# Identifying Realized Jumps on Financial Markets*

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This paper extends the jump detection method based on bi-power variation and swap variance measures to identify realized jumps on financial markets and to estimate parametrically the jump intensity, mean, and variance. Such an approach does not require specifying and estimating the underlying drift and diffusion functions. Finite sample evidence suggests that the jump parameters can be accurately estimated and that the statistical inferences can be reliable relative to the maximum likelihood estimation, under the appropriate choice of jump detection test level and assuming that jumps are rare and large. The bi-power variation approach performs slightly better than the swap variance approach when the jump contribution to total variance is small. Applications to equity market, treasury bond, individual stock, and exchange rate reveal important differences in jump frequencies and volatilities across asset classes over time. For high investment grade credit spread indices, the estimated jump volatility has a better forecasting power than interest rate factors, volatility factors including option-implied volatility, and Fama-French risk factors.

***JEL Classification Numbers:*** C22, G13, G14.
***Keywords:*** Realized Jumps, Realized Variance, Jump-Diffusion Process, Bi-Power Variation, Variance Swap Contract, Jump Volatility, Credit Risk Premium.

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# Identifying Realized Jumps on Financial Markets


## Abstract

This paper extends the jump detection method based on bi-power variation and swap variance measures to identify realized jumps on financial markets and to estimate parametrically the jump intensity, mean, and variance. Such an approach does not require specifying and estimating the underlying drift and diffusion functions. Finite sample evidence suggests that the jump parameters can be accurately estimated and that the statistical inferences can be reliable relative to the maximum likelihood estimation, under the appropriate choice of jump detection test level and assuming that jumps are rare and large. The bi-power variation approach performs slightly better than the swap variance approach when the jump contribution to total variance is small. Applications to equity market, treasury bond, individual stock, and exchange rate reveal important differences in jump frequencies and volatilities across asset classes over time. For high investment grade credit spread indices, the estimated jump volatility has a better forecasting power than interest rate factors, volatility factors including option-implied volatility, and Fama-French risk factors.



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# 1 Introduction

Continuous-time jump-diffusion modeling of asset return process has a long history in finance, dating back since at least Merton (1976). However, the empirical estimation of the jump-diffusion processes has always been a challenge to econometricians, in particular, the identification of actual jumps is not readily available from the time-series data of underlying asset returns. Most of the econometric work relies on complicated numerical methods, or heavy simulation-based procedures, and/or joint identification schemes from both the underlying asset and the derivative prices (see, e.g., Bates, 2000; Andersen, Benzoni, and Lund, 2002; Pan, 2002; Chernov, Gallant, Ghysels, and Tauchen, 2003; Eraker, Johannes, and Polson, 2003, among others). Except for some special cases, the direct likelihood-based estimation of jump parameters is difficult to obtain (Aït-Sahalia, 2002b, 2004). Furthermore, jumps introduce additional risk price parameters in derivative pricing. Given that the identification of actual jump dynamics is already imprecise, the reliable estimation of jump risk parameters and its meaningful interpretation can be even more tenuous (Andersen, Benzoni, and Lund, 2002). The main message from the empirical literature seems to be that jumps are very important in asset pricing, but the estimation of jump parameters and the pricing of jump risk are not easy to implement.

This paper applies a straightforward approach to identify the realized jumps for certain class of jump-diffusion processes, extending the seminal work by Barndorff-Nielsen and Shephard (2004b,c). Recent literature suggests that the realized variation measure from high frequency data provides a more accurate measure of the true variance of the underlying continuous-time process (Andersen, Bollerslev, Diebold, and Labys, 2003b; Barndorff-Nielsen and Shephard, 2004a; Meddahi, 2002). Within the realized variance framework, the continuous and jump part contributions can be separated by comparing the difference between realized variance and bi-power variation (see, Barndorff-Nielsen and Shephard, 2004c; Andersen, Bollerslev, and Diebold, 2004b; Huang and Tauchen, 2005). Considering that jumps in financial markets are usually rare and of large sizes, we further assume that (1) there is at most one jump per day, and (2) jump size dominates daily return when it occurs. We also extend the jump detection test based on the swap variance contract (Jiang and Oomen, 2005) to filter our the realized jumps, under the single jump assumption (1) and using a nonlinear root finding procedure to extract the actual jump size and sign. This allows us to filter out the realized jumps, and further to directly estimate the jump distributions (intensity, mean, and variance). Such an estimation strategy based on identified jumps, is in contrast with the existing literature that relies on latent jumps hence the identification from likelihood function of daily returns.[1]

In the Monte Carlo experiment, we examine four settings where the jump contribution to total variance ranges from 10%, 20%, 50%, to 80%. In these situations, our jump identification and estimation approach works well, in that the parameter estimates are efficient, accurate, and converging as the sample size increases (long-span asymptotics). In addition, the asymptotic Chi-square test based on the Wald standard error converges, as the sampling frequency increases (in-fill asymptotics). Typically as jump contribution to total variance increases, the estimation efficiency of jump parameters also increases. One important caveat is that these convergence results are depending on choosing the level of jump detection test. For the bi-power variation based approach, the significance level for the jump detection test need to be set rather loose at 0.99 when jump contribution to total variance is low (10% and 20%), but set rather tight at 0.999 when the jump contribution is high (50% and 80%). In contrast, the swap variance based approach always favors a tighter test level of 0.999, but its performance when jump contribution is small (10%) is not as good as the bi-power variation based one. This seems to be related to the fact that the criterion function of swap variance test is less responsive to smaller positive jumps, which turn out to be the main empirical finding in literature (see, Andersen, Bollerslev, and Diebold, 2004b; Huang and Tauchen, 2005, e.g.).

The proposed jump identification mechanism is implemented for S&P500 market index, 10-year US treasury bond, Microsoft company stock, and Brazilian exchange rate, to cover a wide spectrum of asset classes. The total realized volatility is higher for individual stock and emerging market exchange rate. The jump intensity is the highest for exchange rate (51%), moderate for equity index (23%) and government bond (21%), and the lowest for individual stock (11%). All the jump mean estimates are insiginificant from zero, which is consistent with the finding of large root-mean-squared-error in jump mean estimates in the Monte Carlo experiment. Jump volatility is small for stock market (0.49%) and bond market (0.45%), but large for firm equity (1.12%) and exchange rate (1.13%). Rolling estimates reveal more interesting jump dynamics. The jump probabilities are quite variable for equity index and treasury bond (from 10% to 40%), but relatively stable for Microsoft stock (10%) and Brazilian Real (50%). Although jump means are mostly indistinguishable from zero for all assets considered here, there are obvious deviations from zero for S&P500 index in late

[1]It should be pointed out that such a jump detection and estimation strategy is invalid by construction for certain Lévy process with infinite small jumps in a finite time period (Bertoin, 1996; Barndorff-Nielsen and Shephard, 2001; Carr and Wu, 2004). Our approach is more applicable to the compound Poisson-Normal jump process (Merton, 1976), where the rare and large jumps in financial markets are presumably the responses to significant economic news arrivals.

1990s (positive return jump) and Brazilian Real around 2003 (exchange rate depreciation). Finally, the jump volatilities have not changed much for government bond and individual stock, but elevated significantly for US equity market from 2000 to 2004 and for emerging market currency around 2003.

Being able to identify realized jumps has important implications for estimating financial market risk premia, because we are able to obtain reliable estimation of the objective jump dynamics, which greatly facilitates the estimation of jump risk prices. For Moody's AAA and BAA credit spread indices, we find that the rolling estimates of stock market jump volatility can predict the spread variation with R-squares 0.61 and 0.72, which are considerably higher than standard interest rate factor, volatility factors including option-implied volatility, and the systematic Fama-French factors. This result is important, since forecasting high investment grade credit spreads has not been very successful and the empirical role of jumps in explaining these credit spreads has not been validated in literature so far. This evidence is also consistent with the finding in Zhang, Zhou, and Zhu (2005) that credit default swap (CDS) spreads of individual firms are well explained by the realized jump risk measures estimated similarly from high frequency individual equity prices.

The rest of the paper is organized as following, the next section introduces the jump identification mechanism based on high frequency intraday data, then Section 3 provides some Monte Carlo evidence on the small sample performance of such a mechanism, Section 4 illustrates the approach with four financial market assets, the following section discusses some implications for predicting credit risk spreads, and Section 6 concludes.

# 2 Identifying Realized Jumps

Jumps are important for asset pricing (Merton, 1976), yet the estimation of jump distribution is very difficult, especially when only low frequency data is available (Bates, 2000; Andersen, Benzoni, and Lund, 2002; Pan, 2002; Chernov, Gallant, Ghysels, and Tauchen, 2003; Eraker, Johannes, and Polson, 2003; Aït-Sahalia, 2004). In recent years, Andersen and Bollerslev (1998), Andersen, Bollerslev, Diebold, and Labys (2001); Andersen, Bollerslev, and Diebold (2005a), Barndorff-Nielsen and Shephard (2002a,b), and Meddahi (2002), have advocated the use of so-called realized variance measures by utilizing the information in the intra-day data for measuring and forecasting volatilities. More recent work on bi-power variation measures, which are developed in a series of papers by Barndorff-Nielsen and Shephard (2003, 2004b,c), allows for the use of high-frequency data to disentangle realized volatility into a continuous and a jump components (see, Andersen, Bollerslev, and Diebold, 2004b; Huang and Tauchen, 2005, as well). Further more, a related approach based on the difference between arithmetic and geometric returns can be used to identify realized jumps (Jiang and Oomen, 2005). In this paper, we rely on the economic intuition that jumps on financial markets are rare and large, to extract the realized jumps and to explicitly estimate the jump intensity, mean, and volatility parameters.

## 2.1 Filtering Jumps from Bi-Power Variation

Let $p_t = \log(P_t)$ denotes the time $t$ logarithmic price of the asset, and it evolves in continuous time as a jump diffusion process:

$$dp_t = \mu_t dt + \sigma_t dW_t + J_t dq_t \tag{1}$$

where $\mu_t$ and $\sigma_t$ are the instantaneous drift and diffusion functions that are completely general and may be stochastic (subject to the regularity conditions), $W_t$ is the standard Brownian motion, $dq_t$ is a Poisson jump process with intensity $\lambda_J$, and $J_t$ refers to the corresponding (log) jump size distributed as Normal$(\mu_J, \sigma_J)$. Note that our approach also allows for time-varying jump rate $\lambda_{J,t}$, jump mean $\mu_{J,t}$, and jump volatility $\sigma_{J,t}$, which can be implemented empirically in many different ways once the actual jumps are filtered out. Time is measured in daily units and the intra-daily returns are defined as follows:

$$r_{t,j} \equiv p_{t,j\cdot\Delta} - p_{t,(j-1)\cdot\Delta} \tag{2}$$

where $r_{t,j}$ refers to the $j^{th}$ within-day return on day $t$, and $\Delta$ is the sampling frequency within each day.

Barndorff-Nielsen and Shephard (2004c) propose two general measures for the quadratic variation process—realized variance and realized bipower variation—which converge uniformly (as $\Delta \to 0$ or $m = 1/\Delta \to \infty$) to different quantities of the underlying jump-diffusion process,

$$RV_t \equiv \sum_{j=1}^{m} r_{t,j}^2 \to \int_{t-1}^{t} \sigma_s^2 ds + \int_{t-1}^{t} J_s^2 dq_s \tag{3}$$

$$BV_t \equiv \frac{\pi}{2} \frac{m}{m-1} \sum_{j=2}^{m} |r_{t,j}||r_{t,j-1}| \to \int_{t-1}^{t} \sigma_s^2 ds \tag{4}$$

Therefore the difference between the realized variance and bipower variation is zero when there is no jump and strictly positive when there is a jump (asymptotically).

A variety of jump detection techniques are proposed and studied by Barndorff-Nielsen and Shephard (2004c), Andersen, Bollerslev, and Diebold (2004b), and Huang and Tauchen

(2005). Here we adopted the ratio statistics[2]

$$RJ_{(bv)} \equiv \frac{RV_t - BV_t}{RV_t} \tag{5}$$

which converges to a standard normal distribution with appropriate scaling

$$ZJ_{(bv)} \equiv \frac{RJ_{(bv)}}{\sqrt{[(\frac{\pi}{2})^2 + \pi - 5]\frac{1}{m}\max(1, \frac{TP_t}{BV_t^2})}} \xrightarrow{d} \mathcal{N}(0,1) \tag{6}$$

where $TP_t$ is the *Tri-Power Quarticity* robust to jumps, and as shown by Barndorff-Nielsen and Shephard (2004c),

$$TP_t \equiv m\mu_{4/3}^{-3}\frac{m}{m-2}\sum_{j=3}^{m}|r_{t,j-2}|^{4/3}|r_{t,j-1}|^{4/3}|r_{t,j}|^{4/3} \rightarrow \int_{t-1}^{t}\sigma_s^4 ds \tag{7}$$

with $\mu_k \equiv 2^{k/2}\Gamma((k+1)/2)/\Gamma(1/2)$ for $k > 0$. This test has excellent size and power, and tells us whether there is a jump occurred during a particular day and how much the jump-squared contribution to the total realized variance, i.e., $\int_{t-1}^{t} J_s^2 dq_s / RV_t$.

Based on the economic intuition on the nature and source of jumps on financial market (Merton, 1976), we further assume that, (1) there is at most one jump per day, and (2) jump size dominates return when jump occurs. These assumptions allow us to filter out the daily realized jumps as

$$\hat{J}_t^{(bv)} = \text{sign}(r_t) \times \sqrt{(RV_t - BV_t) \times \mathrm{I}_{t,(ZJ_{(bv)} \geq \Phi_\alpha^{-1})}} \tag{8}$$

where $\Phi$ is the cumulative distribution function of a standard Normal, $\alpha$ is the significance level of the $z$-test, and $\mathrm{I}_{t,(ZJ_{(bv)} \geq \Phi_\alpha^{-1})}$ is the resulting indicator function on whether there is a jump during the day. Our approach of filtering out the realized jumps is a simple extension to the concept of "significant jump" in Andersen, Bollerslev, and Diebold (2004b), the signed square-root of which is equivalent to our $J_t$. Of course, our interpretation of $J_t$ as a single jump size during a day relies on the simplifying assumption that jumps on financial markets are rare and of large sizes.

## 2.2 Filtering Jumps from Swap Variance Measure

Alternatively, we can use the variance swap contract to identify the jumps, as proposed by Jiang and Oomen (2005) based on the insight in Neuberger (1990). The basic intuition

[2] Huang and Tauchen (2005) perform extensive Monte Carlo experiments on various jump detection techniques under many different settings. Here we choose the ratio statistics, favored by their findings regarding the power and size properties of various test statistics.

is that for a log-normal asset price process, the difference between the arithmetic mean and geometric mean is one-half of the variance. This key insight holds exactly for general continuous time processes with continuous sample path. Applying Itô's Lemma to equation (1) with $P_t = \exp(p_t)$ and subtracting equation (1), we have

$$2\int_{t-1}^{t}(dP_s/P_s - dp_s) = \int_{t-1}^{t}\sigma_s^2 ds + 2\int_{t-1}^{t}(\exp(J_s) - J_s - 1)dq_s \tag{9}$$

which replicates the integrated variance exactly if there is no jump. Let the intra-day arithmetic returns be defined as:

$$R_{t,j} \equiv \frac{P_{t,j\cdot\Delta} - P_{t,(j-1)\cdot\Delta}}{P_{t,(j-1)\cdot\Delta}} \tag{10}$$

where $R_{t,j}$ refers to the $j^{th}$ within-day arithmetic return on day $t$, and $\Delta$ is the sampling frequency within each day. Then the swap variance (SV) contract can form a basis for testing jumps

$$SV_t \equiv 2\sum_{j=1}^{m}(R_{t,j} - r_{t,j}) \rightarrow \int_{t-1}^{t}\sigma_s^2 ds + 2\int_{t-1}^{t}(\exp(J_s) - J_s - 1)dq_s \tag{11}$$

$$RV_t \equiv \sum_{j=1}^{m} r_{t,j}^2 \rightarrow \int_{t-1}^{t}\sigma_s^2 ds + \int_{t-1}^{t} J_s^2 dq_s \tag{12}$$

with the difference $SV_t - RV_t$ converges in probability to $2\int_{t-1}^{t}(\exp(J_s) - 1/2J_s^2 - J_s - 1)dq_s$.

Similar to the ratio statistics in Barndorff-Nielsen and Shephard (2004c), Andersen, Bollerslev, and Diebold (2004b), and Huang and Tauchen (2005) based on the bi-power variation, here we adopted the ratio statistics as in Jiang and Oomen (2005) based on the swap variance measure

$$RJ_{(sv)} \equiv \frac{SV_t - RV_t}{SV_t} \tag{13}$$

which converges to a standard normal distribution with appropriate scaling

$$ZJ_{(sv)} \equiv \frac{RJ_{(sv)}}{\sqrt{\frac{1}{m^2}\frac{QP_t}{BV_t^2}}} \xrightarrow{d} \mathcal{N}(0,1) \tag{14}$$

where $QP_t$ is the *Quad-Power Sexticity* robust to jumps,

$$QP_t \equiv \frac{\mu_6}{9}\frac{m^3\mu_{6/4}^{-4}}{m-3}\sum_{j=4}^{m}|r_{t,j-3}|^{6/4}|r_{t,j-2}|^{6/4}|r_{t,j-1}|^{6/4}|r_{t,j}|^{6/4} \rightarrow \frac{\mu_6}{9}\int_{t-1}^{t}\sigma_s^6 ds \tag{15}$$

The swap variance based test statistics has a convergence rate of $m$, compared to $\sqrt{m}$ for the test based on the bi-power variation (Jiang and Oomen, 2005).

