

**A No-Arbitrage Analysis of the Economic Determinants  
of the Credit Spread Term Structure\***

LIUREN WU<sup>†</sup>

*Zicklin School of Business, Baruch College*

FRANK XIAOLING ZHANG<sup>‡</sup>

*Federal Reserve Board*

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<sup>†</sup>One Bernard Baruch Way, Box B10-225, New York, NY 10010-5585; Tel: (646) 312-3509; Fax: (646) 312-3451; Email: Liuren.Wu@baruch.cuny.edu; Homepage: <http://faculty.baruch.cuny.edu/lwu/>.

<sup>‡</sup>Division of Research and Statistics, Federal Reserve Board, Washington, DC 20551; Tel: (202) 452-2581; Fax: (202) 452-5295; Email: Xiaoling.Zhang@frb.gov.

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## **ABSTRACT**

We summarize the state of the economy using three default-free interest-rate factors, two macroeconomic factors, and two financial factors. We then build a no-arbitrage model that links the dynamics and market prices of these factors to the term structure of credit spreads for corporate bonds at different credit rating groups. Estimation shows that credit spreads decline with increasing interest-rate levels and flattening term structure slopes, but increase with rising financial leverage and, to a lesser extent, with rising stock market volatility. Upward shocks on inflation strongly narrow the credit spread at short maturities, but their impacts on long-term spreads are close to zero, thus generating a strong slope effect on the credit spread term structure.

*JEL Classification:* E43; G12; G13.

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# **A No-Arbitrage Analysis of the Economic Determinants of the Credit Spread Term Structure**

Numerous empirical studies, mostly based on regression analysis, show that the frequency of credit events and the expected loss from such events depend crucially on the state of the macroeconomy, the current interest-rate levels and the term structure, and the state of the financial market. The objective of this paper is to quantify, in an internally consistent manner, the link between the dynamics and market prices of the macroeconomic factors and the credit spreads on corporate bonds at different maturities and across different credit ratings.

The task at hand is challenging. On the one hand, many macroeconomic numbers, interest rate series, and financial market variables are available. Each series contains some information, and also possibly a tremendous amount of noise, about the state of the economy. It is inefficient to focus merely on one or a few of these variables while discarding many others. Meanwhile, it is unrealistic to incorporate all of them as state variables into a formal model of credit spreads. Therefore, how to identify the systematic movements from the many noise series poses the first challenge. On the other hand, the pricing of credit risk that is embedded in the prices of many corporate bonds is likely to be different for bonds at different maturities and credit ratings. Picking any one maturity and/or credit rating group will not reveal the complete picture on how credit risk is priced across different rating groups and at different investment horizons. Hence, how to consistently summarize and quantify the impacts of the many variables on the whole term structure of credit spreads across different credit rating groups poses another challenge.

In this paper, we handle both challenges by building a dynamic factor model of credit spreads. First, we extract a small number of dynamic factors from a wide array of macroeconomic and financial series. Thus, through a dynamic factor structure, we succinctly summarize the information content in many series into a small number of systematic factors. Second, we propose flexible specifications on how these dynamic economic and financial factors are priced and how the instantaneous credit spreads respond to these factors. Given these specifications, we use no-arbitrage arguments as in Duffie and Singleton (1999) and Duffie, Pedersen, and Singleton (2003) to derive the whole term structures of

credit spreads as functions of these dynamic factors. Therefore, through a small number of dynamic factors, we are able to make an internally consistent analysis of the impacts of the large number of macroeconomic and financial variables on the credit spreads across the whole spectrum of maturities and credit rating groups.

Our estimation involves two sequential steps. During the first step, we extract three default-free interest-rate factors, two macroeconomic factors, and two financial factors from 30 observed series using maximum likelihood method joint with Kalman filter. Specifically, we extract three risk-free interest-rate factors from 12 Treasury yields with maturities from three months to ten years. We also use these three interest-rate factors as our base to construct the credit spreads. In addition, we extract one inflation factor from year-over-year percentages changes on seven inflation indexes: the consumer price index (CPI), the core CPI, the producer price index (PPI), the core PPI, the personal consumption expenditure (PCE) deflator, the core PCE deflator, and the gross domestic production (GDP) deflator. We extract one real output growth factor from four year-to-year growth rate series on the real GDP, industrial production, non-farm payrolls, and the real PCE. We extract a financial leverage factor from five financial leverage measures: the market value debt/net worth ratio, the book value debt/net worth ratio, the debt/equity ratio, the financing gap/GDP ratio, and ratio of change in total debt over GDP. Finally, we extract a financial market volatility factor from two volatility indexes: the VXO index computed from options on S&P 100 index, and the VIX volatility index computed from options on S&P 500 index. For stability, we identify the three interest-rate factors, and each of the four macroeconomic and financial factors separately. Furthermore, the three default-free interest-rate factors are cast into a dynamic term structure model so that the factor loadings across the whole term structure of Treasury yields are internally consistent and exclude arbitrage opportunities.

During the second step, we cast the seven factors in a no-arbitrage framework to derive the whole term structure of credit spreads at different credit rating groups as a function of these factors. The factor loadings on the credit spreads depend on the factor dynamics, their market prices, and the instantaneous credit spread as a function of these factors at each credit rating group. We estimate the model parameters with maximum likelihood method and Kalman filter using corporate bond yield data

with maturities from one to ten years at each of the four credit rating groups: AAA, AA, A, and BBB. Based on the estimated model parameters, we derive the impacts of the seven factors on the whole term structure of credit spreads at each of the four credit rating groups.

Estimation shows that credit spreads decline with increasing interest-rate levels and flattening term structure slopes, but increase with rising financial leverage and, to a lesser extent, with rising stock market volatility. Upward shocks on inflation strongly narrow the credit spread at short maturities, but their impacts on long-term spreads are close to zero, thus generating a strong slope effect on the credit spread term structure. On the other hand, the impacts of real output growth and the interest-rate curvature factors are fairly small.

Our work in this paper synergizes two strands of extant literature. The first strand uses regressions to analyze the determinants of changes in credit spreads. A partial list of studies along this line of research includes, Bevan and Garzarelli (2000), Frye (2000), Carey (1998), Pedrosa and Roll (1998), Collin-Dufresne, Goldstein, and Martin (2001), Elton, Gruber, Agrawal, and Mann (2001), and Altman, Brady, Resti, and Sironi (2004). Though rich in economic intuition, the results of these studies often depend on the specific choices of the explanatory variables, as well as the choices of the maturity and credit rating of the credit spreads used as the dependent variable. Given the correlations among the many commonly used explanatory variables, the regression coefficient estimates often change when one leads another variable in or out of the regression. Furthermore, the coefficients also change when the credit spread dependent variable switches maturities or credit rating groups. One needs a dramatic leap of faith to extend the regression results from one maturity to other maturities.

The second strand of literature uses a small number of dynamic factors to summarize the variation on the term structure of interest rates and credit spreads via no arbitrage arguments. Prominent examples include Jones, Mason, and Rosenfeld (1984), Longstaff and Schwartz (1995), Duffie and Singleton (1997), Duffee (1999), Nickell, Perraudin, and Varotto (2000), Liu, Longstaff, and Mandell (2000), Delianedis and Geske (2001), Bangia, Diebold, Kronimus, Schagen, and Schuermann (2002), Collin-Dufresne, Goldstein, and Helwege (2003), Huang and Huang (2003), Bakshi, Madan, and Zhang (2004), Longstaff, Mithal, and Neis (2004), Eom, Helwege, and Huang (2003), and Longstaff, Mithal,

and Neis (2004). Different from the regression analysis, these studies can derive the impacts of the dynamic factors on the whole term structure of interest rates and credit spreads in an internally consistent manner. However, most of these studies rely on latent factors, directly derived from the yield curve and credit spread term structure. The economic meanings of these latent factors are not clear. In the few studies that try to incorporate economic variables, often only a small number of observable variables are included for tractability reasons, and other valuable economic variables are conspicuously left out. In this paper, we exploit the advantages of both strands of studies. On the one hand, we use a few dynamic factors to summarize the information and suppress the noises in many observed macroeconomic and financial time series. On the other hand, we use the no-arbitrage framework to provide an internally consistent analysis on the impacts of these macroeconomic and financial series across the whole term structure of credit spreads.

