedged spot position-risk of spot position with no hedge)/ (risk of spot position with no hedge)*

First, the risk reduction performances of the framework with state-varying correlation are positive and greater than the setting with constant correlation for all cases. Moreover, in the row of "Average" of Table 6, we present the average value of risk reduction percentage of ten hedged spot positions for each of the U.K. and U.S. markets. Remarkably, the setting with state-varying correlation designed by our paper can offer an extra 11.55% (36.68-25.12) risk reduction effect relative to the model with constant correlation for the U.K. market. Moreover, it can offer extra an 11.80% (40.00-28.20) risk reduction performance for the U.S. market.

V. Conclusion

Many prior studies have demonstrated non-constant relationships between spot and futures markets and created dynamic hedge ratios approaches. In this paper, we focus on analyzing asymmetric co-movement behaviors between futures and spot positions under various volatility regimes conditions. Specifically, we employ a Markov-switching technique to identify the high/low volatility state of both futures and spot positions to create four possible volatility state combinations. Our empirical results are consistent with the following notions. First, the situation of both the futures and spot positions in the high volatility state is associated the maximum correlation measures. Second, the framework of state-varying correlations established in this paper can make futures hedge ratio more adaptable for various volatility regime circumstances and provide a more efficient hedging strategy with less risk.

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Table 1 Summary of Main Descriptive Statistics and Correlation Matrix of various Daily Index Returns: U.K. Market

(a) The Descriptive Statistics for Various Sub Stock Indices and Futures Index

	FTSE 100 Index Futures	*Sub Stock Indices*									
		Basic Industries	*Cycle Consumer GDS*	*Cycle Services*	*Financial*	*General Industrials*	*Info Technology*	*Non-cycle Consumer GDS*	*Non-cycle Services*	*Resources*	*Utilities*
Mean	0.025	0.037	0.010	0.042	0.031	0.021	0.018	0.032	0.020	0.005	0.023
S.E.	1.158	1.406	2.063	1.279	1.047	1.731	0.978	1.171	1.376	1.168	0.941
Kurtosis	4.983	5.776	9.739	5.805	6.741	4.832	5.431	5.881	8.732	8.115	5.463
Skewness	-0.086	-0.042	-0.633	-0.083	-0.205	0.150	-0.179	-0.077	-0.040	-0.542	-0.358
Maximum	5.618	8.814	11.301	6.193	6.497	7.681	4.216	7.637	11.088	5.826	3.999
Minimum	-5.311	-7.657	-18.069	-8.257	-8.650	-7.093	-5.667	-5.918	-9.928	-10.144	-5.258

(b) The Correlation Matrix

	FTSE 100 Index Futures	*Basic Industries*	*Cycle Consumer GDS*	*Cycle Services*	*Financial*	*General Industrials*	*Info Technology*	*Non-cycle Consumer GDS*	*Non-cycle Services*	*Resources*	*Utilities*
FTSE 100 Index Futures	1.000	0.640	0.537	0.873	0.546	0.750	0.799	0.720	0.483	0.615	0.604
Basic Industries		1.000	0.235	0.523	0.371	0.346	0.438	0.444	0.337	0.420	0.444
Cycle Consumer GDS			1.000	0.489	0.211	0.577	0.663	0.226	0.344	0.457	0.409
Cycle Services				1.000	0.491	0.591	0.734	0.620	0.501	0.594	0.605
Financial					1.000	0.365	0.507	0.501	0.325	0.375	0.414
General Industrials						1.000	0.644	0.417	0.321	0.433	0.411
Info Technology							1.000	0.528	0.536	0.657	0.686
Non-cycle Consumer GDS								1.000	0.363	0.414	0.445
Non-cycle Services									1.000	0.554	0.555
Resources										1.000	0.672
Utilities											1.000

Notes:

1. *The provided data are from the Data Stream database. The data were obtained daily from January 2nd, 1995 to December 31st, 2004, which included 2,610 observations.*
2. *We use daily returns in US dollars for all stock indices.*

Table 2 Summary of Main Descriptive Statistics and Correlation Matrix of various Daily Index Returns: U.S. Market

(a) The Descriptive Statistics for various Sub Spot Stock Indices and Futures Index