Even with rare and large jumps, the swap variance based measure is more involved for filtering out the realized jumps. Assuming that at most one jump happens per day,

$$(SV_t - RV_t) \times \mathrm{I}_{t,(ZJ_{(sv)} \geq \Phi_\alpha^{-1})} = 2(\exp(J_t) - \frac{1}{2}J_t^2 - J_t - 1) \tag{16}$$

Using a third order Taylor expansion of the right-hand-side, we can solve approximately the realized jump size as

$$\hat{J}_t^{(sv)} \approx \mathrm{sign}(SV_t - RV_t) \times \left[3(SV_t - RV_t) \times \mathrm{I}_{t,(ZJ_{(sv)} \geq \Phi_\alpha^{-1})}\right]^{1/3} \tag{17}$$

However, the approximation error is not small here. An exact solution can be found by minimizing the difference between the observed swap variance measure and its asymptotic function value,

$$\hat{J}_t^{(sv)} = \arg\min_{J_t} \left[(SV_t - RV_t) - 2(\exp(J_t) - \frac{1}{2}J_t^2 - J_t - 1)\right]^2 \times \mathrm{I}_{t,(ZJ_{(sv)} \geq \Phi_\alpha^{-1})} \tag{18}$$

conditional on a significant jump occurring. Our experiment suggests that the most accurate and efficient result can be achieved by using the third order Taylor approximation as the starting value and using the nonlinear root-finding procedure to get the final result.

One clear advantage of the swap variance based jump filtering method, is that the sign of the assumed single jump would be the same as the swap variance test statistics $(SV_t - RV_t)$; while for bi-power variation based jump filtering method, we have to use the sign of daily return to approximate the jump sign. On the other hand, jump size is easier to extract in the case of bi-power variation, as a simple square-root of the difference realized variance and bi-power variation; while for swap variance based method, a nonlinear root finding routine is required to extract the jump size accurately. There is a clear trade-off between these two approaches.

## 2.3 Estimating the Jump Distribution

Once the individual jump size is filtered out, we can further estimate the jump intensity, mean, and variance, by imposing a simple model of Poisson-mixing-Normal jump specification,

$$\hat{\lambda}_J = \frac{\text{Number of Jump Days}}{\text{Number of Trading Days}} \tag{19}$$

$$\hat{\mu}_J = \text{Mean of } \hat{J}_t^{(bv)} \text{ or } \hat{J}_t^{(sv)} \tag{20}$$

$$\hat{\sigma}_J = \text{Standard Deviation of } \hat{J}_t^{(bv)} \text{ or } \hat{J}_t^{(sv)} \tag{21}$$

with usual formula for the standard error estimates. Such an approach for estimating jumps is robust to the specifications of time-varying or even stochastic drift and diffusion functions. It also allows us to specify flexible dynamic structures of the underlying jump arrival rate or jump volatility processes, similar to those in Andersen, Bollerslev, and Huang (2005b). Realized jumps therefore can help us to avoid those estimation methods that rely heavily on numerical simulations.

# 3 Finite Sample Experiment

It is important to evaluate whether the proposed jump identification scheme works well under our assumption of large and rare jumps. In particular, we want to know whether the jump parameters can be accurately estimated and whether the correct inferences can be made, as both the sample size increases and the sampling interval decreases.

## 3.1 Experimental Design

Here we adopt the following benchmark specification of a stochastic volatility jump-diffusion process,

$$dp_t = \mu_t dt + \sigma_t dW_{1t} + J_t dq_t \tag{22}$$

$$d\sigma_t^2 = \beta(\theta - \sigma_t^2)dt + \gamma\sqrt{\sigma_t^2}dW_{2t} \tag{23}$$

with log price drift $\mu_t = 0$; volatility mean reversion $\beta = 0.10$ and volatility-of-volatility $\gamma = 0.05$; jump parameters $\lambda_J = 0.05$, $\mu_J = 0.20$, $\sigma_J = 1.40$; and leverage coefficient $\rho \equiv \mathrm{corr}(dW_{1t}, dW_{2t}) = -0.50$. The volatility long-run mean parameter $\theta$ is chosen for four scenarios to cover a possible range of financial asset classes. Scenario (a) has $\theta = 0.9$ such that the discontinuous part contribution to total variance is 10%. Such a scenario applies more likely to the US equity market, major currencies, and blue chip stocks. In fact, 10% is about the average empirical findings in Andersen, Bollerslev, and Diebold (2004b) and Huang and Tauchen (2005). Scenario (b) with $\theta = 0.4$, may be suited for more volatile and less liquid markets, like medium size stocks or certain commodity futures. Under this scenario the jump part contributes to total variance about 20%. Scenario (c) with $\theta = 0.1$, featuring 50% of jump contribution to realized variance, perhaps better describes those illiquid small stocks and emerging market stock indices or exchange rates. Finally, Scenario (d) with $\theta = 0.025$ and 80% jump contribution to variance, resembles the most illiquid and infrequently traded assets, like some corporate bond and municipal bond markets. The choice of jump parameters also reflect the empirical findings in literature that (1) jumps are rare, (2) jumps are large in terms of standard deviation, and (3) jump mean is hard to distinguish from zero.

The Monte Carlo experiment is designed as follows. Each day one simulates the jump-diffusion process, using 1-second as a tick totaling six and a half trading hours, imitating the US equity market in recent years. The diffusion process with stochastic volatility is simulated by the Euler scheme, the jump timing is simulated from an Exponential distribution, and the jump size is simulated from a Normal distribution. Then the realized jumps are combined with the realized diffusion, and sampled by an econometrician at both 1-minute and 5-minute intervals, illustrating the in-fill asymptotics. To contrast the long-span asymptotics of sample sizes, we use both T = 1000 days and T = 4000 days. Further, the choice of significance level in the jump detection test is also compared between $\alpha = 0.99$ and $\alpha = 0.999$. The appropriate choice of the pre-test level seems to be critical to achieve consistent parameter estimates, given varying degree of jump contribution to the total variance. In addition, the simulation provides us the exact jump timing (Exponential) and jump size (Normal), therefore a maximum likelihood estimator (MLE) can be used as a benchmark for judging the estimation efficiency of two jump-size extraction methods.

## 3.2 Parameter Estimation

The finite sample results on various jump parameter estimates are presented in Tables 1-4. The first column of each table gives the true parameter values, and the second column gives the mean biases and root-mean-squared-errors of the maximum likelihood estimator (MLE). Note that the MLE results do not vary: (1) across the four scenarios (since only the diffusion volatility level is altered), (2) across the pre-test $\alpha = 0.99$ and $\alpha = 0.999$ levels (since no pre-estimation filtering is involved), (3) across the 5-minute and 1-minute sampling intervals (since jumps are observed exactly in simulations). The estimation biases at both 1000 and 4000 days are negligible for all three parameters, relative to their true values. In terms of the estimation efficiency, both jump rate $\lambda_J$ and jump volatility $\sigma_J$ can very accurately estimated with RMSE's much smaller than the parameter values. However, for the jump mean parameter $\mu_J$, the estimate is not accurate at 1000 days (RMSE about the size of parameter value), but can be accurate at 4000 days (RMSE about half the size of parameter value). In addition, all the RMSE's are converging almost exactly at the rate of $\sqrt{4}$, as predicted by the asymptotic theory that MLE is unbiased, efficient, and root-$T$ consistent.

For the jump filtering mechanism based on the bi-power variation measure, the parameter estimation efficiency seem to approach that of MLE very differently, depending whether the jump contribution to total variance is small or large. In Scenario (a) or (b) where jump contribution to variance is as small as 10% or 20%, the RMSE's of parameter estimates are all closer to those of MLE and the convergence rates are closer to $\sqrt{4}$, as the sample size increases from T = 1000 to T = 4000, when we set the pre-test level $\alpha = 0.99$ but not $\alpha = 0.999$. In other words, when jumps are small, long-span asymptotics seems to work better when the pre-test level is less stringent. In contrast, for Scenario (c) or (d) where the jump contribution to total variance is as large as 50% or 80%, the long-span asymptotics seem to work much better when we set $\alpha = 0.999$ rather than $\alpha = 0.99$, where the RMSE's can almost match those of MLE. These findings are intuitive in the following sense. It is clearly more difficult to detect jumps when they are relatively small, therefore loosening the jump detection standard can reveal more jumps that otherwise would have been missed (minimizing the type-I error). On the other hand, when jumps are large they are easier to detect, so we want a more stringent jump filtering standard, such that false revelation of jumps can be avoided as much as possible (minimizing the type-II error). In short, the jump filtering approach based on the bi-power variation measure, can bring us efficient parameter estimates relative to MLE, provided that we appropriately choose the pre-test levels according to the relative sizes of jump contribution to total variance.

In contrast, the jump identification approach based on the swap variance measure, seem to produce more efficient estimation result (close to MLE), only when we set $\alpha = 0.999$ instead of $\alpha = 0.99$, regardless whether the jump contribution to variance is large or small. To be more specific, if we choose $\alpha = 0.999$, RMSE's of swap variance based method generally outperform the those of bi-bower variation based method when jumps are small (Scenario (a) and (b)), while both approaches produces similar efficiencies when jumps are large (Scenario (c) and (d)). If we choose $\alpha = 0.99$, the RMSE's from the swap variance regime are much worse than those from the bi-bower variation regime, for the jump rate and jump volatility parameters but not for the jump mean parameter, uniformly across the four scenarios. These results may be driven by the differences between the swap variance approach and bi-power variation method: (1) two-sided test versus one-sided test, (2) convergence rate at $T$ versus root-$T$, (3) jump signed by the test statistics versus by the daily return. However, the non-uniform performance of the bi-power variation approach as the choice of pre-test level $\alpha$ across scenarios, seem to be related to the small jump size mean parameter $\mu_J = 0.2$ relative to the jump standard deviation parameter $\sigma_J = 1.2$, which is dictated by the empirical evidence that jump mean is indeed hard to distinguish from zero.

The empirical literature so far has found that jump contribution to total variance is around 10% or lower (Andersen, Bollerslev, and Diebold, 2004b; Huang and Tauchen, 2005), which corresponds to our Scenario (a). In this case, the bi-power variation approach with

$\alpha = 0.99$ and swap variance approach with $\alpha = 0.999$ are the best choices, with the former has a better RMSE convergence rate (see Table 1).

### 3.3 Statistical Inference

In addition to the parameter estimation efficiency, we also care about whether asymptotic standard error estimated in finite samples, can provide a reliable statistical inference about the true parameter value. To set the right benchmark, Figure 1 plot the finite sample rejection rates from the Monte Carlo replications against the asymptotic test size. The rejection rate is based on the Chi-square (1) test statistics of each parameter. The deviation between the dashed line (Monte Carlo finite sample result) and dotted diagonal line (asymptotic result), indicates how big is the size distortion. It is clear from Figure 1 that the MLE asymptotic variance estimated in finite sample behaves extremely well, so there is effectively no size distortion at all.

The Wald test statistics based on bi-power variation approach are reported in Figures 2-5. In general, the t-test for jump mean $\mu_J$ is well behaved, while the result of jump rate $\lambda_J$ and jump volatility $\sigma_J$ varies a lot in different settings. In Scenario (a) or (b) where jumps contributes 10% or 20% of the total variance, the chi-square statistics for jump intensity and volatility under the choice of $\alpha = 0.999$ have much higher over-rejection bias compared to the choice $\alpha = 0.99$. In Scenario (c) or (d) with relative jump contribution ranging from 50% to 80%, there is almost no over-rejection bias at $\alpha = 0.999$ level, while the chi-square test does not converge at all for $\alpha = 0.99$. In short, if jumps are small then less stringent jump detection test generates more reliable inferences about the jump parameters, while if jumps are large then more stringent test generates more reliable inferences.

For the swap variance based approach, the asymptotic Wald test always performs better for $\alpha = 0.999$ than for $\alpha = 0.99$. As seen from Figures 6-9, the over-rejection biases for jump rate $\lambda_J$ and jump volatility $\sigma_J$ are exceptionally high under the choice of $\alpha = 0.99$, and do not improve at all from Scenario (a) to (d) as the jump contribution to total variance increases from 10% to 80%. On the other hand, If we choose $\alpha = 0.999$, the size distortions are quite visible under small jump scenarios (a) and (b) but converge to negligible under large jump scenarios (c) and (d). It is an advantage for the swap variance method to favor uniformly the test level of $\alpha = 0.999$, however, under the small jump contribution scenario (a) which is typically found in the empirical literature (Andersen, Bollerslev, and Diebold, 2004b; Huang and Tauchen, 2005), the bi-power variation approach with $\alpha = 0.99$ (upper two panels of Figure 2) seems to have the right convergence of test statistics, as opposed to the swap variance method with $\alpha = 0.999$ (lower two panels of Figure 6). Even under the large jump contribution scenario (d) with the preferred choice of $\alpha = 0.999$, if the sampling interval is restricted to 5-minute, then the test statistics clear has less over-rejection bias for the bi-power variation approach (lower two panels of Figure 5) than the swap variance approach (lower two panels of Figure 9).

### 3.4 Criterion Function of Test Statistics

The differential performance between the bi-power variation and swap variance based approach is estimating the jump parameters, when choosing different pre-test level $\alpha$ under the scenarios with small or large jump contribution to variance, is related to the power of these jump detection test statistics. As illustrated in Figure 10, the bi-power variation test is one-sided while the swap variance test is two-sided. When jump is negative, the criterion function of test statistics is larger for the bi-power variation method than for the swap variance method. When jump is positive, small jumps are more easily detected by swap variance measure, while large jumps are more affecting bi-power variation measure. In our realistic Monte Carlo setting, the jumps have a small positive mean which is hard to distinguish from zero in estimation. So the finding in Jiang and Oomen (2005) may be interpreted as that for large fixed jumps the swap variance based test is more effective, but for small random jumps as in the Scenario (a) of this paper the bi-power variation based test can be more powerful.

## 4 Application to Financial Markets

We select four financial assets to illustrate the proposed methodology in filtering out realized jumps and estimating jump dynamics. The intraday high frequency data for S&P500 index (1986-2005) is obtained from the Institute of Financial Market, the 10-year US treasury bond (1991-2005) from the Federal Reserve Board, Microsoft stock (1993-2002) from NYSE TAQ data base, and the Brazil exchange rate (1999-2005) from the Federal Reserve Bank of New York. These choices are meant to give a representative view of available asset classes, in particular, with a reasonable range of jump contribution to total variance. All the data are transformed to five minute log returns. We eliminate days with less than 60 trades or quotes. We also drop the after-hour tradings due to the liquidity concern, except for the Brazil exchange rate, from which we drop the stale quotes.

Summary statistics for daily percentage returns and realized volatility (square-root of realized variance) are reported in Table 5. The sample means suggest annualized returns of 8.85% for S&P500, 4.10% for t-bond, 11.20% for Microsoft, and 12.85% for the Real. The average realized volatility is higher for individual stock (2.19%) and exchange rate (1.14%), and lower for market index (0.73%) and t-bond (0.56%). The return skewness is negative for S&P500 index and government bond, while positive for Microsoft and Brazilian Real. Kurtosis suggests all return and volatilities are quite deviant from the Normal distribution. The returns are approximately serially uncorrelated, while the volatility series exhibit pronounced own temporal dependencies. In fact, the first ten autocorrelations reported in the bottom part of the table are all highly significant with the gradual, but very slow, decay suggestive of long-memory type features. This is also evident from the time series plots of realized volatility series given in the top panels of Figures 11-14.

## 4.1 Unconditional Jump Parameter Estimates

As shown in the top panel of Table 6, jump contribution to total variance is about 7.85% for S&P500, 10.37% for t-bond, 2.60% for Microsoft, and 30.07% for the exchange rate. The numbers on market index and treasury bond are very close to the finding in Andersen, Bollerslev, and Diebold (2004b) and Huang and Tauchen (2005). Since these numbers are closer to Scenario (a) or (b) of smaller jump contribution to variance in our Monte Carlo study section, we expect that the jump filtering and estimation based the bi-power variation approach would outperform the swap variance approach. In the rest of the empirical exercise, we will focus on the bi-power variation approach.

The realized jumps filtered by our method are plotted in the second panels of Figures 11-14. Jumps in S&P500 index is clearly more frequent with maximum sizes between -2% and +2%. Treasury bond has less frequent jump with similar maximum range. Microsoft stock has rare jumps but the sizes are larger than the market return and government bond. Brazilian currency has the most frequent jumps with sizes as large as -4.5% and +6.5%. The bottom panel of Table 6 reports the parametric distribution estimates based on the filtered realized jumps. Except for the S&P500 index ($\mu_J$ = 0.05 with s.e. = 0.01), all the jump mean estimates are statistically indifferent from zero. The jump intensity estimates are highly significant and vary across assets; with exchange rate being the highest (0.51 with s.e. 0.02), individual stock the lowest (0.11 with s.e. 0.02), and S&P500 (0.23 with s.e. 0.01) and bond in between (0.21 with s.e. 0.02). The standard deviations of jumps are estimated most accurately; being moderate for equity market (0.49 with s.e. 0.01) and bond market (0.45 with s.e. 0.01) but fairly large for individual firm (1.12 with s.e. 0.05) and emerging market currency (1.13 with s.e. 0.03).