The rest of the paper is organized as follows. Section I describes the procedure for extracting the dynamic macroeconomic and financial factors. Section II presents a no-arbitrage model that links the macroeconomic and financial factors to the whole term structure of credit spreads. Section III examines the relationship between credit spreads across different maturities and different rating groups and the extracted economic and financial factors. Section IV concludes.

## **I. Extracting Dynamic Economic and Financial Factors**

We use a dynamic factor model to succinctly summarize the information in many observed macroeconomic and financial series. In a latter section, we study how these dynamic factors affect the term structure of credit spreads at different credit rating groups.

### *A. Estimating Dynamic Factor Models with Maximum Likelihood and Kalman Filter*

Earlier regression analysis show that, besides interest rates, variables that relate to the business cycle also impact credit spread. Furthermore, the classical Merton (1974) structural model suggests that credit spread shall also be functions of financial leverage and asset volatility. Based on these

observations, we consider seven dynamic factors that include (i) two macroeconomic factors, one of which is associated with inflation and the other with real output growth, (ii) two financial factors, one of which is associated with aggregate financial leverage, and the other with the financial market volatility, and (iii) three risk-free interest-rate factors that explain the term structure of Treasury yields.

Formally, we describe the economy by fixing a filtered probability space  $\{\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{0 \leq t \leq \mathcal{T}}\}$ , with some fixed long horizon  $\mathcal{T}$ . We use  $X \in \mathbb{R}^n$  to denote an  $n$ -dimensional vector Markov process that represents the systematic state of the economy. In this paper, we choose  $n = 7$ . We further assume that the state vector  $X$  follows simple VAR(1) dynamics. Under continuous-time notation,  $X$  follows a multi-variate Ornstein-Uhlenbeck process under the statistical measure  $\mathbb{P}$ ,

$$dX_t = -\kappa X_t dt + dW_t, \quad (1)$$

where  $W_t$  denotes an  $n$ -dimensional standard Brownian motion and  $\kappa$  controls the mean-reversion speed of the states. For identification purpose, we normalize the long-run mean of the states  $X$  to zero and also normalize the instantaneous covariance matrix to be an identity matrix.

Next, let  $y \in \mathbb{R}^N$  denote a set of macroeconomic and financial time series. The dimension  $N$  can be very large, and much larger than the dimension of the state of the economy,  $N \gg n$ . In this paper, we choose  $N = 30$ , which includes seven inflation-related series, four output-related series, five aggregate financial-leverage measures, two financial market volatility gauges, and 12 Treasury yields with maturities from three months to ten years. We summarize the systematic movements underlying these economic numbers using the following linear factor structure,

$$y_t = HX_t + e_t, \quad (2)$$

where  $H$  is an  $(N \times n)$  matrix of factor loading coefficients and  $e_t$  denotes an  $(N \times 1)$  vector of idiosyncratic risks, or measurement noises, of the data series. We use  $\mathcal{R}^y = \mathbb{E}[e_t e_t^\top]$  to denote the covariance matrix of the measurement errors. We assume that the measurement errors are independent of the state vector.

If we know the parameters that govern the factor dynamics ( $\kappa$ ), the factor loadings ( $H$ ), and the measurement error variance ( $\mathcal{R}^y$ ), we can infer the systematic states of the economy from the observed data series, with the technique of Kalman filtering.

We rewrite the state dynamics in its discrete-time analog,

$$X_t = \Phi X_{t-1} + \sqrt{Q} \varepsilon_t, \quad (3)$$

where  $\Phi = \exp(-\kappa \Delta t)$ ,  $Q = I \Delta t$ ,  $\varepsilon_t$  denotes an  $(n \times 1)$  iid standard normal random vector,  $\Delta t$  denotes the discrete time interval, and  $I$  denotes an identity matrix of the relevant dimension. With monthly time interval, we set  $\Delta t = 1/12$ .

For Kalman filtering, we regard equation (3) as our state-propagation equation and equation (2) as our measurement equation. Let  $\bar{X}_t, \bar{V}_t, \bar{y}_t, \bar{A}_t$  denote the time- $(t-1)$  ex ante forecasts of time- $t$  values of the systematic factors, the covariance matrix of the systematic factors, the measurement series, and the covariance matrix of the measurement series. Let  $\hat{X}_t$  and  $\hat{V}_t$  denote the ex post update, or filtering, on the systematic factors and their covariance at time  $t$  based on observations ( $y_t$ ) at time  $t$ . The Kalman filter provides the efficient updates on these quantities. Specifically, we have the ex ante predictions as

$$\bar{X}_t = \Phi \hat{X}_{t-1}; \quad (4)$$

$$\bar{V}_t = \Phi \hat{V}_{t-1} \Phi^\top + Q; \quad (5)$$

$$\bar{y}_t = H \bar{X}_t; \quad (6)$$

$$\bar{A}_t = H \bar{V}_t H^\top + \mathcal{R}^y. \quad (7)$$

The ex post filtering updates are,

$$\hat{X}_{t+1} = \bar{X}_{t+1} + K_{t+1} (y_{t+1} - \bar{y}_{t+1}); \quad (8)$$

$$\hat{V}_{t+1} = \bar{V}_{t+1} - K_{t+1} \bar{A}_{t+1} K_{t+1}^\top, \quad (9)$$

where  $K_{t+1} = \bar{V}_{t+1} H^\top (\bar{A}_{t+1})^{-1}$  is the Kalman gain.

Thus, we can obtain a time series of the ex ante forecasts and ex post updates on both the mean and covariance of the systematic factors and the macroeconomic series, via the iterative procedure defined by equations (4) to (9). To estimate model parameters  $\Theta \equiv [\kappa, H, \mathcal{R}^y]$  that govern the factor dynamics and factor loading, we define the monthly log likelihood function by assuming that the forecasting errors on the observed time series are normally distributed,

$$l_{t+1}(\Theta) = -\frac{1}{2} \log |\bar{A}_{t+1}| - \frac{1}{2} \left( (y_{t+1} - \bar{y}_{t+1})^\top (\bar{A}_{t+1})^{-1} (y_{t+1} - \bar{y}_{t+1}) \right). \quad (10)$$

The parameters are estimated by maximizing the sum of the monthly log likelihood values,

$$\Theta = \arg \max_{\Theta} \sum_{t=1}^{T-1} l_{t+1}(\Theta), \quad (11)$$

where  $T$  denotes the number of observations for each series.

In principle, factors can rotate and the loadings can change accordingly without impacting the final result. Such rotations make it difficult to interpret the meanings of the dynamic factors. To improve identification and enhance the economic meaning of the factors, we put constraints on the factor loading matrix. Specifically, we constrain the first factor to have nonzero loadings only on the seven inflation variables, the second factor to have nonzero loadings only on the four output variables, the third factor to have nonzero loadings only on the five financial leverage variables, and the fourth factor to have nonzero loadings only on the two financial market volatility indexes. With these constraints, we extract the four dynamic factors separately, one at a time.

Finally, to extract the three default-free interest-rate factors from Treasury yields, we apply no-arbitrage constraints on the factor loading matrix via a dynamic term structure model. We assume that the instantaneous default-free interest rate is affine in the three interest-rate factors,

$$r(X_{rt}) = a_r + b_r^\top X_{rt}, \quad (12)$$

where  $X_{rt}$  is a subset of  $X_t$  and refers the three interest-rate factors. Furthermore, to derive the Treasury yields as purely a function of these three interest-rate factors, we assume the following dependence structure for the seven factors,

$$\kappa = \begin{bmatrix} \kappa_r & 0 \\ \kappa_{or} & \kappa_o \end{bmatrix} \quad (13)$$

where  $\kappa_r \in \mathbb{R}^{3 \times 3}$  controls the mean-reversion property of the three interest-rate factors,  $\kappa_{or} \in \mathbb{R}^{4 \times 3}$  controls the feedback of the interest-rate factors on other four factors, and  $\kappa_o \in \mathbb{R}^{4 \times 3}$  controls the feedback of the four other factors on themselves. Since the dynamics of the three interest-rate factors do not depend on the other factors, we can derive the Treasury yield as a function of the interest-rate factors only. For identification, we further constrain  $\kappa_r$  to be a lower triangular matrix.