	S&P 500 Index Futures	*Sub Stock Indices*									
		Consumer Discretionary	*Consumer Staples*	*Energy*	*Financials*	*Health Care*	*Info Technology*	*Industrials*	*Materials*	*Telecom Service*	*Utility*
Mean	0.037	0.013	0.010	0.046	0.048	0.054	0.033	0.041	0.039	0.041	0.023
S.E.	1.181	1.147	1.536	2.161	1.304	1.473	1.036	1.345	1.307	1.239	1.330
Kurtosis	6.726	10.388	6.325	6.027	6.548	5.785	9.424	5.505	8.150	7.340	6.390
Skewness	-0.133	-0.384	-0.083	0.184	-0.177	0.086	-0.226	0.024	-0.148	-0.225	0.111
Maximum	5.755	8.483	8.027	16.077	7.656	8.388	7.589	7.942	8.468	7.208	6.978
Minimum	-7.762	-8.996	-10.320	-10.008	-9.173	-8.042	-9.296	-7.212	-10.327	-9.599	-9.121

(b) The Correlation Matrix

	S&P 500 Index Futures	*Consumer Discretionary*	*Consumer Staples*	*Energy*	*Financials*	*Health Care*	*Info Technology*	*Industrials*	*Materials*	*Telecom Service*	*Utility*
S&P 500 Index Futures	1.000	0.454	0.661	0.780	0.665	0.829	0.601	0.492	0.841	0.863	0.625
Consumer Discretionary		1.000	0.362	0.220	0.383	0.447	0.406	0.451	0.370	0.436	0.365
Consumer Staples			1.000	0.507	0.419	0.556	0.403	0.330	0.570	0.565	0.402
Energy				1.000	0.364	0.549	0.254	0.245	0.640	0.646	0.398
Financials					1.000	0.582	0.649	0.410	0.555	0.593	0.428
Health Care						1.000	0.572	0.406	0.746	0.782	0.597
Info Technology							1.000	0.406	0.531	0.582	0.491
Industrials								1.000	0.395	0.464	0.460
Materials									1.000	0.813	0.636
Telecom Service										1.000	0.710
Utility											1.000

Note: All notations are same with Table 1.

Table 3 The g_2 Estimates of SWARCH Model for Various Futures and Spot Positions

	g_2
U.K. FTSE 100 Index Futures	3.8712 (0.2991)
Basic Industries	3.5161 (0.3282)
Cycle Consumer GDS	4.3044 (0.2757)
Cycle Services	4.1482 (0.2900)
Financial	3.7081 (0.3529)
General Industrials	3.7773 (0.2742)
Info Technology	10.7567 (0.7635)
Non-cycle Consumer GDS	3.6882 (0.2709)
Non-cycle Services	5.2724 (0.3814)
Resources	4.0682 (0.2998)
Utilities	3.9071 (0.4179)
U.S. S&P 500 Index Futures	4.4788 (0.3142)
Consumer Discretionary	4.8546 (0.3218)
Consumer Staples	3.6139 (0.2573)
Energy	3.1767 (0.4037)
Financials	4.1799 (0.2939)
Health Care	3.8368 (0.3191)
Info Technology	3.8006 (0.2764)
Industrials	4.2989 (0.3153)
Materials	3.9245 (0.2987)
Telecom Service	4.6288 (0.3317)
Utility	5.3768 (0.4594)

Notes:

1. The value of the parameter g_2 denotes the volatility times of state 2 relative to state 1. The standard error estimate of parameter estimate is in the parenthesis.

2. The g_2 estimates are significantly greater than 1 in all cases. We conclude that state 2 is the high volatility state and that state 1 is low volatility.

Table 4 Correlation Coefficient Estimates between the Futures and Spot Positions under various Volatility States Combinations

	Whole Period	*Futures=HV and Spot=HV*	*Futures=HV and Spot=LV*	*Futures=LV and Spot=HV*	*Futures=LV and Spot=LV*
FTSE 100 Index Futures					
Basic Industries	0.604	0.685*	0.479	0.630	0.562
Cycle Consumer GDS	0.483	0.568*	0.493	0.363	0.479
Cycle Services	0.799	0.834*	0.805	0.828	0.750
Financial	0.873	0.905*	0.723	0.843	0.839
General Industrials	0.615	0.647*	0.455	0.584	0.622
Info Technology	0.538	0.594	0.449	0.622*	0.413
Non-cycle Consumer GDS	0.720	0.796*	0.650	0.588	0.718
Non-cycle Services	0.750	0.815*	0.725	0.729	0.727
Resources	0.640	0.713*	0.592	0.417	0.671
Utilities	0.546	0.642*	0.519	0.642	0.468
S&P 500 Index Futures					
Consumer Discretionary	0.841	0.875*	0.864	0.541	0.783
Consumer Staples	0.601	0.602	0.582	0.562	0.686*
Energy	0.492	0.502	0.584*	0.321	0.521
Financials	0.829	0.854*	0.855	0.509	0.798
Health Care	0.665	0.705*	0.611	0.535	0.609
Info Technology	0.780	0.828*	0.734	0.745	0.660
Industrials	0.863	0.890*	0.855	0.629	0.798
Materials	0.625	0.699*	0.653	0.384-	0.578
Telecom Service	0.661	0.702*	0.650	0.427	0.580
Utility	0.454	0.457	0.495	0.292	0.538*