These results differ from the usual jump estimation results in empirical finance, in that most of the literature focus on latent unobserved jumps, while ours are based on filtered realized jumps. Our findings regarding jump frequency and jump size may be reconciled with the notion that significant jumps on financial markets are related to the surprise responses to the macroeconomic news announcement (Andersen, Bollerslev, Diebold, and Vega, 2003a, 2004a). That's why October 87 market crash is not classified as a big jump, since there were no particular news announcement and market surprise as evidenced in the literature.

### 4.2 Time-Varying Jump Parameter Estimates

Another interesting feature that can be seen from the second panels of Figures 11-14, is that the clustering and amplitude of jumps are changing over time, which leads to the usual conjecture of time-varying jump rate and jump size distribution. To get a first handle of such a possibility, we perform a two-year rolling estimation (except for exchange rate with one-year rolling) of the jump parameters $\lambda_{J,t}$, $\mu_{J,t}$, and $\sigma_{J,t}$, with corresponding 95% standard error bands. As seen from Figure 11, the jump intensity of S&P500 index was fairly high during the early 1990s (30-40%), then dropped considerably during the late 1990s (10-20%), and started to rise again since 2002. Jump size mean have been mostly close to zero, except for the late 1990s when positive jump means are statistically significant and coinciding with the stock market bubble. Jump volatility had been largely stable from late 1980s to late 1990s around 40%, but has been elevated since 1999 and peaked around 2002-2003 at high 80%. As Figure 12 shows, the jump intensity of the bond market was high around 1998 and 2001-2002, while jump mean is mostly zero and jump volatility is little changed around its unconditional level. Microsoft stock, as seen from Figure 13, has nearly constant jump intensity, zero jump mean, and constant jump volatility. For the Brazilian currency in Figure 14, jump intensity is stable, while jump mean is statistically above zero around 2002 to 2003 when jump volatility is also at its peak.

Time-varying jump intensity and jump volatility are very important risk factors in asset pricing, but until recently most of the evidence are coming from the option implied or latent jump specifications (see, for example, Duffie, Pan, and Singleton, 2000; Eraker, Johannes, and Polson, 2003, among others). A recent paper by Andersen, Bollerslev, and Huang (2005b) use the realized jump timing to examine the temporal dependency in jump durations.

## 5 Implications for Risk Premia

A direct identification of realized jumps and an easy characterization of jump distributions have important implications for quantifying financial market risk premia. The reason is that jump parameters are generally very hard to pin down even with both underlying and derivative assets prices, due to the fact that jumps are latent and are rare events in financial markets. Inaccurate estimates of the underlying jump dynamics makes the jump risk premia even harder to quantify. However, as demonstrated bellow, a reliable estimate of stock market jump volatility based on identified realized jumps, can have a superior predicting power for the bond credit risk premium.

## 5.1 Predicting Corporate Bond Spread Indices

Here we examine the daily forecasting powers for Moody's AAA and BAA bond spreads, using the estimated S&P 500 jump volatility from the identified realized jumps of the past two years, which is discussed in Section 4. It has been puzzling to explain high investment grade bond spreads, since those firms entertain very little default risk historically, yet the credit spreads over default-free treasuries are sizable positive (Amato and Remolona, 2003). Although jump risk has been proposed as a possible source of such a credit premium puzzle (Zhou, 2001; Huang and Huang, 2003), the empirical validation in literature has met with mixed and unsatisfactory results (Collin-Dufresne, Goldstein, and Martin, 2001; Collin-Dufresne, Goldstein, and Helwege, 2003; Cremers, Driessen, Maenhout, and Weinbaum, 2004a,b). Here we use an alternative jump risk measure, based on identified realized jumps as opposed to latent or implied jumps, to provide some contrasting positive evidence in explaining high investment grade credit spread indices. For comparison purpose, we also include standard predictors like the short rate and term spread as in Longstaff and Schwartz (1995) and all empirical studies, long-run realized volatility (Campbell and Taksler, 2003) and short-run realized volatility (Zhang, Zhou, and Zhu, 2005), and option implied volatility (Carr and Wu, 2005; Wu and Zhang, 2005).

Table 7 presents the univariate forecasting regressions for Moody's AAA and BAA bond spreads. The OLS coefficients show remarkable similarity between the two spread indices. To be more precise, one percentage increase in short rate lowers credit spreads 14 and 16 basis points; positive term spread increases default premium 5 and 12 basis points. Short rate predict 44% and 36% of spread variation, while term spread by itself has very little forecasting power. Short-run volatility (1-day) has R-squares around 30% with marginal impact around 4 basis points, while long-run volatility (2-year) has higher R-squares about 50-60% and higher impact coefficient 7 to 9 basis points. It is worth pointing out that option implied volatility (VIX index) has about the same predicting power and marginal effect as the long-run and short-run volatilities. In comparison, the S&P500 jump volatility not only has a larger impact on credit spreads — one percentage increase raises spreads about 200 basis points, but also has the highest forecasting power — with R-squares being 61% for the AAA bond spread and 72% for the BAA bond spread. The close association between credit risk premium and market jump volatility can be more vividly seen in Figure 15. Although the daily credit spread is very noisy, there clearly exist both long term trends and short term cycles from 1988 to 2004. It is obvious that time-varying jump volatility trace closely these trends and cycles, while discarding the day-to-day fluctuations in credit spreads.

Given the common finding that the typical default risk factors can only account for a very small fraction of the corporate bond spreads, recent effort has been directed to the role of systematic risk premia in the economy (see, Elton, Gruber, Agrawal, and Mann, 2001; Huang and Huang, 2003; Chen, Collin-Dufresne, and Goldstein, 2005, e.g.). However, those business cycle effects can easily explain the spread variations of low investment grade or speculative grade credit spreads, but has little or no explaining power for the high investment grade credit spreads. As Table 8 shows, the systematic risk factors—market return, SMB, and HML Fama-French variables—have zero predicting capability for the high investment grade credit spread at the daily frequency. The fact that these bonds have little default risk yet commands a sizable risk premium constitutes a major challenge in credit risk pricing. In comparison, jump volatility risk measure stands out as the most powerful instrument in forecasting the credit spread indices, suggesting that a systematic jump risk factor may be important in pricing the top quality credit.

Table 9 presents multiple regressions in forecasting the bond spreads. It seems that two interest rate factors are complementary, in that the combined R-square is much higher that the sum of two univariate regressions. The signs of both short rate and term spread are now negative and larger. Intuitively when economy is in expansion, short rate and term spread tend to be rising, and the credit default condition is also improving. Note that when combining short- and long-run volatilities or implied and jump volatilities, the coefficient magnitude and significance level mostly remain the same. It suggests that two volatility components may be needed in explaining the risk premium dynamics (Adrian and Rosenberg, 2005). Further experiments with multiple predictors suggest that interest rate factors are more important for forecasting the AAA credit spread, while volatility factors more important for BAA credit spread. Therefore the last columns in Table 9 are the multiple regressions that have the highest adjusted R-squares which are close to 80%.

In short, contrary to the negative finding in empirical literature about the jump impact on credit spread, our measure of realized jump volatility have strong predictability for high investment grade credit spreads. The forecasting power is higher than the interest rate factors, short-run and long-run volatility factors, or even the option implied volatility factor.

### 5.2 Explaining Credit Default Spreads of Individual Firms

In a related paper, Zhang, Zhou, and Zhu (2005) apply the jump identification strategy of this paper to individual firms, and find that the realized jump risk measures (intensity, mean, and volatility) all have strong explaining power for credit default swap (CDS) spreads. In particular, jump risk alone can predict about 19% variation of the CDS spreads. By separating realized volatility and jump measures, Zhang, Zhou, and Zhu (2005) also strengthen the forecasting power of equity volatility measures (Campbell and Taksler, 2003), and increase the overall forecasting R-square to 77%. Furthermore, they find that the nonlinear effects of jump and volatility risk measures on credit spreads are largely consistent with a structural model with stochastic volatility and jumps.

### 5.3 Econometric Estimation of Jump-Diffusion Processes

Being able to filter out the realized jumps, provides us a new scope in estimating and testing the continuous-time asset return process, along the lines of Chernov, Gallant, Ghysels, and Tauchen (2003). Observable jumps can also strengthen the likelihood based estimation method for jump-diffusion process (Aït-Sahalia, 2002a). Further, the measurement error problem in detecting and estimating jumps, when jump contribution to total variance is relatively small, certainly need to be addressed in the second stage estimation of jump parameters, as in Andersen, Bollerslev, and Meddahi (2005c). Finally, the discrete-time joint dynamics of realized variance and jumps may serve as an ideal score generator in an Efficient Method of Moments setting, for estimating richer and more complex jump-diffusion specifications in continuous-time (Bollerslev, Kretschmer, Pigorsch, and Tauchen, 2005)

## 6 Conclusion

Disentangling jumps from diffusion has always been a challenge for pricing financial assets and for estimating the jump-diffusion processes. Building on the recent jump detection literature by differentiating realized variance and bi-power variation (Barndorff-Nielsen and Shephard, 2003, 2004b,c; Andersen, Bollerslev, and Diebold, 2004b; Huang and Tauchen, 2005), we extend the methodology to filter out the realized jumps, under two key assumptions typically adopted in financial economics: (1) jumps are rare and there is at most one jump per day, and (2) jumps are large and dominate return signs when occurring. We also extend the jump detection test based on the swap variance contract (Jiang and Oomen, 2005) to filter our the realized jumps, under the single jump assumption and using a nonlinear root finding procedure.

These approximations provide us powerful tools to identify the realized jumps on financial markets. Our Monte Carlo experiment under realistic empirical settings suggests that consistent parameter estimates and converging inference tests can be obtained, if both the sample size and the sampling frequency increase, with an appropriate choice of the significance level of the jump detection pre-test. Of course, the stochastic volatility of the pure diffusion part creates a measurement error problem for detecting the jump timing and for estimating jump parameters, especially when the jump contribution to total variance is relatively small. Although both the bi-power variation approach and the swap variance approach achieve similar estimation efficiency when jump contribution to variance is large, the former seems to perform better than the latter when the jump contribution to total variance is small. This finding seems to be related to the properties of the criterion functions of the two jump test statistics, and is also driven the fact that in our realistic Monte Carlo setting the jumps have a small positive mean that is hard to distinguish from zero.

The proposed jump identification method (based on the bi-power variation approach) is applied to four financial assets — S&P500 index, treasury bond, Microsoft stock, and Brazil Real. We find that the jump intensity varies significantly among these asset classes (from 10% to 50%). All the jump mean estimates are insignificantly from zero, except for the S&P500 index driven by a positive run in late 1990s. Jump volatility is small for equity market and bond market (near 0.5%) but large for individual firm and exchange rate (above 1%). Rolling estimates reveal that the jump probabilities are quite variable for equity index and treasury bond (from 10% to 40%), but relatively stable for Microsoft stock (10%) and Brazil currency (50%). The jump volatilities are little changed for government bond and individual stock, but elevated a great deal for stock market from 2000 to 2004 and for Brazil currency around 2003.

The identification of realized jumps and direct estimation of jump distributions have important implications in assessing financial market risk premia. Given more reliable estimates of the objective jump dynamics, the impact on jump risk premia can be more precisely estimated. For example, the Moody's AAA and BAA credit risk premia can be predicted by the realized jump volatility measure, much better than the interest rate factors, volatility factors, and Fama-French risk factors. Explaining the credit spreads of high investment grade entities has always been a challenge in credit risk pricing, and a systematic jump risk factor holds some promise in resolving such an puzzle. Individual firm's credit spreads can also be better predicted by the realized jump risk measures from each firm's equity returns (Zhang, Zhou, and Zhu, 2005). Finally, transforming jumps from latent to realized, even with a considerable measurement error problem, may significantly enhance the econometrician's tool box in estimating the underlying jump distribution dynamics (Bollerslev, Kretschmer, Pigorsch, and Tauchen, 2005).

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Table 1: Monte Carlo Experiment with Scenario (a)

| Method | Maximum Likelihood Estimation | | | | Based on Bi-Power Variation | | | | Based on Swap Variance Contract | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Statistics | Mean Bias | | RMSE | | Mean Bias | | RMSE | | Mean Bias | | RMSE | |
| Sample Size | 1000 | 4000 | 1000 | 4000 | 1000 | 4000 | 1000 | 4000 | 1000 | 4000 | 1000 | 4000 |
| | Sampling Frequency $\Delta$ = 5-minute, Level of Significance $\alpha = 0.99$ | | | | | | | | | | | |
| $\lambda_J = 0.05$ | 0.0006 | 0.0003 | 0.0073 | 0.0034 | -0.0066 | -0.0063 | 0.0093 | 0.0072 | 0.0266 | 0.0267 | 0.0278 | 0.0271 |
| $\mu_J = 0.2$ | -0.0010 | -0.0019 | 0.1927 | 0.0987 | -0.0002 | -0.0078 | 0.2048 | 0.1030 | 0.0090 | 0.0097 | 0.1264 | 0.0658 |
| $\sigma_J = 1.2$ | -0.0047 | -0.0007 | 0.1406 | 0.0710 | -0.0048 | -0.0010 | 0.1442 | 0.0702 | -0.2493 | -0.2400 | 0.2760 | 0.2467 |
| | Sampling Frequency $\Delta$ = 1-minute, Level of Significance $\alpha = 0.99$ | | | | | | | | | | | |
| $\lambda_J = 0.05$ | 0.0006 | 0.0003 | 0.0073 | 0.0034 | -0.0007 | -0.0006 | 0.0069 | 0.0035 | 0.0166 | 0.0171 | 0.0182 | 0.0175 |
| $\mu_J = 0.2$ | -0.0010 | -0.0019 | 0.1927 | 0.0987 | -0.0168 | -0.0196 | 0.1856 | 0.0955 | -0.0079 | -0.0070 | 0.1420 | 0.0744 |
| $\sigma_J = 1.2$ | -0.0047 | -0.0007 | 0.1406 | 0.0710 | -0.0390 | -0.0341 | 0.1404 | 0.0750 | -0.1951 | -0.1883 | 0.2326 | 0.1992 |
| | Sampling Frequency $\Delta$ = 5-minute, Level of Significance $\alpha = 0.999$ | | | | | | | | | | | |
| $\lambda_J = 0.05$ | 0.0006 | 0.0003 | 0.0073 | 0.0034 | -0.0204 | -0.0203 | 0.0210 | 0.0204 | -0.0064 | -0.0061 | 0.0091 | 0.0069 |
| $\mu_J = 0.2$ | -0.0010 | -0.0019 | 0.1927 | 0.0987 | 0.0893 | 0.0763 | 0.3082 | 0.1669 | 0.0706 | 0.0733 | 0.2278 | 0.1328 |
| $\sigma_J = 1.2$ | -0.0047 | -0.0007 | 0.1406 | 0.0710 | 0.2458 | 0.2533 | 0.2946 | 0.2654 | 0.0795 | 0.0864 | 0.1682 | 0.1113 |
| | Sampling Frequency $\Delta$ = 1-minute, Level of Significance $\alpha = 0.999$ | | | | | | | | | | | |
| $\lambda_J = 0.05$ | 0.0006 | 0.0003 | 0.0073 | 0.0034 | -0.0116 | -0.0115 | 0.0132 | 0.0119 | -0.0060 | -0.0056 | 0.0088 | 0.0064 |
| $\mu_J = 0.2$ | -0.0010 | -0.0019 | 0.1927 | 0.0987 | 0.0366 | 0.0303 | 0.2401 | 0.1237 | 0.0405 | 0.0391 | 0.2191 | 0.1169 |
| $\sigma_J = 1.2$ | -0.0047 | -0.0007 | 0.1406 | 0.0710 | 0.1310 | 0.1352 | 0.1943 | 0.1526 | 0.0718 | 0.0760 | 0.1633 | 0.1066 |