Finally, to close the model, we assume that the factors have affine market price of risk,

$$\gamma(X_t) = \gamma_0 + \gamma_1 X_t, \quad (14)$$

where  $\gamma_0 \equiv [\gamma_{r0}, \gamma_{o0}]^\top$  is an  $(n \times 1)$  vector and  $\gamma_1$  is an  $(n \times n)$  matrix, which has the following structure,

$$\gamma_1 = \begin{bmatrix} \gamma_{r1} & 0 \\ 0 & \gamma_{o1} \end{bmatrix} \quad (15)$$

with  $\gamma_{r1}$  further constrained to be a lower-triangular matrix and  $\gamma_{o1}$  being a diagonal matrix. Under this market price of risk specification, the factor dynamics remain Ornstein-Uhlenbeck under the risk-neutral measure  $\mathbb{Q}$ ,

$$dX_t = \kappa^\mathbb{Q} (\theta^\mathbb{Q} - X_t) dt + dW_t^\mathbb{Q}, \quad (16)$$

with  $\kappa^\mathbb{Q} = \kappa + \gamma_1$  and  $\kappa^\mathbb{Q} \theta^\mathbb{Q} = -\gamma_0$ . Given the structural assumption on (15), the factor dependence has an analogous form to (13) under the risk-neutral measure  $\mathbb{Q}$ , with  $\kappa_r^\mathbb{Q}$  being lower triangular.

The fair value of the default-free zero-coupon bonds can be written as

$$B(X_{rt}, \tau) = \mathbb{E}^\mathbb{Q} \left[ \exp \left( - \int_t^{t+\tau} r(X_{rs}) ds \right) \middle| \mathcal{F}_t \right], \quad (17)$$

where  $\mathbb{E}^{\mathbb{Q}}[\cdot | \mathcal{F}_t]$  denotes the expectation operator under the risk-neutral measure, conditional on the time- $t$  filtration. The specification of the  $\mathbb{Q}$ -dynamics for the factors  $X_t$  in (16) and the instantaneous default-free interest-rate function in (12) satisfy the affine condition of Duffie and Kan (1996) and Duffie, Pan, and Singleton (2000). The fair values of default-free zero-coupon bonds can be solved as exponential-affine functions of the state vector,

$$B(X_{rt}, \tau) = \exp\left(-a(\tau) - b(\tau)^\top X_{rt}\right), \quad (18)$$

where the coefficients  $[a(\tau), b(\tau)]$  are solutions to the following ordinary differential equations:

$$\begin{aligned} a'(\tau) &= a_r - b(\tau)^\top \gamma_{r0} - b(\tau)^\top b(\tau)/2, \\ b'(\tau) &= b_r - (\kappa_r + \gamma_{r1})^\top b(\tau), \end{aligned} \quad (19)$$

subject to the boundary conditions  $b(0) = 0$  and  $c(0) = 0$ . In equation (19), we use the subscript  $r$  to denote the sub-matrix or vector that corresponds to the three interest-rate factors.

Like the instantaneous interest rate, the continuously compounded spot rates are also affine functions of the interest-rate factors,

$$R(X_{rt}, \tau) \equiv -\frac{\ln B(X_t, \tau)}{\tau} = \left[\frac{a(\tau)}{\tau}\right] + \left[\frac{b(\tau)}{\tau}\right]^\top X_{rt}. \quad (20)$$

In extracting the three interest-rate factors, we replace the measurement equation (2) with equation (20). Hence, instead of estimating the loading  $H$  directly, we estimate the parameters related to interest-rate factor dynamics ( $\kappa_r$ ), the market prices of risk ( $\gamma_{r0}, \gamma_{r1}$ ), and instantaneous interest-rate function ( $a_r, b_r$ ). For identification, we assume a lower-triangular matrix for  $\kappa_r$ .

## B. Data Description

Our estimation is based on a monthly sample from January 1988 to June 2004, 198 monthly observations for each series. Monthly or quarterly data series on output, inflation and financial leverage

are from the Federal Reserve Board. First, we extract an inflation factor from seven inflation-related variables: the consumer price index (CPI), the core CPI, the producer price index (PPI), the core PPI, the personal consumption expenditure (PCE) deflator, the core PCE deflator, and the gross domestic production (GDP) deflator. The GDP deflator is available at quarterly frequency. All other variables are available in monthly frequency. We convert the price indexes into year-over-year percentage changes. We then standardize each series by subtracting the sample mean and dividing the series by its sample standard deviation. We extract the inflation factor at monthly frequency from these seven standardized series. Since the GDP deflator is available at quarterly frequency, we fill the gaps with missing values. Our estimation approach can readily handle missing data. The ex post updates in equations (8) and (9) are based on the available subset of  $y_t$ .

Second, we extract a real growth factor from four output/employment macroeconomic series. They are the real GDP, industrial production, non-farm payrolls, and the real PCE. The real GDP is available in quarterly frequency. The other three series are available in monthly frequency, but the data on real PCE start at a later date in January 1991. Again, we first turn the four series into year-over-year growth rates and then standardize them before we extract the real growth factor.

Third, we extract a financial leverage factor from the following five financial leverage measures: the market value debt/net worth ratio, the book value debt/net worth ratio, the debt/equity ratio, the financing gap/GDP ratio, and ratio of change in total debt over GDP. These data are available in quarterly frequency. We extract the financial leverage factor in monthly frequency from these five quarterly series. In months when we have no observations, we do not update the factor and just use its predicted values.

Fourth, we extract a financial market volatility factor from two volatility indexes: the VXO index computed from options on S&P 100 index, and the VIX volatility index computed from options on S&P 500 index. Both series are available from Bloomberg in daily frequency, but the VIX series starts at a later date in January 1990. We first compute the yearly moving average of the daily volatility series and then sample the moving average number at the end of each and extract the volatility factor in monthly

frequency. Given the documented level dependence on the volatility of volatility, we first take logs on two series and then standardize them before we extract the volatility factor.

Finally, we also extract three interest-rate factors from 12 Treasury yields series with maturities of three months, six months, and every year from one to ten years. The Treasury yields data are monthly continuously compounded spot rates, obtained from the Federal Reserve Board.

### *C. Default-free Interest-Rate Factors and Factor Dynamics*

Table I reports the estimates on the default-free interest-rate factor dynamics, their market prices, and their links to the instantaneous default-free interest rate. The  $\kappa_r$  matrix measures the persistence of the three interest-rate factors and their interactions under the statistical measure  $\mathbb{P}$ . The small diagonal elements of the matrix show the extreme persistence of the interest rate series, with the first factor being the most persistent one and the last factor being the least persistent. The significantly negative estimates on the two off-diagonal elements in the last row of  $\kappa_r$  indicate that the third factor feeds back strongly and negatively on the first two factors.

The matrix  $\gamma_{r1}$  measures the coefficient on the proportional component of the market price of risk, with  $\kappa_r^{\mathbb{Q}} = \kappa_r + \gamma_{r1}$  determining the persistence under the risk-neutral measure  $\mathbb{Q}$ . The estimate for the first diagonal element is negative, albeit insignificant, but the estimates for the other two diagonal elements are increasing positive. Combining these estimates for  $\gamma_{r1}$  with the  $\kappa_r$  estimates, we observe that under the risk-neutral measure  $\mathbb{Q}$  the first factor becomes even more persistent while the other two factors become even less persistent, resulting in an increase in persistence difference among the three factors. Furthermore, the two off-diagonal elements in the last row of  $\gamma_{r1}$  are both strongly negative, reinforcing the negative estimates in  $\kappa_r$  to generate even stronger feedback effects under the risk-neutral measure.

The vector  $\gamma_{r0}$  reports the constant portion of the market prices of risk. The estimates are negative for the first two factors, but positive for the last factor. Nevertheless, all estimates show large standard error so that none of the  $t$ -statistics are significant. Finally, given that all three factors are normalized

to have zero long-run mean,  $a_r$  measures the long run mean of the short rate. The  $b_r$  vector measures the loading of the three factors on the instantaneous interest rate.