Notes:

1. * denotes the maximum value in the row. Except for one cases of the U.K. market and three cases of the U.S. market, the maximum value appears when futures and spot positions are in the high volatility states both.
2. The values of correlation coefficient estimate for the entire sample period are among these values from four alternative situations in all cases.

Table 5 Standard Error of Hedged Spot Positions with Index Futures

	No Hedge	Hedge via one constant correlation framework	Hedge via state-varying correlation framework
FTSE 100 Index Futures			
Basic Industries	0.9413	0.7640	0.6721*
Cycle Consumer GDS	1.3769	1.2229	1.1504*
Cycle Services	0.9785	0.6004	0.4306*
Financial	1.2799	0.6328	0.4399*
General Industrials	1.1684	0.9399	0.8008*
Info Technology	2.0645	1.7756	1.5091*
Non-cycle Consumer GDS	1.1720	0.8273	0.7242*
Non-cycle Services	1.7318	1.1731	0.8422*
Resources	1.4065	1.0991	1.0198*
Utilities	1.0468	0.8903	0.7884*
S&P 500 Index Futures			
Consumer Discretionary	1.3074	0.7194	0.4769*
Consumer Staples	1.0366	0.8396	0.7216*
Energy	1.3458	1.1848	1.1346*
Financials	1.4735	0.8371	0.6472*
Health Care	1.3051	0.9909	0.8452*
Info Technology	2.1620	1.3796	1.0198*
Industrials	1.2401	0.6359	0.4625*
Materials	1.3309	1.0565	0.9558*
Telecom Service	1.5366	1.1751	0.9844*
Utility	1.1478	1.0364	0.9251*

Note:

1. The hedge ratio that minimizes the variance of spot position can be expressed as:

$$HR = \frac{Cov(\Delta S_t, \Delta F_t)}{Var(\Delta F_t)} = \rho_{S,F} \times \frac{SD(\Delta S_t)}{SD(\Delta F_t)}$$

   where ΔS_t and ΔF_t denote change rates the prices of futures and spot positions, respectively. $COV(\Delta S_t, \Delta F_t)$ and $Var(\Delta F_t)$ are the covariance of ΔS_t and ΔF_t and the variance of ΔF_t,, respectively. Moreover, $SD(\Delta S_t)$ and $SD(\Delta F_t)$ are the standard deviations of ΔS_t and ΔF_t, respectively. $\rho_{S,F}$ is the correlation coefficient between ΔS_t and ΔF_t.
2. To focus on discussing change in volatility regime conditions to make the dynamics spot-futures relationship and hedge ratios. In this paper, we use the absolute value of realized return rate to serve as the proxy of standard deviation of futures and spot positions.
3. In the setting with one measure of correlation, we use the entire sample to estimate the one constant correlation. In contrast, in the setting of the four measures of correlation, we estimate the state-varying correlations for various volatility state combinations.
4. * presents the minimum value in the row.

Table 6 Risk Reduction Performance of Hedged Spot Positions with Index Futures

	No Hedge	Hedge via one constant correlation framework	Hedge via state-varying correlation framework
FTSE 100 Index Futures			
Basic Industries	-	18.84%	28.60%*
Cycle Consumer GDS	-	11.18%	16.45%*
Cycle Services	-	38.64%	55.99%*
Financial	-	50.56%	65.63%*
General Industrials	-	19.56%	31.46%*
Info Technology	-	13.99%	26.90%*
Non-cycle Consumer GDS	-	29.41%	38.21%*
Non-cycle Services	-	32.26%	51.37%*

Resources	-	21.86%	27.49%*
Utilities	-	14.95%	24.68%*
Average		25.12%	36.68%*
S&P 500 Index Futures			
Consumer Discretionary	-	44.97%	63.52%*
Consumer Staples	-	19.00%	30.39%*
Energy	-	11.96%	15.69%*
Financials	-	43.19%	56.08%*
Health Care	-	24.07%	35.24%*
Info Technology	-	36.19%	52.83%*
Industrials	-	48.72%	62.70%*
Materials	-	20.62%	28.18%*
Telecom Service	-	23.53%	35.94%*
Utility	-	9.71%	19.40%*
Average		28.20%	40.00%*

Note:

1. In this paper, we use the risk of spot positions with no hedge as a benchmark to calculate the risk reduction performance. Specifically, the risk reduction % can be expressed as:

Risk reduction %=-100(risk of hedged spot position-risk of spot position with no hedge)/(risk of spot position with no hedge)*

2. In the row of "Average", we present the average value of risk reduction % of ten hedged spot positions for the U.K. and U.S. markets.