Table 2: Monte Carlo Experiment with Scenario (b)

| Method | Maximum Likelihood Estimation | | | | Based on Bi-Power Variation | | | | Based on Swap Variance Contract | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Statistics | Mean Bias | | RMSE | | Mean Bias | | RMSE | | Mean Bias | | RMSE | |
| Sample Size | 1000 | 4000 | 1000 | 4000 | 1000 | 4000 | 1000 | 4000 | 1000 | 4000 | 1000 | 4000 |
| | Sampling Frequency $\Delta$ = 5-minute, Level of Significance $\alpha = 0.99$ | | | | | | | | | | | |
| $\lambda_J = 0.05$ | 0.0006 | 0.0003 | 0.0073 | 0.0034 | -0.0008 | -0.0007 | 0.0072 | 0.0035 | 0.0298 | 0.0299 | 0.0310 | 0.0302 |
| $\mu_J = 0.2$ | -0.0010 | -0.0019 | 0.1927 | 0.0987 | -0.0097 | -0.0133 | 0.1974 | 0.0984 | -0.0362 | -0.0395 | 0.1251 | 0.0740 |
| $\sigma_J = 1.2$ | -0.0047 | -0.0007 | 0.1406 | 0.0710 | -0.0521 | -0.0511 | 0.1517 | 0.0897 | -0.2806 | -0.2741 | 0.3054 | 0.2804 |
| | Sampling Frequency $\Delta$ = 1-minute, Level of Significance $\alpha = 0.99$ | | | | | | | | | | | |
| $\lambda_J = 0.05$ | 0.0006 | 0.0003 | 0.0073 | 0.0034 | 0.0027 | 0.0030 | 0.0079 | 0.0046 | 0.0180 | 0.0183 | 0.0196 | 0.0187 |
| $\mu_J = 0.2$ | -0.0010 | -0.0019 | 0.1927 | 0.0987 | -0.0171 | -0.0216 | 0.1913 | 0.0963 | -0.0300 | -0.0327 | 0.1420 | 0.0799 |
| $\sigma_J = 1.2$ | -0.0047 | -0.0007 | 0.1406 | 0.0710 | -0.0635 | -0.0649 | 0.1512 | 0.0979 | -0.2023 | -0.1975 | 0.2397 | 0.2082 |
| | Sampling Frequency $\Delta$ = 5-minute, Level of Significance $\alpha = 0.999$ | | | | | | | | | | | |
| $\lambda_J = 0.05$ | 0.0006 | 0.0003 | 0.0073 | 0.0034 | -0.0137 | -0.0135 | 0.0150 | 0.0138 | -0.0018 | -0.0016 | 0.0070 | 0.0038 |
| $\mu_J = 0.2$ | -0.0010 | -0.0019 | 0.1927 | 0.0987 | 0.0596 | 0.0509 | 0.2754 | 0.1411 | 0.0322 | 0.0271 | 0.2013 | 0.1054 |
| $\sigma_J = 1.2$ | -0.0047 | -0.0007 | 0.1406 | 0.0710 | 0.1507 | 0.1502 | 0.2196 | 0.1710 | 0.0176 | 0.0244 | 0.1420 | 0.0747 |
| | Sampling Frequency $\Delta$ = 1-minute, Level of Significance $\alpha = 0.999$ | | | | | | | | | | | |
| $\lambda_J = 0.05$ | 0.0006 | 0.0003 | 0.0073 | 0.0034 | -0.0075 | -0.0073 | 0.0101 | 0.0080 | -0.0034 | -0.0031 | 0.0076 | 0.0045 |
| $\mu_J = 0.2$ | -0.0010 | -0.0019 | 0.1927 | 0.0987 | 0.0285 | 0.0218 | 0.2395 | 0.1177 | 0.0285 | 0.0219 | 0.2060 | 0.1070 |
| $\sigma_J = 1.2$ | -0.0047 | -0.0007 | 0.1406 | 0.0710 | 0.0822 | 0.0818 | 0.1699 | 0.1137 | 0.0401 | 0.0428 | 0.1513 | 0.0859 |

Table 3: Monte Carlo Experiment with Scenario (c)

| Method | Maximum Likelihood Estimation | | | | Based on Bi-Power Variation | | | | Based on Swap Variance Contract | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Statistics | Mean Bias | | RMSE | | Mean Bias | | RMSE | | Mean Bias | | RMSE | |
| Sample Size | 1000 | 4000 | 1000 | 4000 | 1000 | 4000 | 1000 | 4000 | 1000 | 4000 | 1000 | 4000 |
| | Sampling Frequency $\Delta$ = 5-minute, Level of Significance $\alpha = 0.99$ | | | | | | | | | | | |
| $\lambda_J = 0.05$ | 0.0006 | 0.0003 | 0.0073 | 0.0034 | 0.0055 | 0.0054 | 0.0093 | 0.0065 | 0.0338 | 0.0338 | 0.0349 | 0.0341 |
| $\mu_J = 0.2$ | -0.0010 | -0.0019 | 0.1927 | 0.0987 | -0.0287 | -0.0307 | 0.1859 | 0.0915 | -0.0687 | -0.0709 | 0.1327 | 0.0923 |
| $\sigma_J = 1.2$ | -0.0047 | -0.0007 | 0.1406 | 0.0710 | -0.1123 | -0.0994 | 0.1746 | 0.1200 | -0.3175 | -0.3106 | 0.3395 | 0.3162 |
| | Sampling Frequency $\Delta$ = 1-minute, Level of Significance $\alpha = 0.99$ | | | | | | | | | | | |
| $\lambda_J = 0.05$ | 0.0006 | 0.0003 | 0.0073 | 0.0034 | 0.0064 | 0.0064 | 0.0096 | 0.0074 | 0.0199 | 0.0202 | 0.0214 | 0.0206 |
| $\mu_J = 0.2$ | -0.0010 | -0.0019 | 0.1927 | 0.0987 | -0.0305 | -0.0315 | 0.1848 | 0.0907 | -0.0477 | -0.0507 | 0.1432 | 0.0872 |
| $\sigma_J = 1.2$ | -0.0047 | -0.0007 | 0.1406 | 0.0710 | -0.1048 | -0.0931 | 0.1707 | 0.1148 | -0.2194 | -0.2148 | 0.2535 | 0.2244 |
| | Sampling Frequency $\Delta$ = 5-minute, Level of Significance $\alpha = 0.999$ | | | | | | | | | | | |
| $\lambda_J = 0.05$ | 0.0006 | 0.0003 | 0.0073 | 0.0034 | -0.0066 | -0.0066 | 0.0093 | 0.0074 | 0.0030 | 0.0032 | 0.0078 | 0.0048 |
| $\mu_J = 0.2$ | -0.0010 | -0.0019 | 0.1927 | 0.0987 | 0.0185 | 0.0155 | 0.2381 | 0.1104 | -0.0020 | -0.0051 | 0.1796 | 0.0927 |
| $\sigma_J = 1.2$ | -0.0047 | -0.0007 | 0.1406 | 0.0710 | 0.0502 | 0.0639 | 0.1523 | 0.0974 | -0.0464 | -0.0411 | 0.1467 | 0.0804 |
| | Sampling Frequency $\Delta$ = 1-minute, Level of Significance $\alpha = 0.999$ | | | | | | | | | | | |
| $\lambda_J = 0.05$ | 0.0006 | 0.0003 | 0.0073 | 0.0034 | -0.0036 | -0.0036 | 0.0076 | 0.0049 | -0.0008 | -0.0005 | 0.0070 | 0.0035 |
| $\mu_J = 0.2$ | -0.0010 | -0.0019 | 0.1927 | 0.0987 | 0.0064 | 0.0049 | 0.2233 | 0.1033 | 0.0126 | 0.0069 | 0.1933 | 0.0998 |
| $\sigma_J = 1.2$ | -0.0047 | -0.0007 | 0.1406 | 0.0710 | 0.0246 | 0.0385 | 0.1440 | 0.0814 | 0.0025 | 0.0066 | 0.1422 | 0.0726 |

Table 4: Monte Carlo Experiment with Scenario (d)

| Method | Maximum Likelihood Estimation | | | | Based on Bi-Power Variation | | | | Based on Swap Variance Contract | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Statistics | Mean Bias | | RMSE | | Mean Bias | | RMSE | | Mean Bias | | RMSE | |
| Sample Size | 1000 | 4000 | 1000 | 4000 | 1000 | 4000 | 1000 | 4000 | 1000 | 4000 | 1000 | 4000 |
| | Sampling Frequency $\Delta$ = 5-minute, Level of Significance $\alpha = 0.99$ | | | | | | | | | | | |
| $\lambda_J = 0.05$ | 0.0006 | 0.0003 | 0.0073 | 0.0034 | 0.0089 | 0.0091 | 0.0112 | 0.0098 | 0.0378 | 0.0381 | 0.0389 | 0.0383 |
| $\mu_J = 0.2$ | -0.0010 | -0.0019 | 0.1927 | 0.0987 | -0.0469 | -0.0385 | 0.1689 | 0.0891 | -0.0803 | -0.0830 | 0.1345 | 0.1002 |
| $\sigma_J = 1.2$ | -0.0047 | -0.0007 | 0.1406 | 0.0710 | -0.1294 | -0.1254 | 0.1902 | 0.1419 | -0.3455 | -0.3403 | 0.3651 | 0.3453 |
| | Sampling Frequency $\Delta$ = 1-minute, Level of Significance $\alpha = 0.99$ | | | | | | | | | | | |
| $\lambda_J = 0.05$ | 0.0006 | 0.0003 | 0.0073 | 0.0034 | 0.0085 | 0.0085 | 0.0109 | 0.0092 | 0.0216 | 0.0220 | 0.0230 | 0.0223 |
| $\mu_J = 0.2$ | -0.0010 | -0.0019 | 0.1927 | 0.0987 | -0.0446 | -0.0360 | 0.1702 | 0.0890 | -0.0549 | -0.0581 | 0.1430 | 0.0901 |
| $\sigma_J = 1.2$ | -0.0047 | -0.0007 | 0.1406 | 0.0710 | -0.1156 | -0.1105 | 0.1791 | 0.1287 | -0.2348 | -0.2303 | 0.2659 | 0.2390 |
| | Sampling Frequency $\Delta$ = 5-minute, Level of Significance $\alpha = 0.999$ | | | | | | | | | | | |
| $\lambda_J = 0.05$ | 0.0006 | 0.0003 | 0.0073 | 0.0034 | -0.0028 | -0.0027 | 0.0068 | 0.0042 | 0.0064 | 0.0067 | 0.0099 | 0.0076 |
| $\mu_J = 0.2$ | -0.0010 | -0.0019 | 0.1927 | 0.0987 | -0.0089 | 0.0017 | 0.2038 | 0.1005 | -0.0152 | -0.0203 | 0.1688 | 0.0892 |
| $\sigma_J = 1.2$ | -0.0047 | -0.0007 | 0.1406 | 0.0710 | 0.0167 | 0.0216 | 0.1470 | 0.0747 | -0.0890 | -0.0845 | 0.1610 | 0.1082 |
| | Sampling Frequency $\Delta$ = 1-minute, Level of Significance $\alpha = 0.999$ | | | | | | | | | | | |
| $\lambda_J = 0.05$ | 0.0006 | 0.0003 | 0.0073 | 0.0034 | -0.0019 | -0.0016 | 0.0064 | 0.0036 | 0.0008 | 0.0012 | 0.0072 | 0.0037 |
| $\mu_J = 0.2$ | -0.0010 | -0.0019 | 0.1927 | 0.0987 | -0.0117 | -0.0018 | 0.1999 | 0.0985 | 0.0046 | -0.0017 | 0.1875 | 0.0961 |
| $\sigma_J = 1.2$ | -0.0047 | -0.0007 | 0.1406 | 0.0710 | 0.0139 | 0.0155 | 0.1474 | 0.0727 | -0.0197 | -0.0172 | 0.1422 | 0.0736 |

Table 5: Summary Statistics for Daily Returns and Realized Variances

| Asset Type | S&P500 Index (%) | | T-Bond (%) | | Microsoft (%) | | Brazil Reais (%) | |
|---|---|---|---|---|---|---|---|---|
| Statistics | $\text{Return}_t$ | $\sqrt{\text{RV}_t}$ | $\text{Return}_t$ | $\sqrt{\text{RV}_t}$ | $\text{Return}_t$ | $\sqrt{\text{RV}_t}$ | $\text{Return}_t$ | $\sqrt{\text{RV}_t}$ |
| Mean | 0.0348 | 0.7341 | 0.0164 | 0.5598 | 0.0448 | 2.1890 | 0.0514 | 1.1392 |
| Std. Dev. | 1.0868 | 0.4162 | 0.5995 | 0.2894 | 2.0803 | 0.9186 | 1.2774 | 0.9679 |
| Skewness | -2.1087 | 2.2511 | -0.3418 | 3.3803 | 0.1621 | 4.7522 | 0.9040 | 4.0363 |
| Kurtosis | 48.2123 | 13.2551 | 4.3559 | 24.9531 | 4.0724 | 75.5502 | 18.6239 | 29.3109 |
| Minimum | -22.8867 | 0.1309 | -3.3200 | 0.1327 | -8.8421 | 0.8089 | -10.5039 | 0.0383 |
| 5% Qntl. | -1.6453 | 0.2945 | -0.9900 | 0.2755 | -3.2029 | 1.1742 | -1.7590 | 0.3939 |
| 25% Qntl. | -0.4495 | 0.4473 | -0.3300 | 0.3861 | -1.2896 | 1.6107 | -0.4650 | 0.6125 |
| 50% Qntl. | 0.0524 | 0.6330 | 0.0380 | 0.4932 | 0.0000 | 2.0112 | 0.0000 | 0.8509 |
| 75% Qntl. | 0.5660 | 0.9086 | 0.3900 | 0.6550 | 1.3340 | 2.5817 | 0.5404 | 1.3399 |
| 95% Qntl. | 1.6081 | 1.5189 | 0.9488 | 1.0361 | 3.4989 | 3.6994 | 1.8410 | 2.7629 |
| Maximum | 8.3795 | 5.4363 | 2.2200 | 4.0919 | 10.5769 | 20.8284 | 10.6768 | 11.7541 |
| $\rho_1$ | 0.0146 | 0.7533 | 0.0348 | 0.2415 | -0.0602 | 0.4818 | 0.0224 | 0.5615 |
| $\rho_2$ | -0.0474 | 0.7013 | -0.0138 | 0.2011 | -0.0235 | 0.4046 | -0.0079 | 0.5304 |
| $\rho_3$ | -0.0088 | 0.6669 | -0.0446 | 0.1568 | -0.0212 | 0.3502 | 0.0863 | 0.4902 |
| $\rho_4$ | -0.0208 | 0.6465 | -0.0421 | 0.1651 | 0.0149 | 0.3252 | 0.0947 | 0.5037 |
| $\rho_5$ | -0.0182 | 0.6379 | 0.0002 | 0.1959 | -0.0112 | 0.3304 | 0.0147 | 0.4263 |
| $\rho_6$ | -0.0056 | 0.6164 | -0.0079 | 0.1576 | 0.0390 | 0.3301 | 0.0791 | 0.5040 |
| $\rho_7$ | -0.0431 | 0.6062 | 0.0213 | 0.1260 | 0.0208 | 0.3162 | -0.0460 | 0.4047 |
| $\rho_8$ | 0.0116 | 0.6042 | 0.0013 | 0.1767 | 0.0257 | 0.3018 | 0.0370 | 0.3939 |
| $\rho_9$ | 0.0308 | 0.5917 | 0.0020 | 0.1502 | -0.0009 | 0.2674 | 0.0714 | 0.3506 |
| $\rho_{10}$ | 0.0228 | 0.5819 | 0.0189 | 0.1453 | -0.0057 | 0.2516 | 0.0467 | 0.3454 |

Table 6: Jump Parameter Estimation for Four Assets

| Statistics | S&P500 | T-bond | Microsoft | Brazil Real |
|---|---|---|---|---|
| Mean $\sqrt{\mathrm{RV}_t}$ | 0.7341 | 0.5598 | 2.1890 | 1.1392 |
| Mean RJ in Std | 0.1338 | 0.1361 | 0.0542 | 0.3875 |
| Mean RJ in Var | 0.0785 | 0.1037 | 0.0260 | 0.3007 |
| Trading Days | 4752 | 3376 | 2481 | 1469 |
| **Parameter** | **S&P500** | **T-bond** | **Microsoft** | **Brazil Real** |
| $\lambda_J$ | 0.2319 | 0.2062 | 0.1092 | 0.5133 |
| (s.e.) | (0.0127) | (0.0153) | (0.0189) | (0.0182) |
| $\mu_J$ | 0.0500 | 0.0262 | 0.1141 | 0.0418 |
| (s.e.) | (0.0147) | (0.0169) | (0.0682) | (0.0413) |
| $\sigma_J$ | 0.4885 | 0.4454 | 1.1233 | 1.1345 |
| (s.e.) | (0.0104) | (0.0119) | (0.0482) | (0.0292) |