Equation (20) links the risk-neutral dynamics of the interest-rate factors and the instantaneous interest rate function to the factor loadings on the whole spot rate curve.  $a(\tau)/\tau$  measures the mean term structure of the spot rates. As shown in the left panel of Figure 1, the Treasury yields show an upward mean term structure. On the other hand,  $b(\tau)/\tau$  measures the factor loading on the yield curve, i.e., the instantaneous response of the spot rate at maturity  $\tau$  to a unit shock on the three interest-rate factors  $X_r$ . The right panel in Figure 1 plots the loading of the three interest-rate factors on the yield curve. According to equation (19), the factor loading  $b(\tau)/\tau$  is governed by the short rate loading  $b_r$  and the factor persistence  $\kappa^{\mathbb{Q}} = \kappa_r + \gamma_{r1}$  under the risk-neutral measure. Figure 1 shows that the first factor (solid line), which is also the most persistent factor, has its largest impact on long-term yields, whereas the third and also the most transient factor (the dotted line) has its largest impact on short-term yields. The second interest-rate factor (the dashed line) shows an intermediate persistence and its impact is largest at intermediate interest-rate maturities, thus generating a hump-shaped pattern for the loading on the yield curve.

The three interest-rate factors explain the 12 Treasury yields almost perfectly. They explain over 99.9 percent of variation on each series. The explained percentage variation is defined as one minus the variance of the fitting error over the variance of the original interest rate series. Hence, we can safely assume that the three interest-rate factors explain fully the variation in interest rates.

#### *D. Macroeconomic and Financial Factors*

Table II reports the estimates of the factor loading matrix ( $H$ ) and the absolute magnitude of the  $t$ -statistics (in parentheses). All loading parameter estimates are statistically significant. Furthermore, the economic meaning of each factor is clear, except for the financial leverage factor, for which the loadings are negative on market value of debt/net worth, financial gap over GDP, and total debt change over GDP, but are positive on book value of debt/new worth, and debt/equity ratio.

The left panel in Figure 2 plots the two macroeconomic factors. The solid line depicts the inflation factor and the dashed line depicts for real output growth factor. The inflation factor had a steep hike in the early 1991, coinciding with the spike in inflationary pressure caused by energy shocks during the first Gulf War. The inflationary pressure quickly receded and stayed low for the rest of the sample period. The dashed line for the real output growth shows two periods of sharp slowdown and one period of prolonged healthy output growth. The first slowdown coincided with the 1991–1992 recession. From mid 1994 to late 2000, the output growth factor remained at high values with some fluctuations. The factor started another very steep fall in early 2001, reflecting the sharp slowdown of the output growth, and reached the bottom in the second quarter of 2002. The output growth has picked up since then. This upswing is still continuing as of now, and the current level of this factor is still way below its level reached in 2000.

The left panel in Figure 2 plots the two financial factors, with the solid line denoting the financial leverage factor and the dashed line denoting the financial volatility factor. The financial leverage factor started high and had been declining ever since until year 2000, except for a small spike during the first Gulf War. After 2000, the financial leverage picked up again and reached a plateau around 2003 before it started falling again. The volatility factor extracted from stock index options, as shown by the dashed line, also started very high in later 1980s. The first Gulf War caused a spike on the stock market volatility, but otherwise the volatility stayed low between 1992 and 1997. The stock market volatility increased and maintained at a high level since then and until 2003, after which the volatility started to come down together with the financial leverage.

Via simple VAR(1) regressions on the seven dynamic factors, we estimate the time series dynamics of the four macroeconomic and financial factors. Table III reports the parameter estimates from the regressions. The  $\kappa_{r_o}$  matrix captures the feedback impact of the three default-free interest rate factors on the four macroeconomic and financial factors. The  $\kappa_o$  matrix measures the feedback of the four factors on themselves.

## II. A Dynamic Factor Model of the Term Structure of Credit Spreads

In this section, we incorporate the seven dynamic factors into a no-arbitrage dynamic term structure model of credit spreads. By estimating the model parameters, we provide an internally consistent analysis on the impacts of the seven factors on the whole term structure of credit spreads at different credit rating groups.

### A. A No-Arbitrage Model of Credit Spreads

We follow Duffie and Singleton (1999) and Duffie, Pedersen, and Singleton (2003) and write the fair values of defaultable zero-coupon bonds in terms of future instantaneous default-free rates and instantaneous credit spreads,

$$D(X_t, \tau) = \mathbb{E}^{\mathbb{Q}} \left[ \exp \left( - \int_t^{t+\tau} [r(X_{rs}) + s(X_s)] ds \right) \middle| \mathcal{F}_t \right], \quad (21)$$

where  $s(X_t)$  denotes the instantaneous default spread, which can be thought of as a reduced-form product of default probabilities and loss given default. In addition, the instantaneous spread can also be used to capture spreads induced by liquidity and other factors.

In equation (21), we write the instantaneous default-free interest rate as a function of the three default-free interest-rate factors  $X_{rt}$ , but we allow the instantaneous credit spread to be a function of all seven dynamic factors, which also includes two macroeconomic factors and two financial factors. Specifically, we assume the following affine dependence for each credit rating group,

$$s_i(X_t) = a_i + b_i^\top X_t,$$

where the subscript  $i$  denotes the  $i$ th credit rating group. For different ratings groups, we allow both a level difference and a difference in their reactions on the dynamic factors.

Then, the fair value of zero-coupon bonds of a certain credit group  $i$  is also exponential affine in the seven factors,

$$D_i(X_t, \tau) = \exp\left(-a_i(\tau) - b_i(\tau)^\top X_t\right),$$

where the coefficients  $[a_i(\tau), b_i(\tau)]$  are solutions to the following ordinary differential equations:

$$\begin{aligned} a_i'(\tau) &= a_r + a_i - b_i(\tau)^\top \gamma_0 - b_i(\tau)^\top b_i(\tau)/2, \\ b_i'(\tau) &= b_r + b_i - (\kappa + \gamma_1)^\top b_i(\tau), \end{aligned} \quad (22)$$

subject to the boundary conditions  $b_i(0) = 0$  and  $c_i(0) = 0$ . The continuously compounded spot rate on defaultable bonds of the  $i$ th rating group becomes

$$R_i(X_t, \tau) \equiv -\frac{\ln D_i(X_t, \tau)}{\tau} = \left[\frac{a_i(\tau)}{\tau}\right] + \left[\frac{b_i(\tau)}{\tau}\right]^\top X_t. \quad (23)$$

Hence, the credit spreads defined on the continuously compound spot rates can be written as

$$S_i(X_t, \tau) \equiv R_i(X_t, \tau) - R(X_t, \tau) = \left[\frac{a_i(\tau) - a(\tau)}{\tau}\right] + \left[\frac{b_i(\tau) - b(\tau)}{\tau}\right]^\top X_t. \quad (24)$$

Since the default-free bond yields only depend on the interest-rate factors, we stack  $b(\tau)$  with zeros in equation (24) on the other factors. Furthermore, since the three default-free interest-rate factors explain 99.9 percent of the variation on the Treasury yield curve, the spread as defined in (24) fully reflect the credit and possible liquidity component in the corporate bond yield. Thus, via no-arbitrage arguments, we link the credit spreads across all maturities at a certain credit rating group to the seven dynamic factors. The no-arbitrage links are determined by the factor dynamics, the market prices of the factors, and by the instantaneous credit spread as a function of these factors.

### *B. Constructing the Corporate Bond Yield Spreads*

We use month-end prices on corporate bonds that are either in the Merrill Lynch U.S. Corporate Master Index or the Merrill Lynch U.S. High Yield Index. These indices track the prices of U.S. dollar-

denominated investment grade and high yield corporate public debt issued in the U.S. domestic bond market. The Merrill Lynch data set covers the period from January 1997 to June 2004. To construct a long time-series of corporate bond yield sample, we augment the Merrill Lynch data by the Lehman Brothers Fixed Income database from January 1985 to December 1996. The Lehman data covers the period from January 1973 to March 1998, but there are very few noncallable bonds were issued before 1985. We estimate our models based on data from January 1988 to October 2004.