3. In average, the state-varying correlation framework established in this paper can offer an extra 11.55% (36.68-25.12) risk reduction effect relative to the model with one constant correlation for the U.K. market. Moreover, it can offer an extra 11.80% (40.00-28.20) risk reduction performance for the U.S. market.

*4. * present the maximum value in the row.*

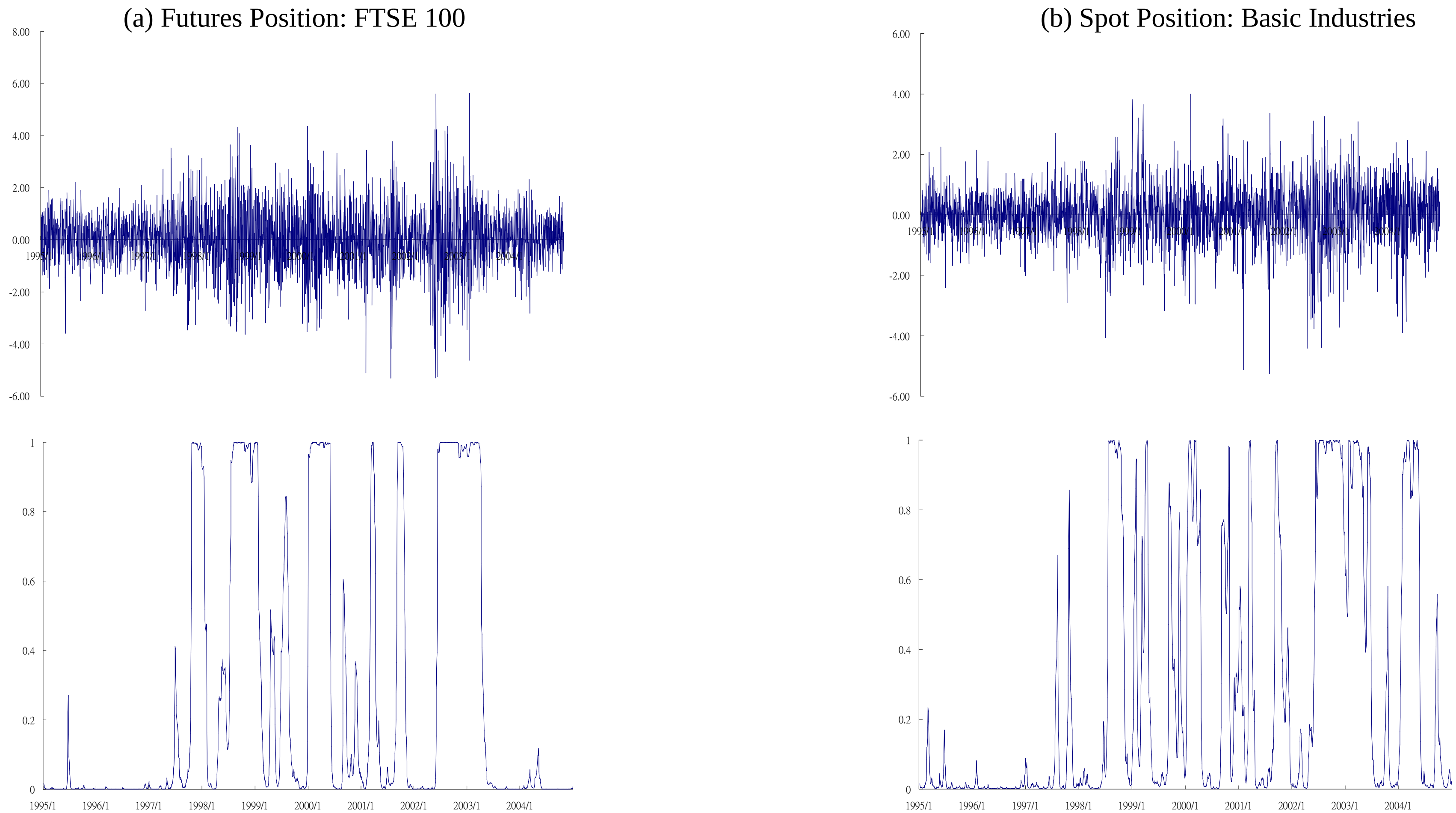

Figure 1 Daily Return Rates and Smoothing Probability of High Volatility States of Futures and Spot Positions: Case of FTSE 100 Index Futures and Sub Index of Basic Industries