Table 7: Univariate Prediction of Credit Spreads with Interest Rate and Volatility Factors

| Regressors | Moody's AAA Bond Yield Spread | | | | | |
|---|---|---|---|---|---|---|
| Constant | 1.8733 | 1.1216 | 0.7971 | 0.3414 | 0.5537 | 0.4138 |
| (s.e.) | (0.0126) | (0.0127) | (0.0115) | (0.0148) | (0.0178) | (0.0108) |
| Short Rate | -0.1425 | | | | | |
| (s.e.) | (0.0025) | | | | | |
| Term Spread | | 0.0546 | | | | |
| (s.e.) | | (0.0062) | | | | |
| Short-Run Volatility | | | 0.0359 | | | |
| (s.e.) | | | (0.0009) | | | |
| Long-Run Volatility | | | | 0.0679 | | |
| (s.e.) | | | | (0.0011) | | |
| Implied Volatility | | | | | 0.0316 | |
| (s.e.) | | | | | (0.0008) | |
| Jump Volatility | | | | | | 1.7213 |
| (s.e.) | | | | | | (0.0214) |
| Adj. R-Square | 0.4352 | 0.0181 | 0.2921 | 0.4816 | 0.2679 | 0.6062 |
| Regressors | Moody's BAA Bond Yield Spread | | | | | |
| Constant | 2.7966 | 1.8608 | 1.5473 | 0.8806 | 1.1193 | 1.0076 |
| (s.e.) | (0.0163) | (0.0151) | (0.0139) | (0.0159) | (0.0201) | (0.0112) |
| Short Rate | -0.1577 | | | | | |
| (s.e.) | (0.0032) | | | | | |
| Term Spread | | 0.1196 | | | | |
| (s.e.) | | (0.0073) | | | | |
| Short-Run Volatility | | | 0.0447 | | | |
| (s.e.) | | | (0.0010) | | | |
| Long-Run Volatility | | | | 0.0923 | | |
| (s.e.) | | | | (0.0012) | | |
| Implied Volatility | | | | | 0.0454 | |
| (s.e.) | | | | | (0.0009) | |
| Jump Volatility | | | | | | 2.2772 |
| (s.e.) | | | | | | (0.0221) |
| Adj. R-Square | 0.3594 | 0.0591 | 0.3056 | 0.5996 | 0.3714 | 0.7156 |

Table 8: Univariate Prediction of Credit Spreads with Fama-French and Jump Risk Factors

| Regressors | Moody's AAA Bond Yield Spread | | | | | |
|---|---|---|---|---|---|---|
| Constant | 1.2175 | 1.2173 | 1.2171 | 1.6213 | 1.2342 | 0.4138 |
| (s.e.) | (0.0067) | (0.0067) | (0.0067) | (0.0211) | (0.0101) | (0.0108) |
| Market Return | -0.0061 | | | | | |
| (s.e.) | (0.0068) | | | | | |
| SMB | | 0.0136 | | | | |
| (s.e.) | | (0.0119) | | | | |
| HML | | | 0.0171 | | | |
| (s.e.) | | | (0.0120) | | | |
| Jump Intensity | | | | -1.7271 | | |
| (s.e.) | | | | (0.0862) | | |
| Jump Mean | | | | | -0.2821 | |
| (s.e.) | | | | | (0.1260) | |
| Jump Volatility | | | | | | 1.7213 |
| (s.e.) | | | | | | (0.0214) |
| Adj. R-Square | 0.0001 | 0.0001 | 0.0002 | 0.0869 | 0.0010 | 0.6062 |
| Regressors | Moody's BAA Bond Yield Spread | | | | | |
| Constant | 2.0709 | 2.0706 | 2.0704 | 2.3399 | 2.1735 | 1.0076 |
| (s.e.) | (0.0081) | (0.0081) | (0.0081) | (0.0266) | (0.0121) | (0.0112) |
| Market Return | -0.0077 | | | | | |
| (s.e.) | (0.0083) | | | | | |
| SMB | | 0.0228 | | | | |
| (s.e.) | | (0.0145) | | | | |
| HML | | | 0.0151 | | | |
| (s.e.) | | | (0.0146) | | | |
| Jump Intensity | | | | -1.1512 | | |
| (s.e.) | | | | (0.1084) | | |
| Jump Mean | | | | | -1.7229 | |
| (s.e.) | | | | | (0.1512) | |
| Jump Volatility | | | | | | 2.2772 |
| (s.e.) | | | | | | (0.0221) |
| Adj. R-Square | 0.0000 | 0.0004 | 0.0000 | 0.0259 | 0.0297 | 0.7156 |

Table 9: Multivariate Prediction of Credit Spreads with Interest Rate and Volatility Factors

| Regressors | Moody's AAA Bond Yield Spread | | | | | | |
|---|---|---|---|---|---|---|---|
| Constant | 2.6484 | 0.3196 | 0.2704 | 1.6572 | 1.5038 | 0.3814 | 1.5123 |
| (s.e.) | (0.0208) | (0.0143) | (0.0134) | (0.0285) | (0.0268) | (0.0139) | (0.0262) |
| Short Rate | -0.2243 | | | -0.1555 | -0.1464 | | -0.1356 |
| (s.e.) | (0.0028) | | | (0.0028) | (0.0029) | | (0.0029) |
| Term Spread | -0.2271 | | | -0.1356 | -0.1452 | | -0.1599 |
| (s.e.) | (0.0053) | | | (0.0048) | (0.0043) | | (0.0043) |
| Short-Run Volatility | | 0.0155 | | 0.0128 | | 0.0098 | |
| (s.e.) | | (0.0008) | | (0.0007) | | (0.0010) | |
| Long-Run Volatility | | 0.0556 | | 0.0282 | | -0.0449 | -0.0338 |
| (s.e.) | | (0.0012) | | (0.0010) | | (0.0026) | (0.0024) |
| Implied Volatility | | | 0.0112 | | 0.0150 | 0.0104 | 0.0173 |
| (s.e.) | | | (0.0007) | | (0.0006) | (0.0010) | (0.0006) |
| Jump Volatility | | | 1.5248 | | 0.7012 | 2.3203 | 1.4642 |
| (s.e.) | | | (0.0236) | | (0.0268) | (0.0541) | (0.0611) |
| Adj. R-Square | 0.6089 | 0.5199 | 0.6319 | 0.7338 | 0.7726 | 0.6659 | 0.7825 |
| Regressors | Moody's BAA Bond Yield Spread | | | | | | |
| Constant | 3.2851 | 0.8590 | 0.7594 | 1.3596 | 1.1138 | 0.8291 | 1.0900 |
| (s.e.) | (0.0311) | (0.0154) | (0.0130) | (0.0360) | (0.0311) | (0.0135) | (0.0215) |
| Short Rate | -0.2093 | | | -0.0756 | -0.0567 | | -0.0456 |
| (s.e.) | (0.0042) | | | (0.0036) | (0.0033) | | (0.0030) |
| Term Spread | -0.1432 | | | 0.0262 | 0.0083 | | |
| (s.e.) | (0.0079) | | | (0.0061) | (0.0050) | | |
| Short-Run Volatility | | 0.0153 | | 0.0180 | | -0.0021 | -0.0037 |
| (s.e.) | | (0.0009) | | (0.0008) | | (0.0010) | (0.0009) |
| Long-Run Volatility | | 0.0801 | | 0.0622 | | -0.0516 | -0.0320 |
| (s.e.) | | (0.0013) | | (0.0013) | | (0.0025) | (0.0028) |
| Implied Volatility | | | 0.0194 | | 0.0253 | 0.0273 | 0.0302 |
| (s.e.) | | | (0.0006) | | (0.0007) | (0.0010) | (0.0009) |
| Jump Volatility | | | 1.9370 | | 1.4392 | 2.9100 | 2.1696 |
| (s.e.) | | | (0.0229) | | (0.0311) | (0.0524) | (0.0702) |
| Adj. R-Square | 0.4058 | 0.6248 | 0.7676 | 0.7127 | 0.7929 | 0.7885 | 0.7997 |

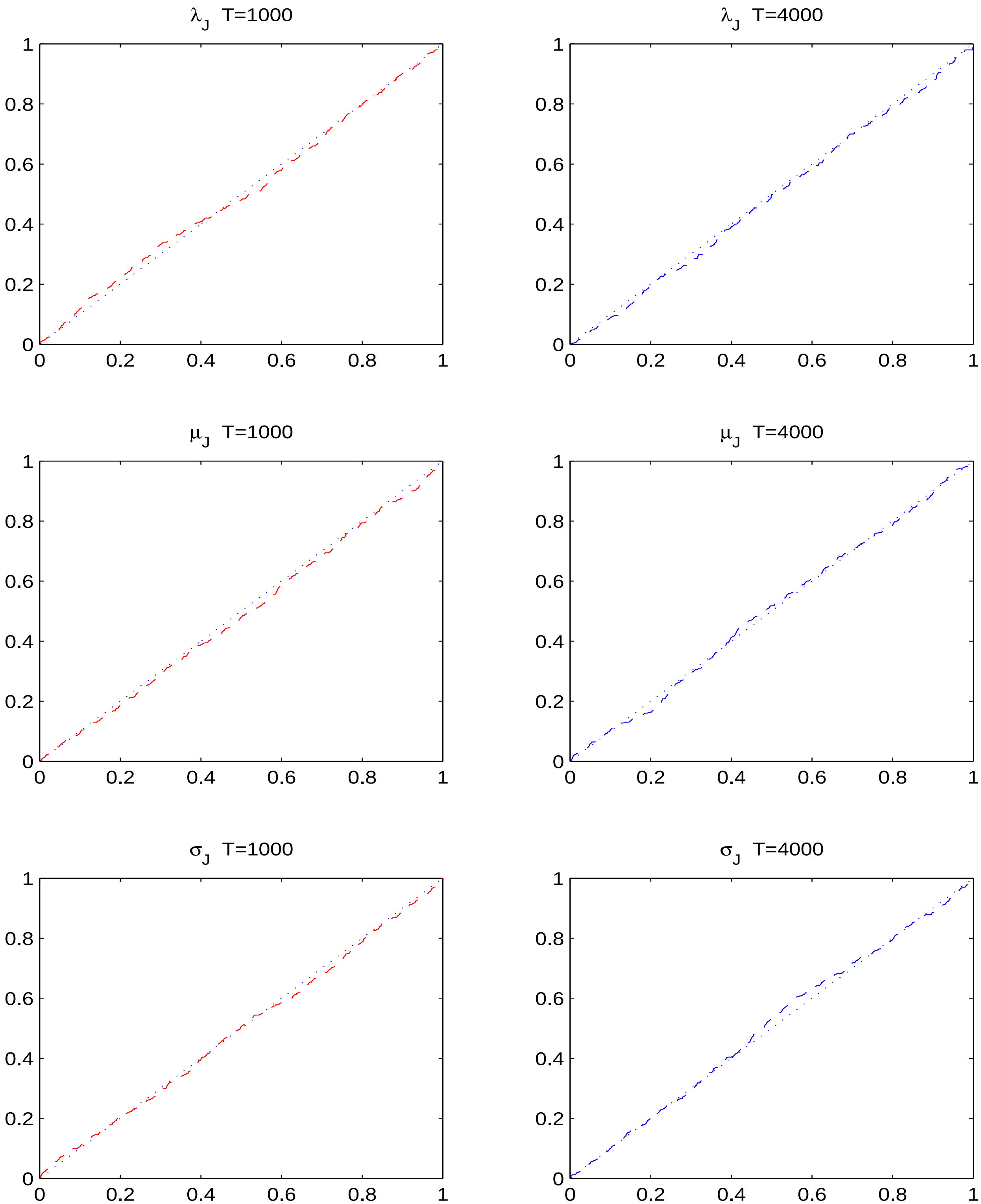



Figure 1: Asymptotic Wald Test with Maximum Likelihood Estimator

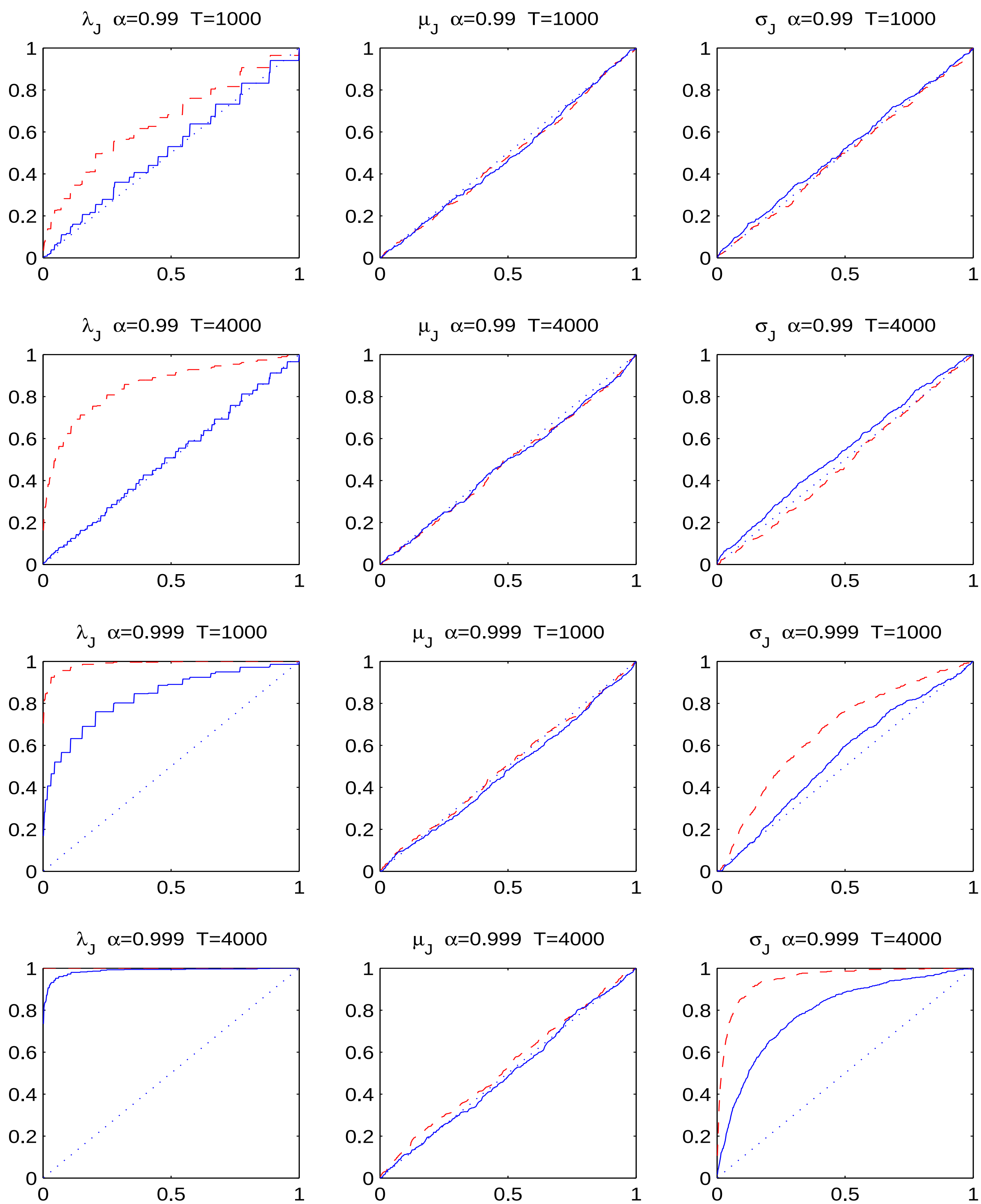



Figure 2: Asymptotic Wald Test for Bi-Power Variation Approach with Scenario (a) The relative contribution of diffusion and jump to variance is 90% versus 10%. The dotted line is the reference Uniform distribution, the dash line is for sampling interval $\Delta = 5$-minute, and the solid line is for sampling interval $\Delta = 1$-minute.

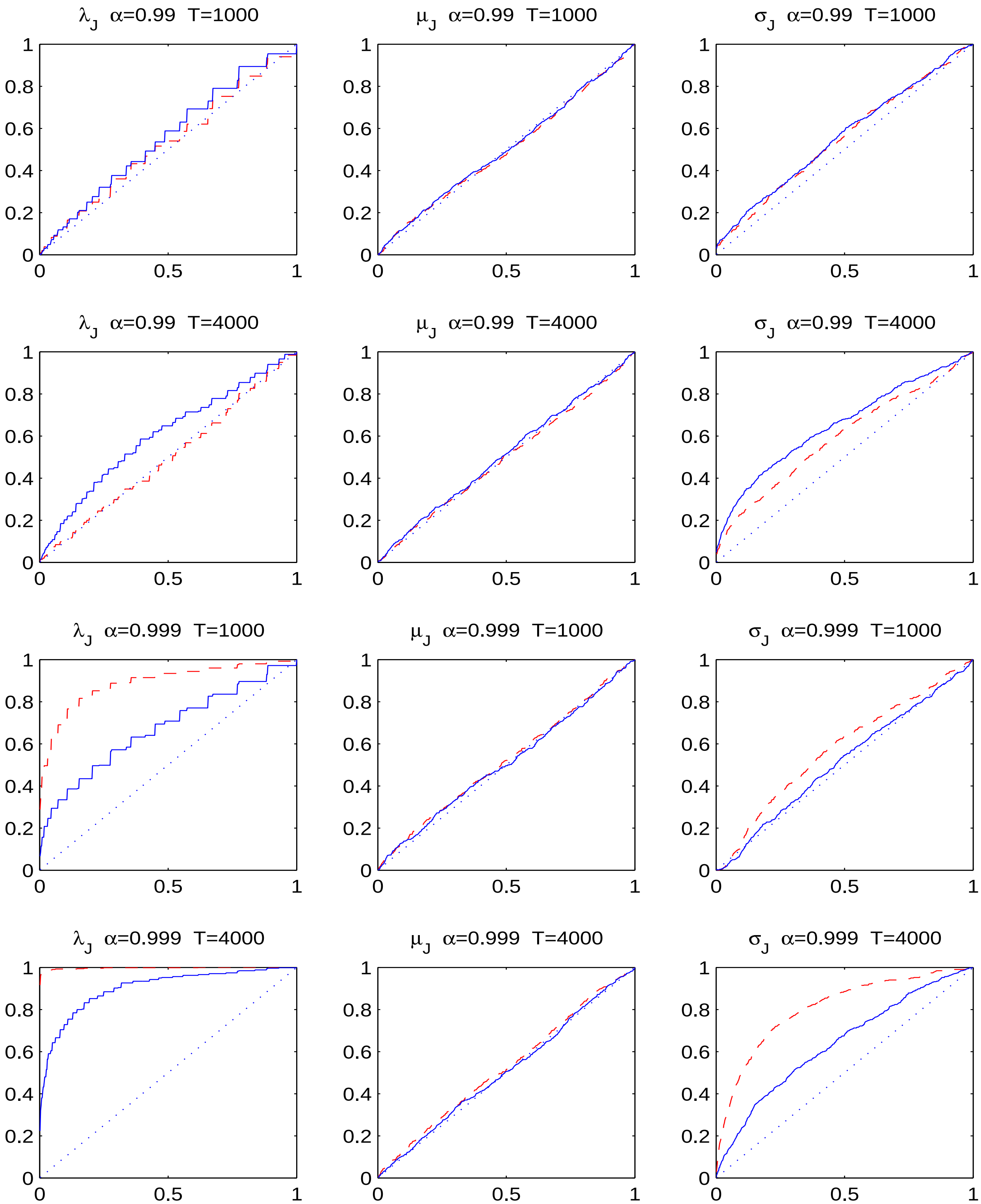



Figure 3: Asymptotic Wald Test for Bi-Power Variation Approach with Scenario (b) The relative contribution of diffusion and jump to variance is 80% versus 20%. The dotted line is the reference Uniform distribution, the dash line is for sampling interval $\Delta$ = 5-minute, and the solid line is for sampling interval $\Delta$ = 1-minute.