We enforce the following bond selection criteria. First, we consider only straight bonds without option features. Callable, puttable, convertible and bonds with sinking fund clause are dropped from our sample. Second, bonds with remaining maturities less than one year or greater than 35 years are eliminated. Third, only those bonds that have fixed coupon schedule and pay fixed rate semi-annual coupons are included. Fourth, we include only senior unsecured bonds, where bond seniority information are obtained from Moody's Investors Services. Finally, for the Lehman data, bond prices that are calculated using a matrix method are excluded. The resulting bond sample has 337,990 bond-month observations.

Spot continuously compounded corporate yields for each letter-grade rating class are then estimated using Nelson-Siegel method from the corporate bond sample.<sup>1</sup> For example, for credit rating AA, there are a total of 47031 bond-month observations. Nelson-Siegel method is implemented for each month on this sub-sample of AA bonds to extract the spot yield curves for credit rating AA. The same procedure is repeated for credit ratings AAA, A, and BBB. Yield spread for each rating class is calculated as the difference between the spot yield of the rating class and the maturity-matched Treasury yield.

Table IV reports the summary statistics of the credit spreads at different maturities and rating groups. The mean credit spread increases with declining credit ratings, and there is a jump in magnitude from A to BBB rating. The mean term structure pattern at each rating class is relatively flat. The standard deviations of credit spreads on AAA, AA, and A bonds are in the same range while the standard deviation estimates on BBB bonds are much larger. Credit spreads also show high persistence.

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<sup>1</sup>See Bolder and Streliski (1999) for details of the procedure.

The monthly autocorrelation estimates increase with maturity and reach a plateau at around five-year maturity.

### C. *Estimating the No-Arbitrage Links*

For estimation, we again cast the model into a state-space form, extract the distributions of the states at each date by using an efficient filtering technique, and then estimate the model using quasi maximum likelihood method, assuming normal forecasting errors on corporate bond credit spreads.

In this estimation, the state propagation equation remains the same as in (3). For a fixed credit rating group  $i$ , the measurement equation is now defined on the corporate spreads assuming additive normal pricing errors on each series,

$$y_{it} = S_i(X_t, \tau) + e_t, \quad cov(e_t) = \mathcal{R}^i, \quad \tau = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \text{ years.} \quad (25)$$

Since the corporate spreads are linear functions of the state vector under our model specification, we can again use the Kalman filter technique to update the conditional mean and variance of the states and the forecasting errors on the credit spreads. Furthermore, since we have already extracted the dynamic factors  $X$ , we now regard them as observables. Hence, the ex post updates are,

$$\hat{X}_{t+1} = \hat{X}_{t+1}^m; \quad \hat{V}_{t+1} = 0, \quad (26)$$

where  $\hat{X}_{t+1}^m$  denotes the dynamic factors extracted in the earlier sections. The ex post variance is zero because we treat  $\hat{X}_{t+1}^m$  as observable. Moreover, given that the factors are observable, we estimate the parameters governing the time-series factor dynamics via simple regression analysis. We estimate the market prices of dynamic factors and the instantaneous credit spread function using maximum likelihood method, assuming that the forecasting errors on the credit spreads are normally distributed. We estimate the term structure of credit spreads for each of the four credit rating classes separately.

### III. Economic Determinants of the Credit Spread Term Structure

By estimating the dynamic term structure model of credit spreads, we can now quantify the impacts of each dynamic factor on the term structure of credit spreads at different credit rating groups.

To gauge the performance of the seven dynamic factors in explaining the variation of the term structure of Treasury and corporate bond yields, Table V reports the forecasted percentage variance on the Treasury and corporate yields, defined as one minus the ratio of the forecasting error variance over the spot rate variance at each maturities. We capture the term structure of the Treasury yields using three latent factors. Table V shows that these three interest-rate factors predict over 96 percent of the monthly variation in the Treasury yields.

With four additional macroeconomic and financial factors, the model can predict over 90 percent of most corporate bond yields under AAA, AA, and A credit rating groups. The seven dynamic factors predict a lower percentage between 70 to 90 percent on the BBB bond yields. Table IV show that the BBB bond spreads show dramatically larger standard deviations than spreads at other credit ratings. Our estimation suggests that a large portion of this extra variation is due to idiosyncratic movements. Overall, the dynamic factor model that we propose works well.

Table VI reports the parameter estimates for the instantaneous credit spread function. The estimates for  $a_i$  measure the fixed component of the instantaneous spread. More interesting is the estimates for  $b_i$ , which measure the instantaneous response of the instantaneous credit spread to unit shocks on the seven dynamic factors. The signs of the estimates are consistent across the four credit rating groups. The loading on the first interest-rate factor is significantly negative, more so at lower credit ratings. This factor is generally referred to as an interest-rate level factor. The negative estimate suggests that the instantaneous credit spread declines with increasing interest rate levels. The loading on the second interest rate factor is positive, although its magnitude becomes smaller at lower credit rating groups. This second factor is often referred to as a term structure slope factor. The positive estimate suggests that a steepening of the default-free interest-rate curve is often associated with a widening credit spread. The most transient interest rate factor has positive loadings on the instantaneous credit

spread. Albeit statistically significant, the magnitudes of the estimates are smaller than the loading estimates on the first two factors. This most transient factor is often referred to as a curvature factor. It is also positively related to interest rate volatility (Heidari and Wu (2003)). The positive estimate suggests that an increase in the yield curve curvature and interest rate volatility leads to a widening of the instantaneous credit spread.

The loading estimates on the inflation factor are negative, but the loading estimates on the real output growth factor are all positive, suggesting that inflation increase leads to a declining short-term spread whereas high real growth leads to a widening credit spread. On the other hand, the loading on the two financial factors are both positive, suggesting that both financial leverage and stock market volatility are associated with increases in credit spreads, at least at short maturities.

The risk-neutral factor dynamics and the instantaneous credit spread function jointly determine the loading the dynamic factors across the whole term structure of credit spreads. The risk-neutral factor dynamics are jointly determined by the time-series dynamics and their market prices of risk. Table I reports the dynamics on the three interest-rate factors. Table III reports the time-series dynamics on the four macroeconomic and financial factors.

Table VII reports the market prices of risks estimates on the four macroeconomic and financial factors. Since we estimate the term structure of credit spread for each credit rating group separately, we obtain four sets of market price estimates, one for each credit rating group. Similar estimates across different rating groups would suggest the robustness for the market prices; different estimates, on the other hand, would suggest either measurement noise or evidence of market segmentation that different rating groups price the risk differently. The market price of risk estimates from the four data sets are quite similar, providing evidence of robustness on the estimates. The  $\gamma_{o0}$  measure the constant portion of the market price. The estimates are positive on inflation, real growth, and financial leverage, but positive on volatility. The  $\gamma_{o1}$  estimates capture the proportional coefficient of the market prices of risk. The estimates suggest that the market price increases with inflation, financial leverage, and volatility risk, but decreases with real growth.

In Figure 3, we plot the factor loading on the whole term structure credit spread under each of the four credit rating groups. The loading,  $b_i(\tau)/\tau$ , is computed based on the parameter estimates of our dynamic term structure model of interest rates and credit spreads.

The left panels plot the impacts of the three interest-rate factors on the term structure of credit spreads across the four rating groups. The impacts of the first interest rate-factor (solid lines) on the credit spreads are negative and increasingly so with increasing maturity. The impacts of the second interest-rate factor (dashed lines) are strongly positive, but the impacts decline with maturity. The impacts of the third interest-rate factor are very small, negative at short maturities but positive at longer maturities.

The impacts of the two macroeconomic factors are plotted in the middle panels. The solid lines depict the impacts of the inflation factor, which show a strong slope effect. The impacts on the credit spreads are strongly negative at short maturities, but are close to zero (or even slightly positive) at long maturities. On the other hand, the impacts of the real growth factor are close to zero, slightly positive at short maturities, but slightly negative at long maturities.

The right panels plot the impacts of the two financial factors. The impacts of the financial leverage factor are strong and positive, but the magnitude declines with increasing maturity. The impacts of the financial market volatility factor is also positive, but much smaller.