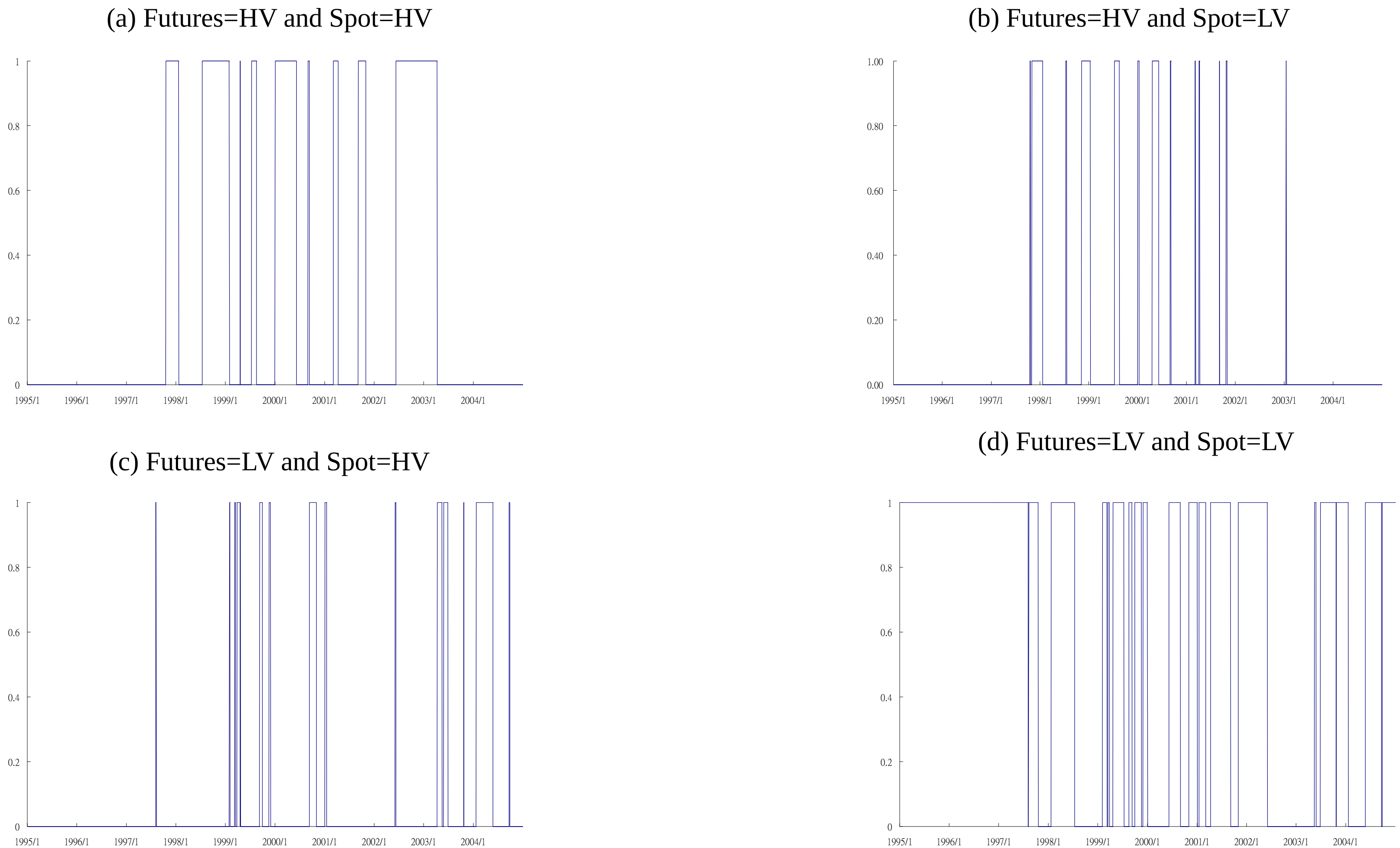

Figure 2 Four Volatility State Combinations of Futures and Spot Positions: Case of FTSE 100 Index Futures and Sub Index of Basic Industries