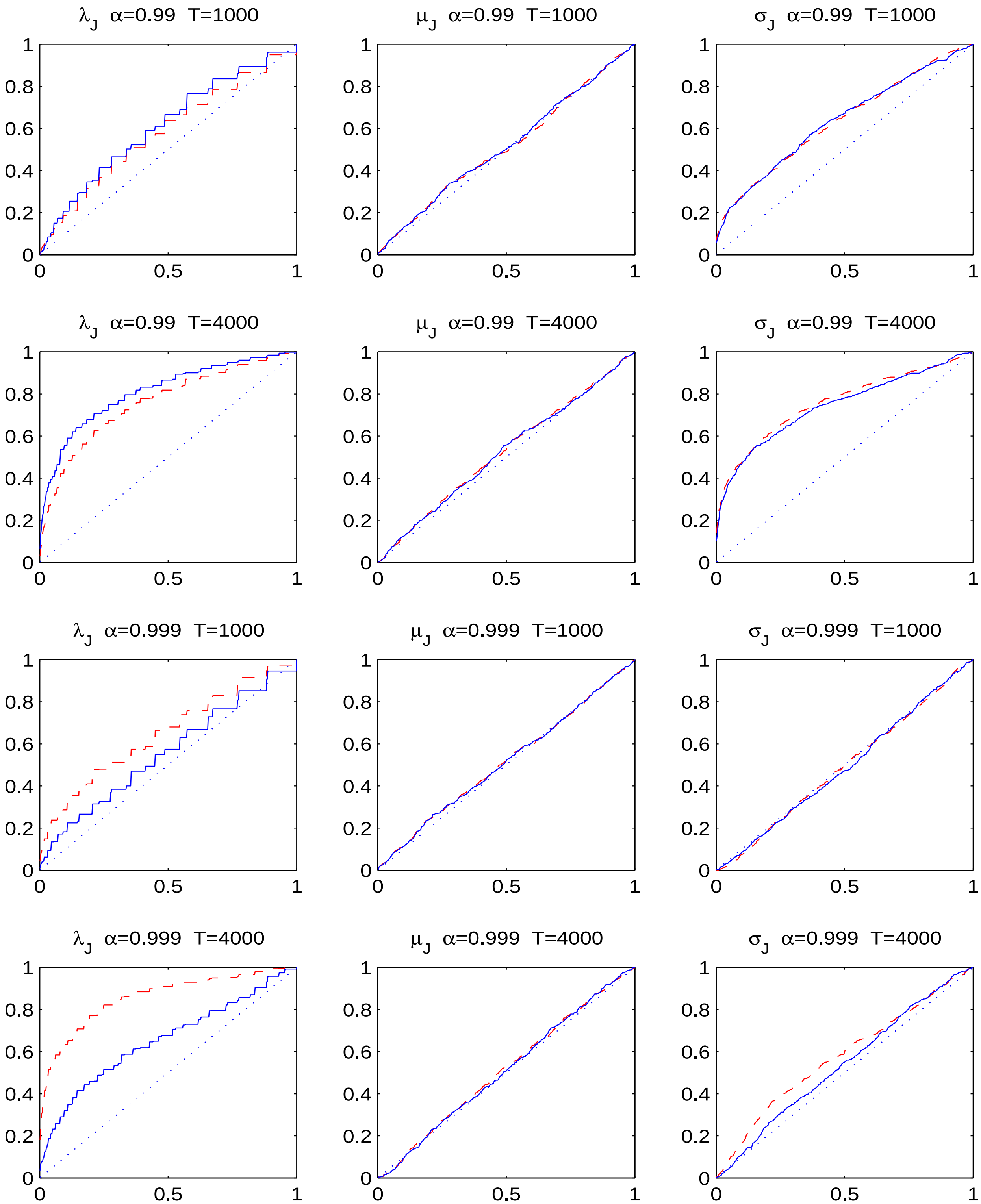



Figure 4: Asymptotic Wald Test for Bi-Power Variation Approach with Scenario (c) The relative contribution of diffusion and jump to variance is 50% versus 50%. The dotted line is the reference Uniform distribution, the dash line is for sampling interval $\Delta = 5$-minute, and the solid line is for sampling interval $\Delta = 1$-minute.

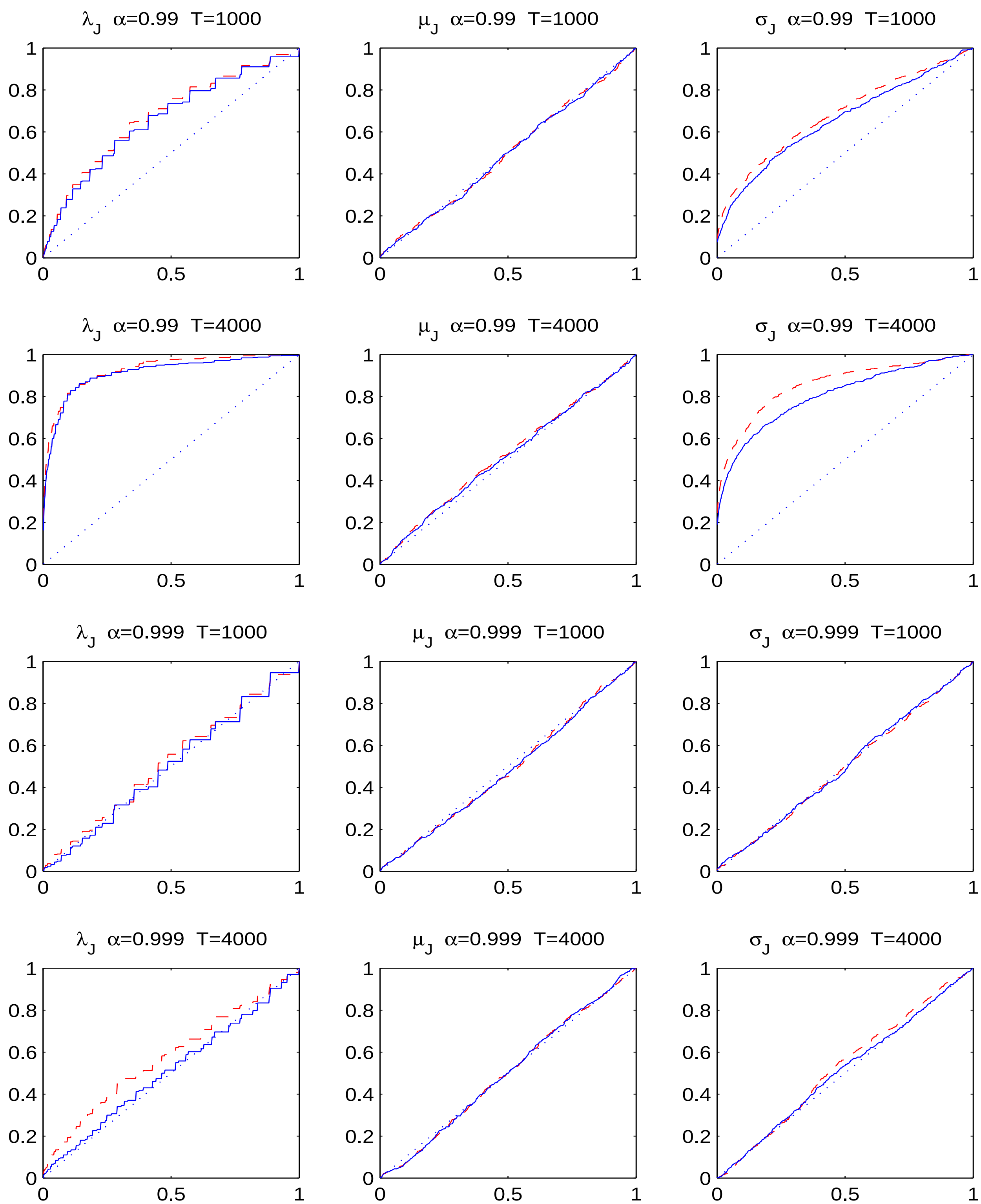



Figure 5: Asymptotic Wald Test for Bi-Power Variation Approach with Scenario (d) The relative contribution of diffusion and jump to variance is 20% versus 80%. The dotted line is the reference Uniform distribution, the dash line is for sampling interval $\Delta = 5$-minute, and the solid line is for sampling interval $\Delta = 1$-minute.

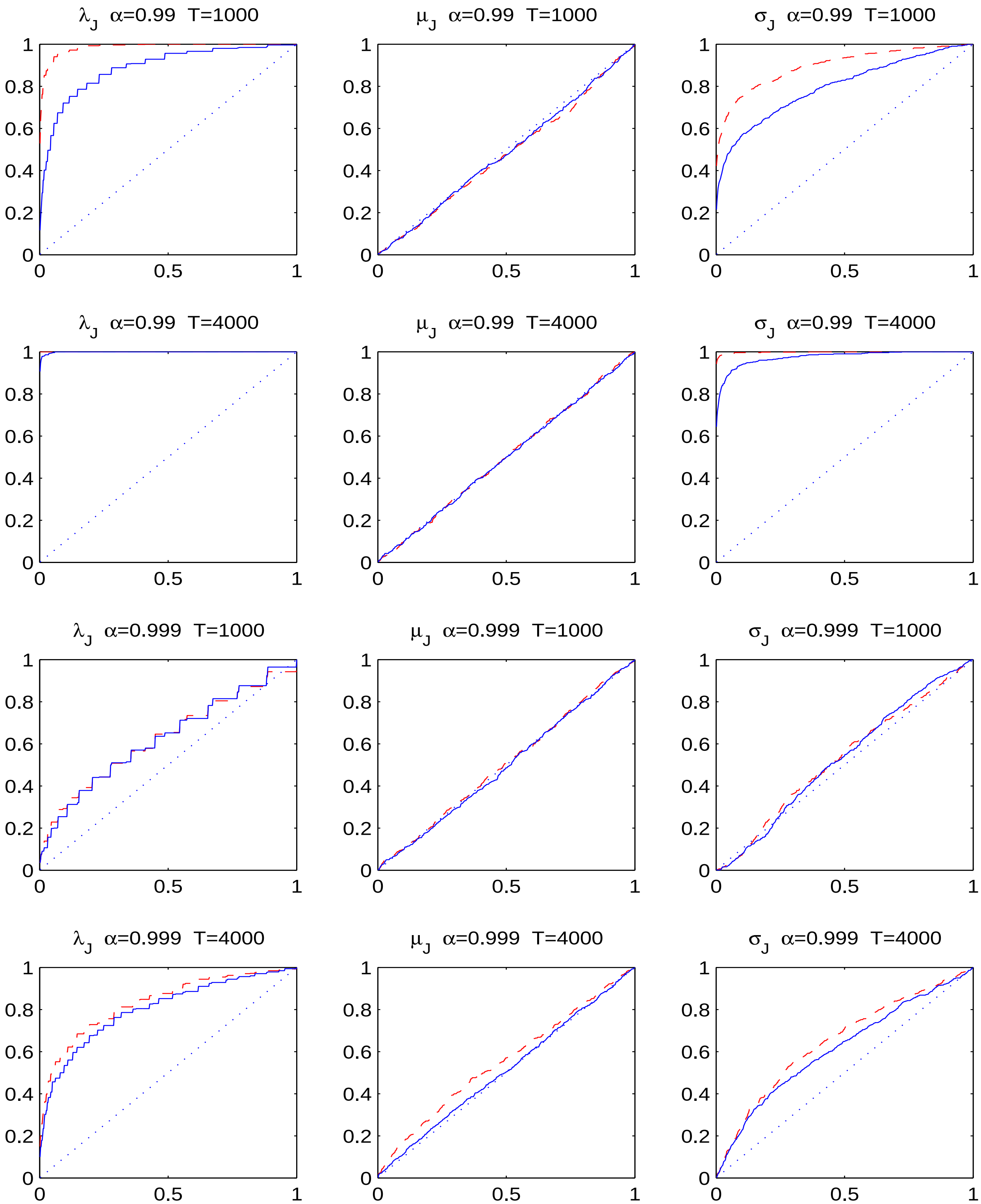



Figure 6: Asymptotic Wald Test for Swap Variance Approach with Scenario (a)
The relative contribution of diffusion and jump to variance is 90% versus 10%. The dotted line is the reference Uniform distribution, the dash line is for sampling interval $\Delta = 5$-minute, and the solid line is for sampling interval $\Delta = 1$-minute.

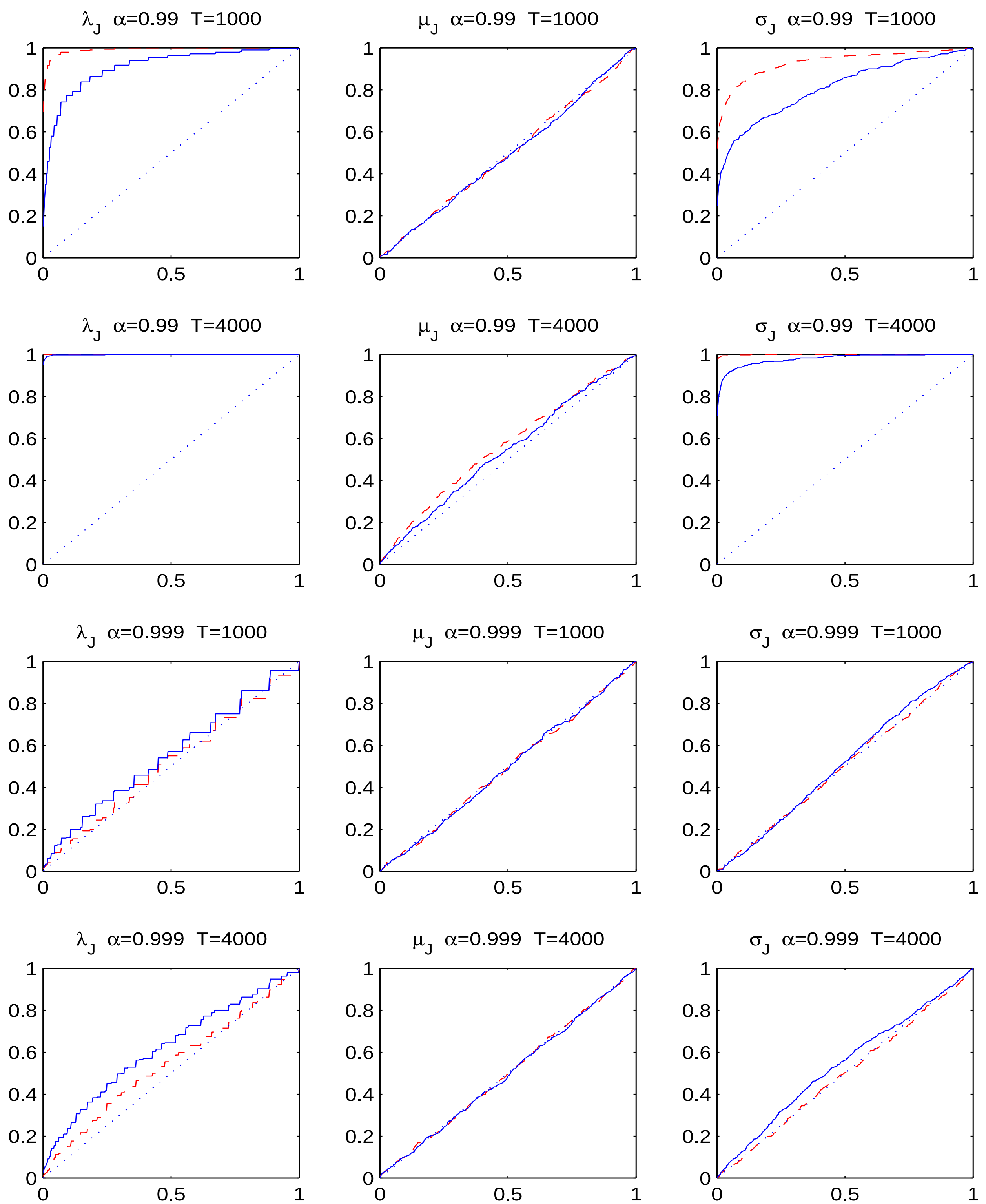



Figure 7: Asymptotic Wald Test for Swap Variance Approach with Scenario (b)
The relative contribution of diffusion and jump to variance is 80% versus 20%. The dotted line is the reference Uniform distribution, the dash line is for sampling interval $\Delta = 5$-minute, and the solid line is for sampling interval $\Delta = 1$-minute.