## IV. Conclusion

We use a dynamic factor model to summarize the information in many observed macroeconomic and financial data series and to provide a no-arbitrage link between the many data series and the term structure of credit spreads at different credit rating groups. By estimating the model, we quantify the impacts of many macroeconomic and financial series on the whole term structure of credit spreads. We find that credit spreads decline with increasing interest-rate levels and flattening term structure slopes, but increase with financial leverage and, to a lesser extent, stock market volatility. Upward shocks on inflation strongly narrow the credit spread at short maturities, but their impacts on long-term spreads

are close to zero, thus generating a strong slope effect on the term structure of credit spreads. On the other hand, the impacts of real output growth and the interest-rate curvature factors are fairly small.

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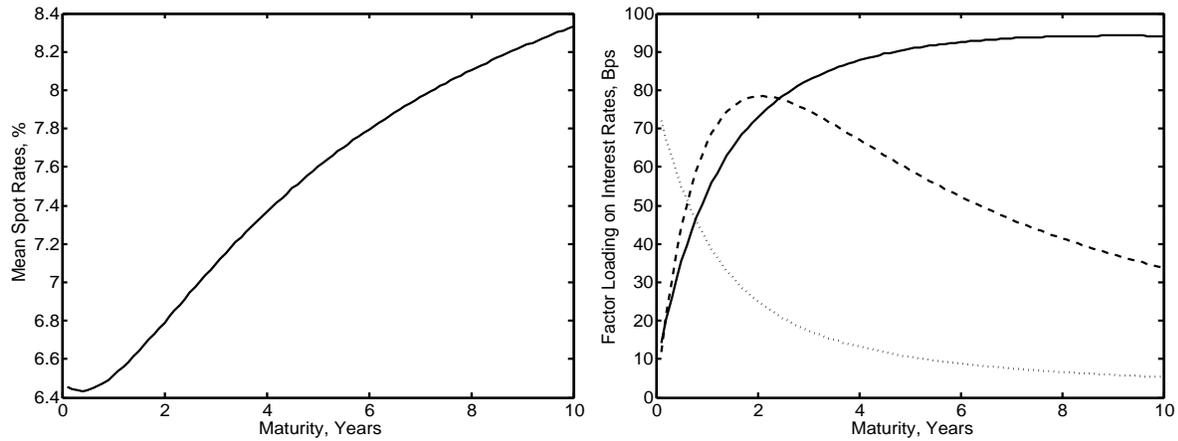
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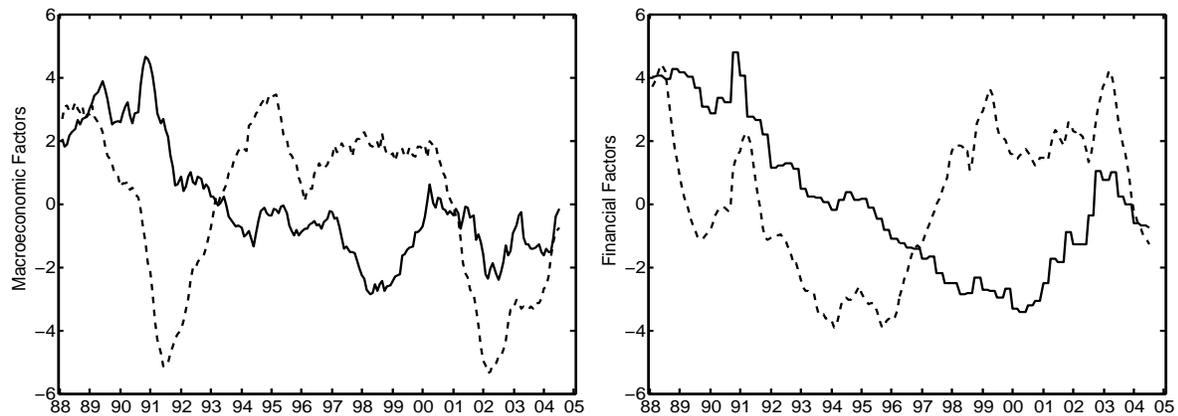
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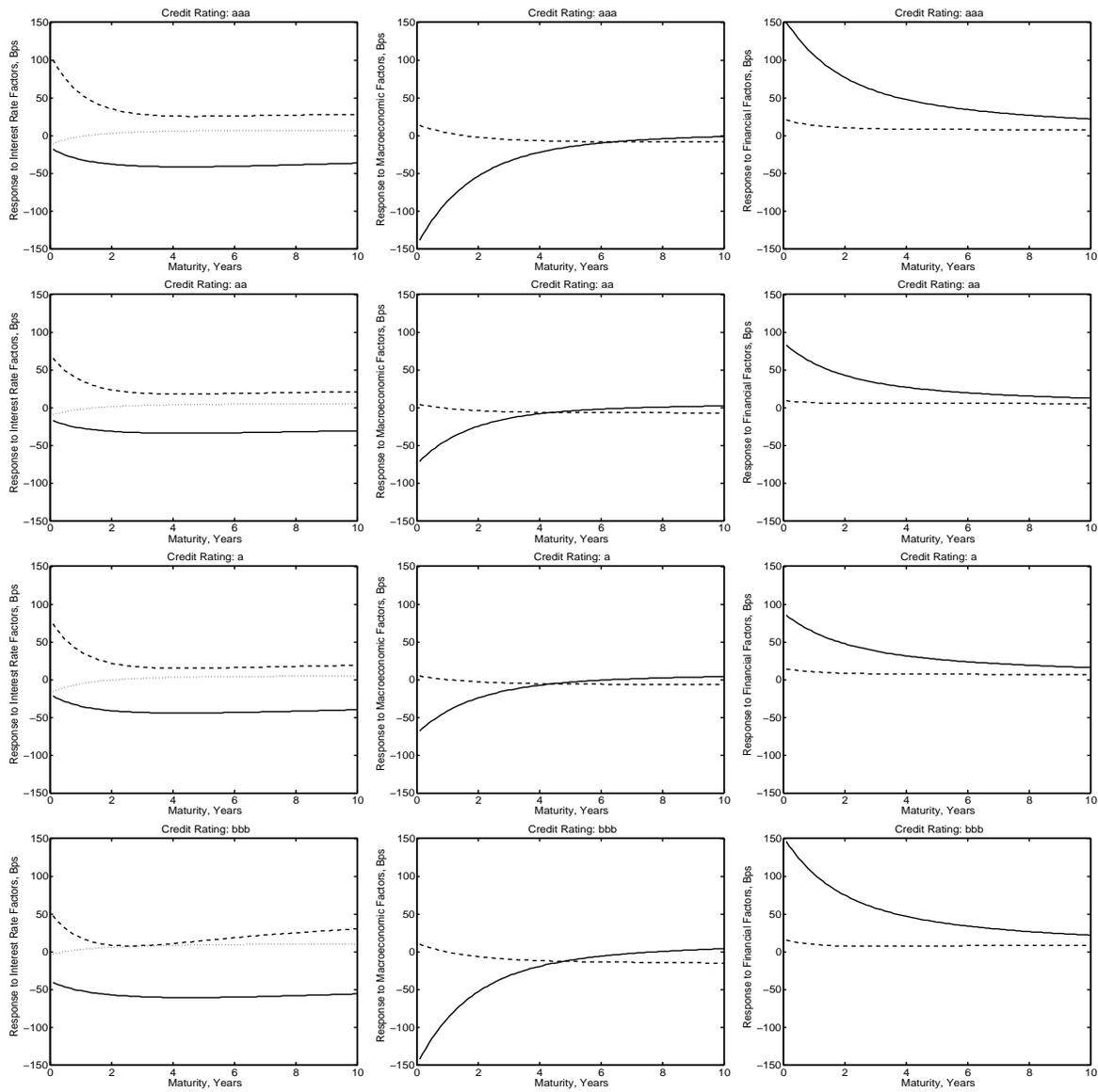
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**Figure 1. Mean term structure and factor loadings on the riskfree yield curve.** The left panel plots the mean term structure of the riskfree spot rate curve,  $a(\tau)/\tau$ . The three lines in the right panel plot the instantaneous response of the riskfree spot rate curve to per unit shock on the three interest-rate factors,  $b(\tau)/\tau$ . Both are computed based on parameter estimates of a three-factor Gaussian affine model. In the right panel, the solid lines denotes the first factor, the dashed line denotes the second factor, and the dotted line denotes the third factor.



**Figure 2. Time series of economic and financial factors.** Lines in the left panel denote the extracted macroeconomic factors, with the solid line denoting for inflation and the dashed line for real growth. Lines in the right panels denote the financial factors, with the solid line for financial leverage and dashed line for financial market volatility.



**Figure 3. Instantaneous response of term structure of credit spreads to unit shocks on the dynamic factors.** Lines denote the instantaneous response of the term structure of credit spreads at different credit rating groups to unit shocks on the three interest rate factors (left panels), two macroeconomic factors (middle panels), and two financial factors (right panels). The three interest-rate factors in the left panels are in solid, dashed, and dotted lines, respectively. Solid lines in the middle panels denote the inflation factor and dashed lines in the middle denote the real growth factor. Solid lines in the right panels denote the financial leverage factor and dashed lines in the right panels denote the volatility factor.

**Table I**  
**Default-free Interest-rate Factor Dynamics**

Entries report the parameter estimates and the absolute values of the  $t$ -statistics (in parentheses) on the default-free interest-rate factors dynamics, their market prices, and their link to the instantaneous default-free interest rate. The parameters are estimated based on maximum likelihood method with Kalman filter using continuously compounded Treasury yields from three months to ten years. Data are monthly from January 1988 to June 2004, obtained from the Federal Reserve Board.

| $\kappa_r$  | $\gamma_{r1}$   | $\gamma_{r0}$  | $a_r$  | $b_r$  |
|---|---|--|--|--|
| $\begin{bmatrix} 0.0251 & 0 & 0 \\ (0.11) & -- & -- \\ 0.0247 & 0.2665 & 0 \\ (0.11) & (1.26) & -- \\ -1.3604 & -1.5300 & 0.7455 \\ (3.49) & (2.42) & (2.68) \end{bmatrix}$ | $\begin{bmatrix} -0.0027 & 0 & 0 \\ (0.07) & -- & -- \\ -0.0562 & 0.2157 & 0 \\ (1.07) & (1.46) & -- \\ -0.4380 & -1.5760 & 0.7166 \\ (3.24) & (4.80) & (2.39) \end{bmatrix}$ | $\begin{bmatrix} -0.1455 \\ (0.29) \\ -0.6866 \\ (0.38) \\ 0.6717 \\ (0.04) \end{bmatrix}$ | $\begin{bmatrix} 0.0648 \\ (0.21) \end{bmatrix}$ | $\begin{bmatrix} 0.0008 \\ (1.19) \\ 0.0000 \\ (0.08) \\ 0.0077 \\ (17.8) \end{bmatrix}$ |

**Table II**  
**Extracting Systematic Factors From Macroeconomic and Financial Data**

Entries report the estimates and the absolute values of the  $t$ -statistics (in parentheses) of parameters that link each observed data series to four systematic factors.  $H_i$  denotes the loading of each series on the  $i$ th systematic factor. The factor loadings are estimated with maximum likelihood method and Kalman filtering using macroeconomic and financial data series listed below. The macroeconomic and financial leverage data are from the Federal Reserve Board, the volatility series are downloaded from Bloomberg. The sample period is from January 1988 to June 2004.

| Series                | $H_1$  |           | $H_2$  |           | $H_3$   |          | $H_4$  |           |
|-----------------------|--------|-----------|--------|-----------|---------|----------|--------|-----------|
| CPI                   | 0.5272 | ( 8.05 )  | —      | —         | —       | —        | —      | —         |
| Core CPI              | 0.4853 | ( 4.78 )  | —      | —         | —       | —        | —      | —         |
| PPI                   | 0.3881 | ( 3.64 )  | —      | —         | —       | —        | —      | —         |
| Core PPI              | 0.4745 | ( 8.26 )  | —      | —         | —       | —        | —      | —         |
| PCE Deflator          | 0.5451 | ( 11.27 ) | —      | —         | —       | —        | —      | —         |
| Core PCE Deflator     | 0.5014 | ( 5.01 )  | —      | —         | —       | —        | —      | —         |
| GDP deflator          | 0.5154 | ( 9.38 )  | —      | —         | —       | —        | —      | —         |
| Real GDP              | —      | —         | 0.2636 | ( 6.19 )  | —       | —        | —      | —         |
| Industrial Production | —      | —         | 0.3055 | ( 9.93 )  | —       | —        | —      | —         |
| Non-farm Payrolls     | —      | —         | 0.3941 | ( 15.08 ) | —       | —        | —      | —         |
| Real PCE              | —      | —         | 0.2149 | ( 7.89 )  | —       | —        | —      | —         |
| Debt/Net Worth        | —      | —         | —      | —         | -0.2499 | ( 3.33 ) | —      | —         |
| Debt/Net Worth (B)    | —      | —         | —      | —         | 0.3500  | ( 4.21 ) | —      | —         |
| Debt/Equity           | —      | —         | —      | —         | 0.4317  | ( 6.32 ) | —      | —         |
| Financing Gap/GDP     | —      | —         | —      | —         | -0.2151 | ( 2.66 ) | —      | —         |
| Total Debt Change/GDP | —      | —         | —      | —         | -0.1548 | ( 1.77 ) | —      | —         |
| VXO                   | —      | —         | —      | —         | —       | —        | 0.4325 | ( 14.56 ) |
| VIX                   | —      | —         | —      | —         | —       | —        | 0.4276 | ( 14.94 ) |

**Table III**  
**Macroeconomic and Financial Factor Dynamics**

Entries report the parameter estimates and the absolute values of the  $t$ -statistics (in parentheses) on time-series dynamics of the four macroeconomic and financial factors.  $\kappa_{or}$  captures the impacts of the three interest-rate factors on the four macroeconomic and financial factors,  $\kappa_o$  captures the feedback of the four factors on themselves. The time-series are estimated via regression analysis on the seven extracted dynamic factors.

| $\kappa_{ro}$ |          |          |          | $\kappa_{ro}$ |          |           |
|---------------|----------|----------|----------|---------------|----------|-----------|
| -0.2081       | -0.0975  | 0.0679   | 0.6338   | -0.2995       | -0.3541  | -0.1739   |
| (-44.12)      | (-30.20) | (68.47)  | (1.86)   | (-79.63)      | (-51.83) | (-104.39) |
| -1.0685       | -0.4071  | 0.7403   | 0.8609   | -0.2104       | -0.3133  | 0.1285    |
| (-46.04)      | (-32.07) | (71.71)  | (38.69)  | (-1.55)       | (-54.81) | (120.48)  |
| 0.0179        | 0.3812   | -0.0326  | -0.4485  | -0.0694       | 0.4924   | -0.1128   |
| (31.12)       | (31.08)  | (-61.13) | (-33.34) | (-82.38)      | (2.02)   | (-102.54) |
| 0.6974        | 0.4635   | -0.6425  | 0.3492   | 0.0959        | 0.0573   | 0.2613    |
| (41.79)       | (30.40)  | (-65.37) | (35.18)  | (84.19)       | (58.22)  | (2.29)    |

**Table IV**  
**Summary Statistics of Credit Spreads on Corporate Bonds**

Entries report mean, standard deviation, and monthly autocorrelation of the credit spreads on corporate bonds. The spreads are defined as the difference in percentage points between continuously compounded spot rates at a certain credit rating group and the corresponding Treasury spot rates. Corporate bond spot rates are extracted using Nelson-Siegel method from the corporate bond data. Data are monthly from January 1988 to June 2004, obtained from the Federal Reserve Board and Merrill Lynch.