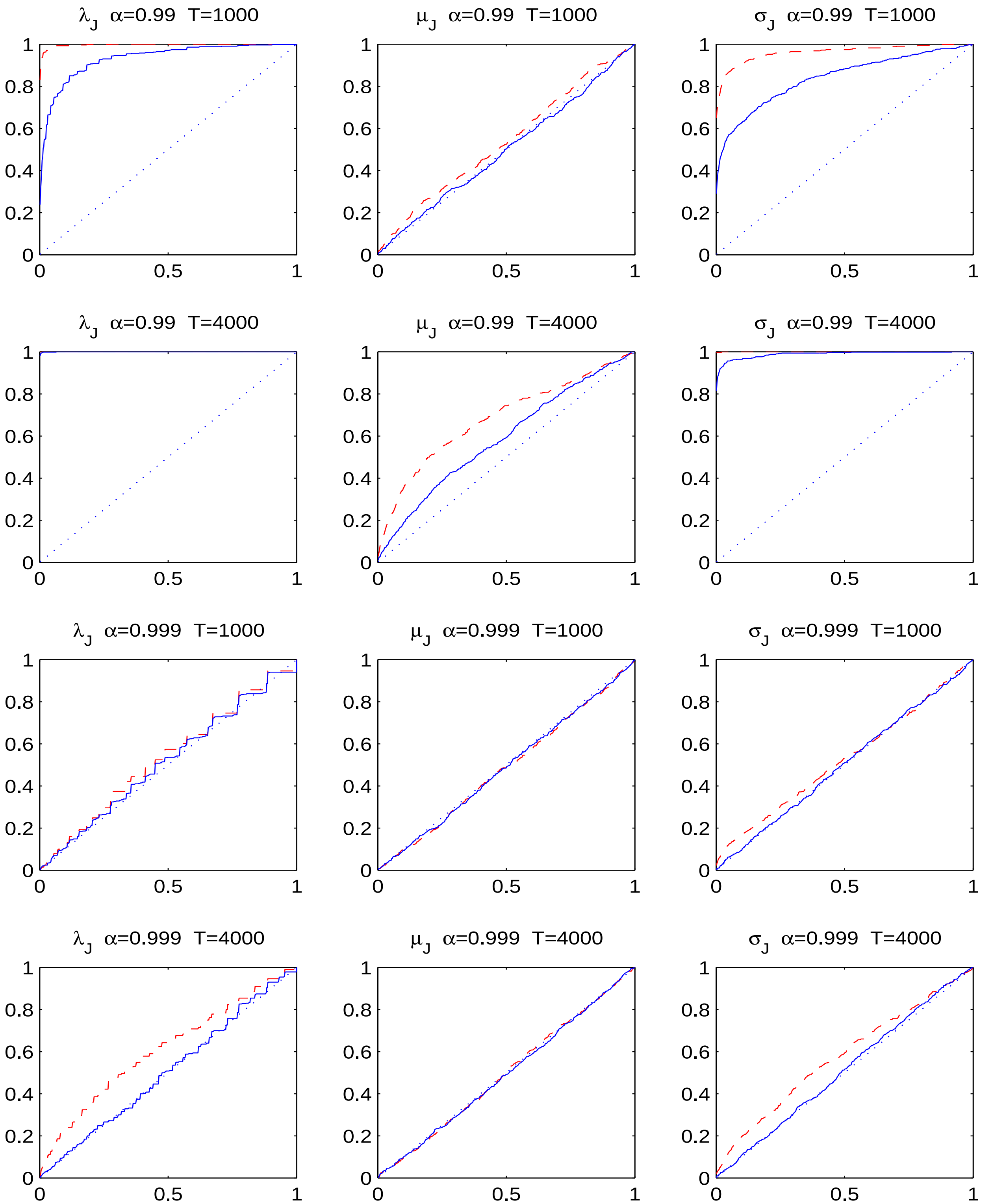



Figure 8: Asymptotic Wald Test for Swap Variance Approach with Scenario (c)
The relative contribution of diffusion and jump to variance is 50% versus 50%. The dotted line is the reference Uniform distribution, the dash line is for sampling interval $\Delta = 5$-minute, and the solid line is for sampling interval $\Delta = 1$-minute.

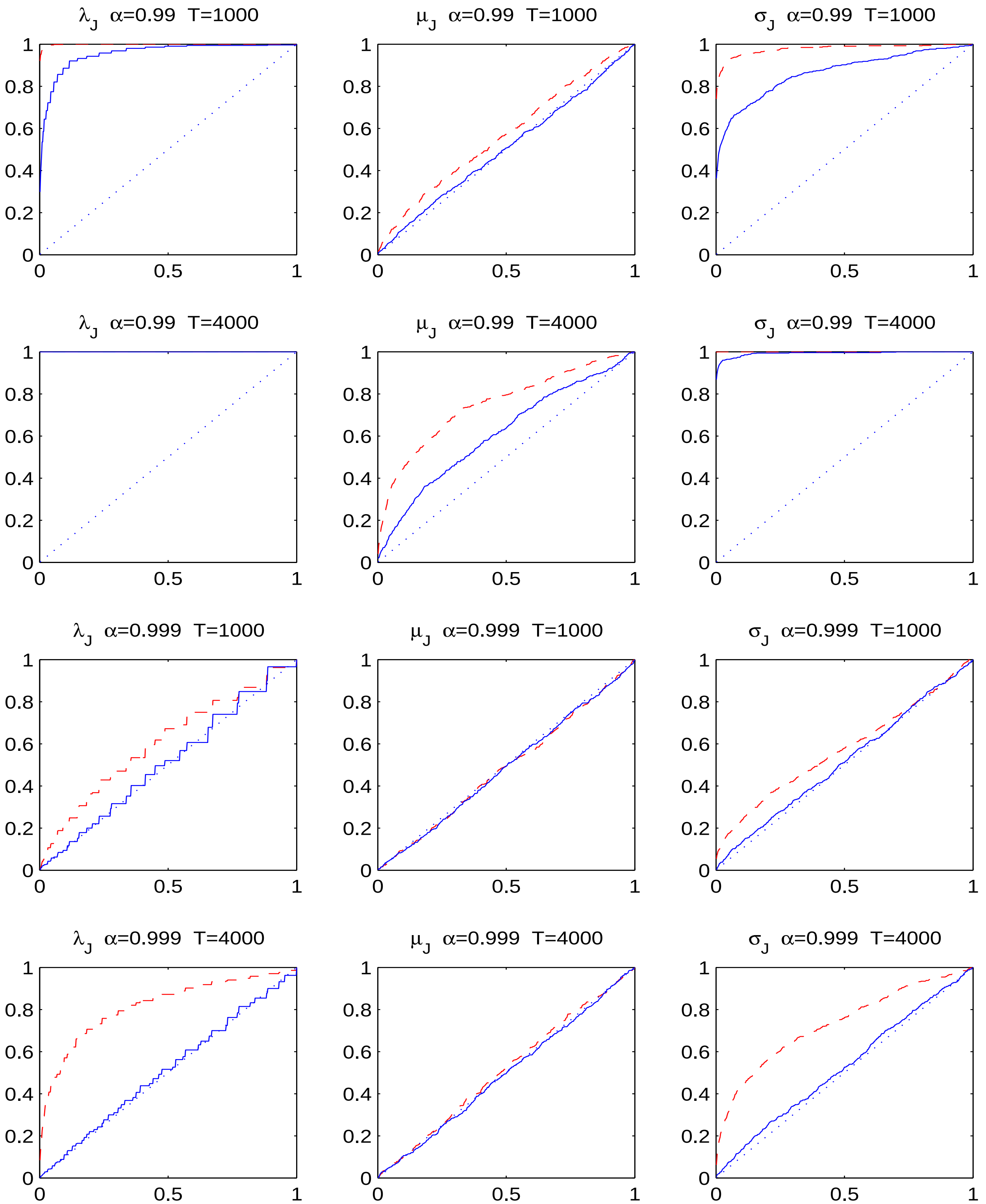



Figure 9: Asymptotic Wald Test for Swap Variance Approach with Scenario (d)
The relative contribution of diffusion and jump to variance is 20% versus 80%. The dotted line is the reference Uniform distribution, the dash line is for sampling interval $\Delta = 5$-minute, and the solid line is for sampling interval $\Delta = 1$-minute.