| Maturity                  | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    |
|---------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Sample Mean               |       |       |       |       |       |       |       |       |       |       |
| AAA                       | 0.705 | 0.715 | 0.734 | 0.751 | 0.760 | 0.760 | 0.753 | 0.740 | 0.721 | 0.699 |
| AA                        | 0.750 | 0.749 | 0.771 | 0.797 | 0.819 | 0.833 | 0.840 | 0.838 | 0.831 | 0.819 |
| A                         | 0.892 | 0.940 | 0.992 | 1.036 | 1.069 | 1.089 | 1.098 | 1.098 | 1.090 | 1.078 |
| BBB                       | 1.521 | 1.474 | 1.479 | 1.506 | 1.537 | 1.565 | 1.587 | 1.600 | 1.607 | 1.607 |
| Sample Standard Deviation |       |       |       |       |       |       |       |       |       |       |
| AAA                       | 0.415 | 0.291 | 0.286 | 0.300 | 0.311 | 0.316 | 0.317 | 0.317 | 0.316 | 0.315 |
| AA                        | 0.278 | 0.251 | 0.266 | 0.279 | 0.290 | 0.299 | 0.306 | 0.313 | 0.319 | 0.324 |
| A                         | 0.306 | 0.325 | 0.345 | 0.351 | 0.353 | 0.352 | 0.352 | 0.352 | 0.353 | 0.355 |
| BBB                       | 0.552 | 0.609 | 0.624 | 0.614 | 0.595 | 0.574 | 0.555 | 0.539 | 0.526 | 0.516 |
| Monthly Autocorrelation   |       |       |       |       |       |       |       |       |       |       |
| AAA                       | 0.923 | 0.949 | 0.967 | 0.972 | 0.973 | 0.973 | 0.972 | 0.971 | 0.968 | 0.964 |
| AA                        | 0.898 | 0.939 | 0.954 | 0.959 | 0.962 | 0.964 | 0.966 | 0.966 | 0.966 | 0.965 |
| A                         | 0.912 | 0.954 | 0.962 | 0.964 | 0.965 | 0.965 | 0.965 | 0.964 | 0.963 | 0.962 |
| BBB                       | 0.910 | 0.955 | 0.966 | 0.969 | 0.970 | 0.969 | 0.968 | 0.966 | 0.963 | 0.961 |

**Table V**  
**The Explanatory Power of the Dynamic Factors on the Term Structure of Treasury and Defaultable Bond Yields**

Entries report one minus the ratio of the forecasting error variance to the original spot rate variance. Treasury and corporate bond yields are forecasted by seven dynamic factors, which include three default-free interest rate factors, two macroeconomic factors, and two financial factors. The term structure of Treasury yields and credit spreads are linked to the dynamic factors via a no-arbitrage dynamic term structure modeling framework.

| Maturity | 1     | 2     | 3     | 5     | 5     | 6     | 7     | 8     | 9     | 10    |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Treasury | 0.982 | 0.975 | 0.970 | 0.967 | 0.965 | 0.964 | 0.963 | 0.963 | 0.961 | 0.960 |
| AAA      | 0.764 | 0.865 | 0.909 | 0.930 | 0.941 | 0.947 | 0.950 | 0.953 | 0.955 | 0.956 |
| AA       | 0.930 | 0.951 | 0.959 | 0.962 | 0.964 | 0.965 | 0.967 | 0.968 | 0.968 | 0.968 |
| A        | 0.906 | 0.924 | 0.930 | 0.933 | 0.936 | 0.939 | 0.941 | 0.944 | 0.947 | 0.949 |
| BBB      | 0.700 | 0.807 | 0.857 | 0.881 | 0.893 | 0.896 | 0.895 | 0.892 | 0.886 | 0.879 |

**Table VI**  
**Parameter Estimates on the Instantaneous Credit Spreads Function**

Entries report the parameter estimates (and absolute magnitudes of the  $t$ -statistics in parentheses) on the instantaneous credit spread function under different rating groups.  $a_i$  is the intercept and  $b_i$  is the loading vector on each of the seven factors. The parameters are estimated with maximum likelihood methods and Kalman filter, using corporate bond yield spreads over the corresponding Treasury yield at maturities from one to ten years. Data are monthly from January 1988 to June 2004, obtained from the Federal Reserve Board and Merrill Lynch.

| Ratings                    | AAA                  | AA                   | A                    | BBB                  |
|----------------------------|----------------------|----------------------|----------------------|----------------------|
| Intercepts ( $a_i$ )       | -0.0079<br>( 11.72 ) | -0.0031<br>( 6.79 )  | -0.0054<br>( 11.15 ) | 0.0035<br>( 5.32 )   |
| Factor Loadings ( $b_i$ ): |                      |                      |                      |                      |
| Interest Factor I          | -0.0016<br>( 5.82 )  | -0.0016<br>( 7.08 )  | -0.0019<br>( 8.44 )  | -0.0039<br>( 12.57 ) |
| Interest Factor II         | 0.0109<br>( 27.72 )  | 0.0070<br>( 32.52 )  | 0.0081<br>( 33.31 )  | 0.0053<br>( 11.93 )  |
| Interest Factor III        | -0.0012<br>( 7.06 )  | -0.0010<br>( 9.37 )  | -0.0018<br>( 17.10 ) | -0.0005<br>( 3.25 )  |
| Inflation Factor           | -0.0146<br>( 22.23 ) | -0.0075<br>( 20.71 ) | -0.0071<br>( 21.01 ) | -0.0151<br>( 26.93 ) |
| Real Growth Factor         | 0.0015<br>( 11.62 )  | 0.0005<br>( 7.55 )   | 0.0005<br>( 8.64 )   | 0.0012<br>( 11.40 )  |
| Leverage Factor            | 0.0157<br>( 27.55 )  | 0.0086<br>( 30.44 )  | 0.0089<br>( 32.02 )  | 0.0152<br>( 39.66 )  |
| Volatility Factor          | 0.0022<br>( 13.11 )  | 0.0010<br>( 10.27 )  | 0.0015<br>( 14.28 )  | 0.0017<br>( 11.18 )  |

**Table VII**  
**Parameter Estimates on the Market Prices of Macroeconomic and Financial Risks**

Entries report the parameter estimates and absolute magnitudes of the  $t$ -statistics (in parentheses) on market prices of macroeconomic and financial risk factors.  $\gamma_{o0}$  denotes the constant component of the market price,  $\gamma_{o1}$  denotes the coefficient for the proportional component of the market price. We estimate the market prices of risks using the term structure of credit spreads at each of four credit rating groups. Data are monthly from January 1988 to June 2004, obtained from the Federal Reserve Board and Merrill Lynch.

| Factors<br>Ratings | $\gamma_{o0}$        |                      |                      |                      | $\gamma_{o1}$       |                     |                      |                      |
|--------------------|----------------------|----------------------|----------------------|----------------------|---------------------|---------------------|----------------------|----------------------|
|                    | AAA                  | AA                   | A                    | BBB                  | AAA                 | AA                  | A                    | BBB                  |
| Inflation          | -2.4770<br>( 7.43 )  | -2.9253<br>( 11.21 ) | -2.4951<br>( 10.49 ) | -2.5045<br>( 10.82 ) | 0.0786<br>( 5.72 )  | 0.0910<br>( 4.52 )  | 0.0126<br>( 0.69 )   | 0.0978<br>( 7.83 )   |
| Real Growth        | -3.7831<br>( 4.32 )  | -3.5488<br>( 3.19 )  | -2.4877<br>( 2.48 )  | -2.8009<br>( 4.56 )  | -0.1233<br>( 8.30 ) | -0.1204<br>( 7.19 ) | -0.2517<br>( 11.02 ) | -0.1241<br>( 10.24 ) |
| Leverage           | -2.5539<br>( 11.08 ) | -2.6936<br>( 16.39 ) | -3.0567<br>( 31.44 ) | -2.4322<br>( 12.15 ) | 0.0564<br>( 5.29 )  | 0.0547<br>( 4.59 )  | -0.0222<br>( 1.83 )  | 0.0242<br>( 2.44 )   |
| Volatility         | 3.5030<br>( 7.44 )   | 3.5105<br>( 3.16 )   | 6.1156<br>( 7.89 )   | 3.2546<br>( 5.64 )   | 0.4562<br>( 8.23 )  | 0.3706<br>( 6.91 )  | 0.4088<br>( 7.48 )   | 0.4746<br>( 12.33 )  |