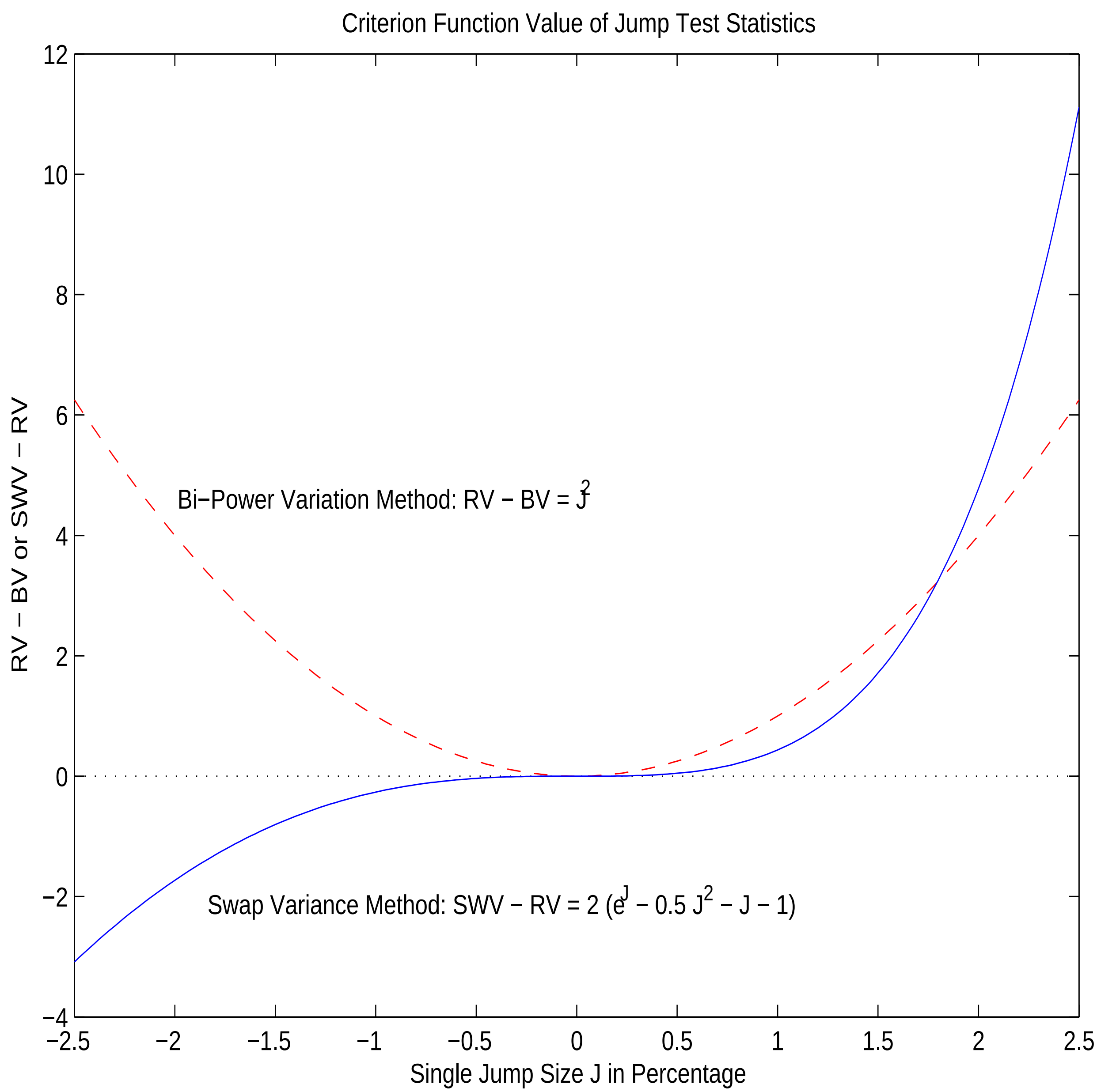



Figure 10: Criterion Function Value of the Jump Test Statistics

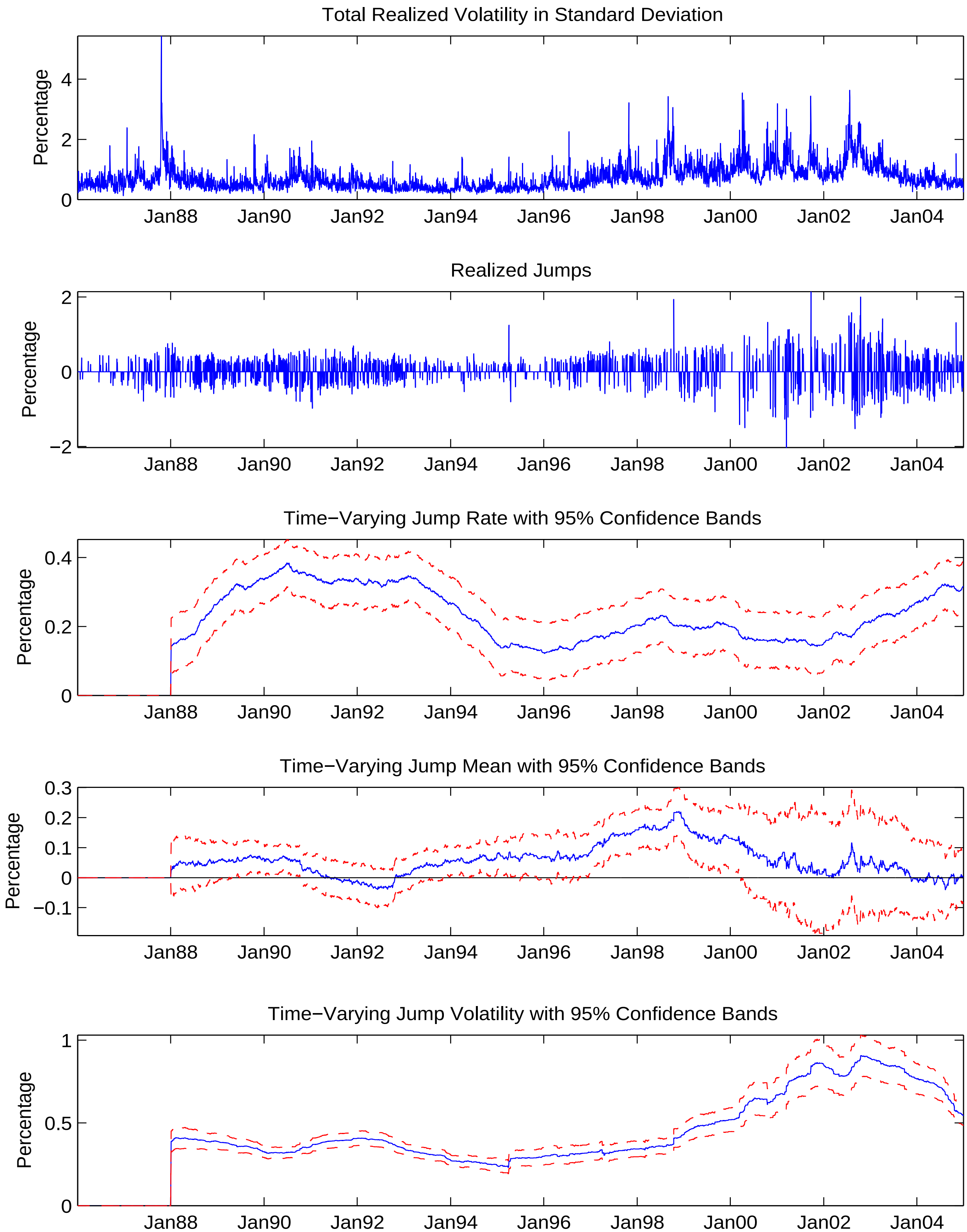



Figure 11: S&P500 Realized Variance, Realized Jumps, and Temporal Variation

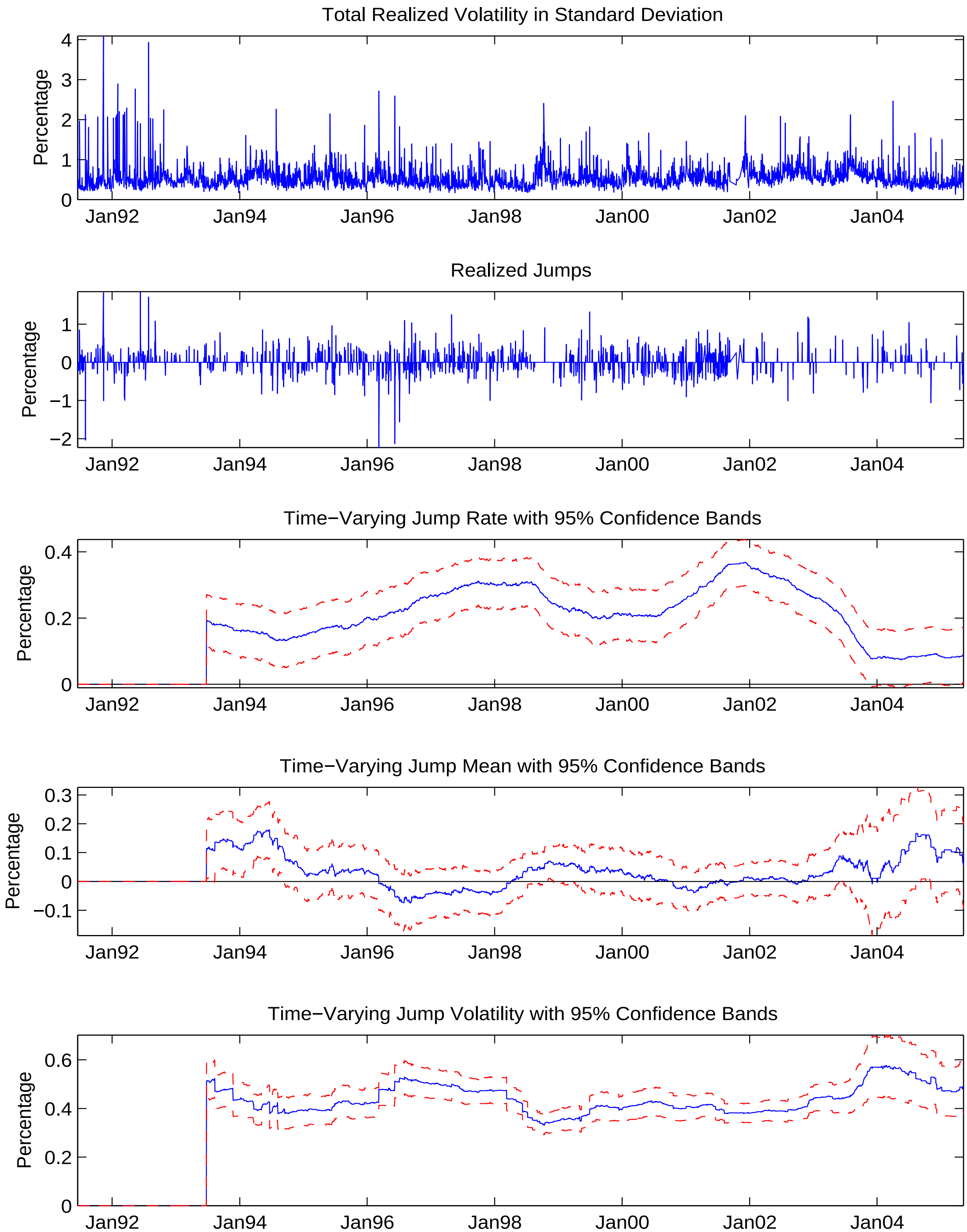



Figure 12: Treasury Bond Realized Variance, Realized Jumps, and Temporal Variation

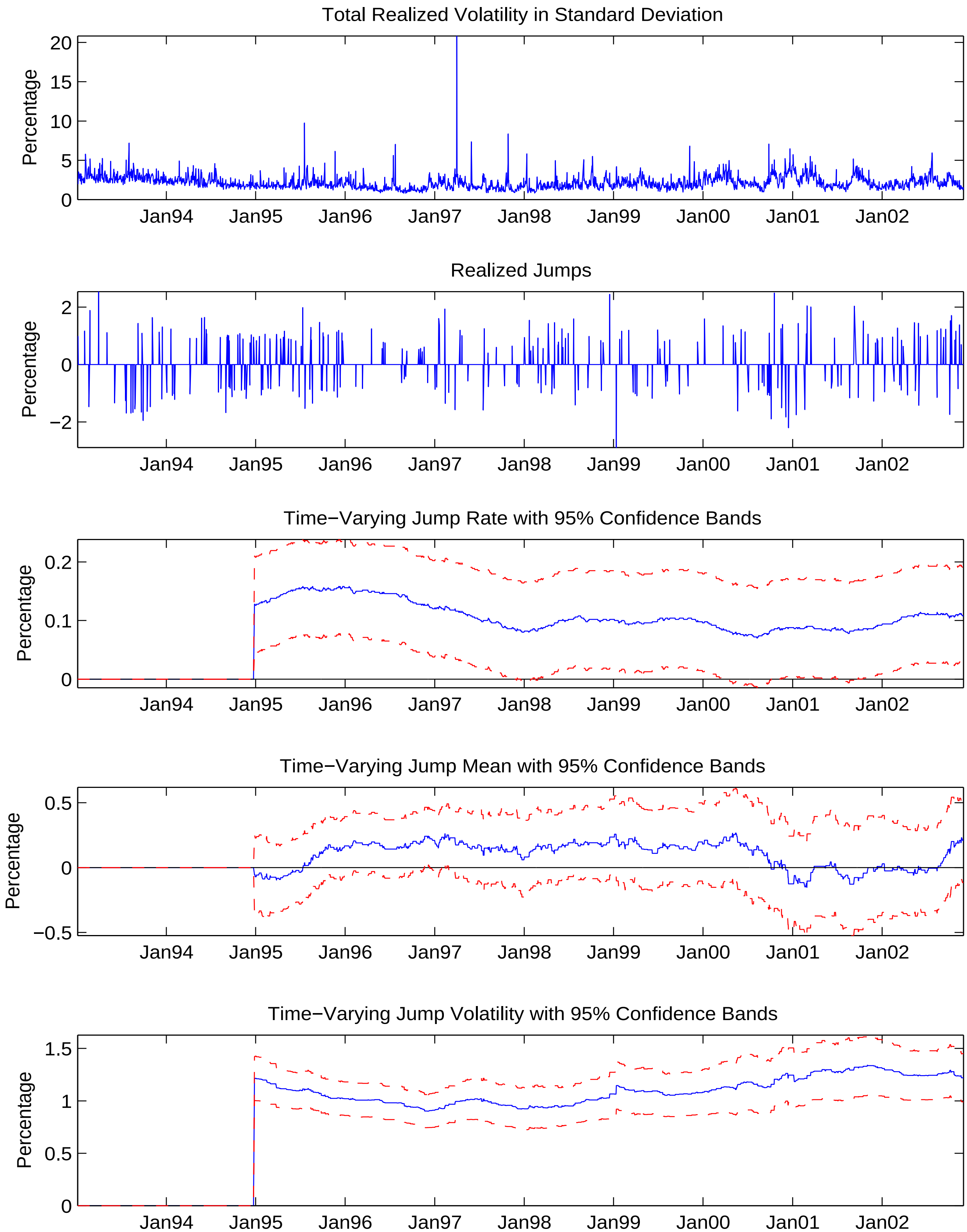



Figure 13: Microsoft Stock Realized Variance, Realized Jumps, and Temporal Variation

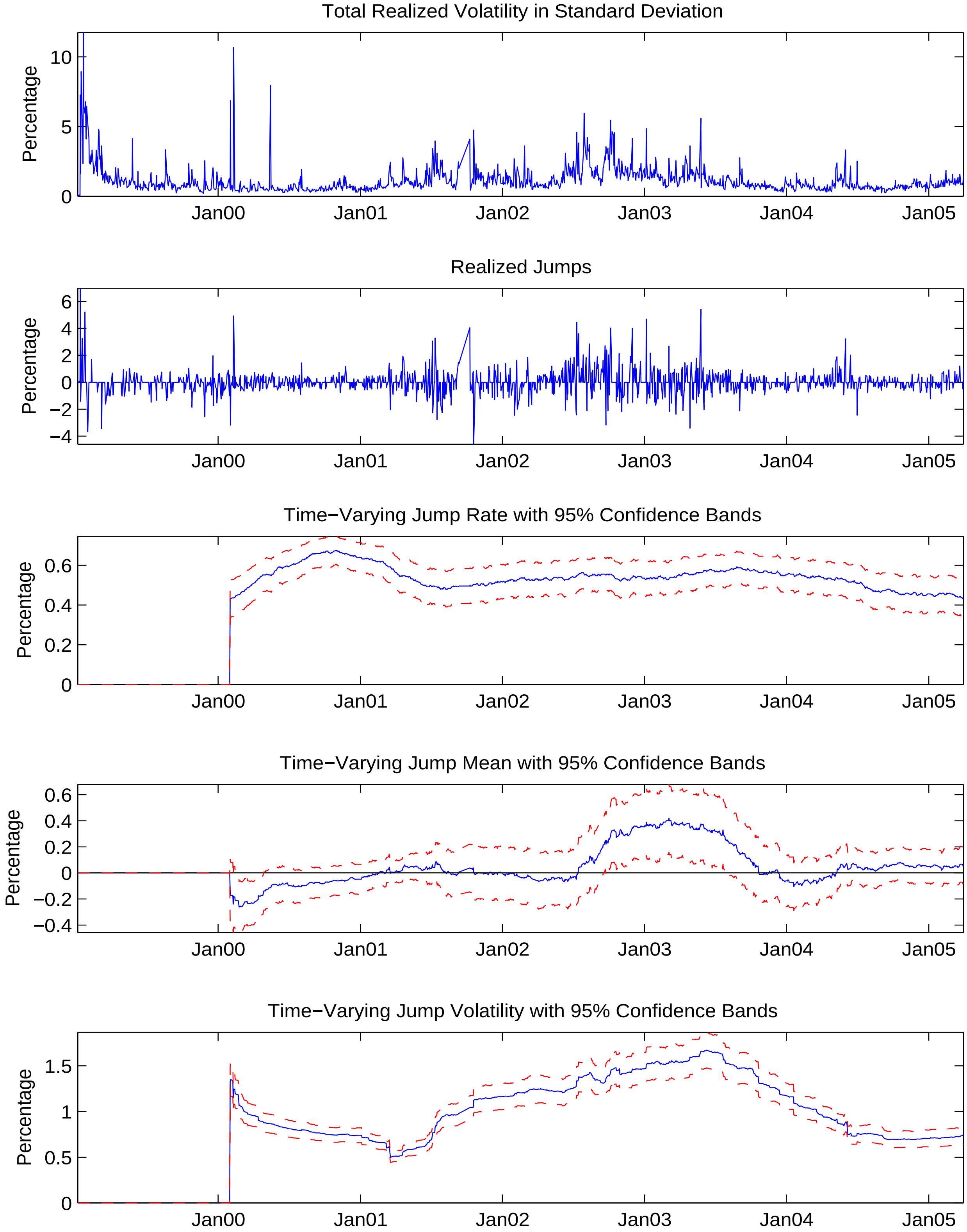
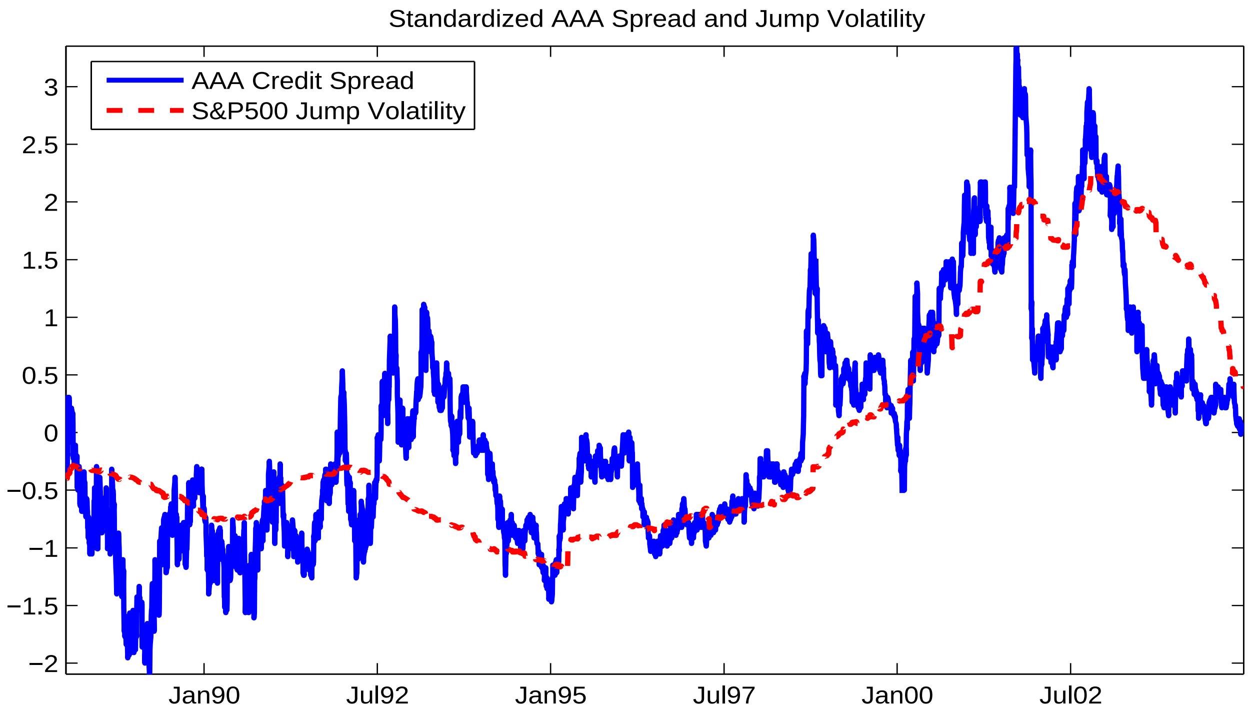



Figure 14: Brazil Real Realized Variance, Realized Jumps, and Temporal Variation



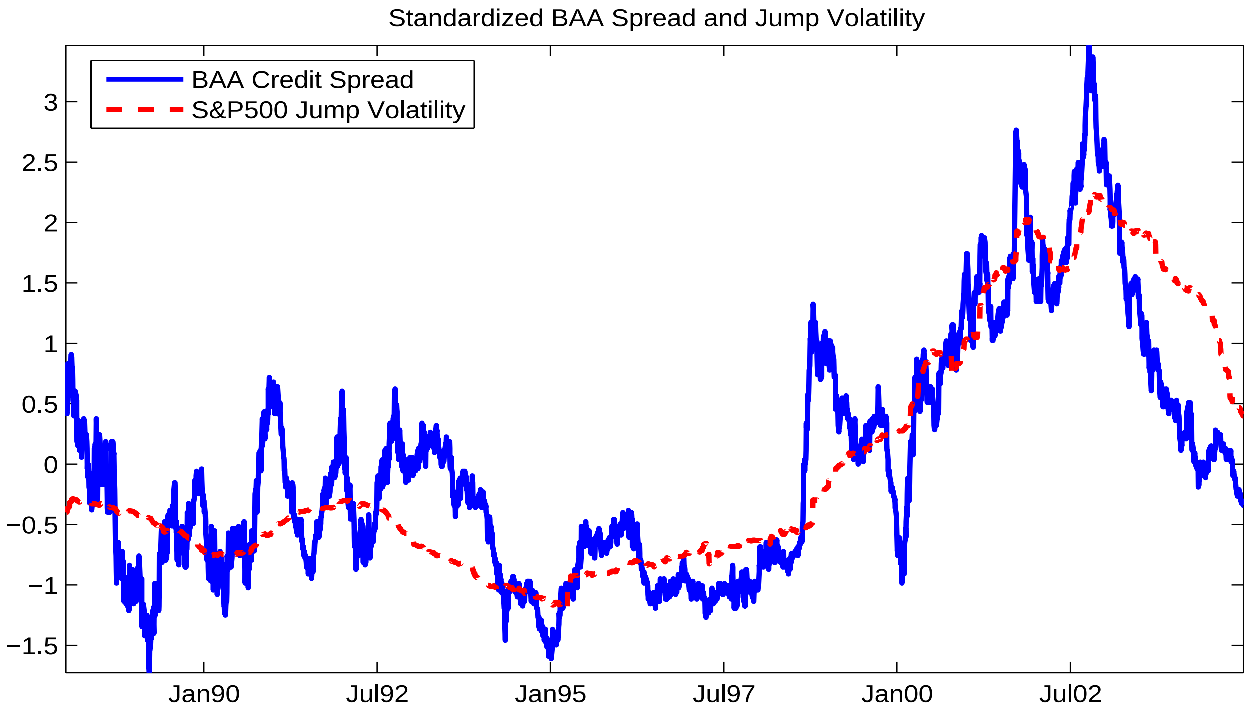




Figure 15: Bond Spread and Jump Volatility